6.3 A coil consists of 100 turns of wire wrapped around a square frame of sides 0.25 m. The coil is centered at the origin with each of its sides parallel to the x- or y-axis. Find the induced emf across the open-circuited ends of the coil if the magnetic field is given by

(a) $\mathbf{B} = \hat{\mathbf{z}}20e^{-3t}$ (T)

(b) $\mathbf{B} = \hat{\mathbf{z}}20\cos x \cos 10^3 t$ (T)

(c) $\mathbf{B} = \hat{\mathbf{z}}20\cos x \sin 2y \cos 10^3 t$ (T)

Solution: Since the coil is not moving or changing shape, $V_{\text{emf}}^{\text{m}} = 0$ V and $V_{\text{emf}} = V_{\text{emf}}^{\text{tr}}$. From Eq. (6.6),

$$V_{\text{emf}} = -N\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} = -N\frac{d}{dt}\int_{-0.125}^{0.125}\int_{-0.125}^{0.125} \vec{B}\cdot(\hat{\mathbf{z}}\,dx\,dy),$$

where $N = 100$ and the surface normal was chosen to be in the $+\hat{\mathbf{z}}$ direction.

(a) For $\vec{B} = \hat{\mathbf{z}}20e^{-3t}$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}\left(20\exp{-3t}\,(0.25)^2\right) = 375e^{-3t} \qquad \text{(V)}.$$

(b) For $\vec{B} = \hat{\mathbf{z}}20\cos x\cos 10^3 t$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}\left(20\cos 10^3 t\int_{x=-0.125}^{0.125}\int_{y=-0.125}^{0.125}\cos x\,dx\,dy\right) = 124.6\sin 10^3 t \qquad \text{(kV)}.$$

(c) For $\vec{B} = \hat{\mathbf{z}}20\cos x\sin 2y\cos 10^3 t$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}\left(20\cos 10^3 t\int_{x=-0.125}^{0.125}\int_{y=-0.125}^{0.125}\cos x\sin 2y\,dx\,dy\right) = 0.$$

6.5 A circular-loop TV antenna with 0.02-m^2 area is in the presence of a uniform-amplitude 300-MHz signal. When oriented for maximum response, the loop develops an emf with a peak value of 30 (mV). What is the peak magnitude of **B** of the incident wave?

Solution: TV loop antennas have one turn. At maximum orientation, Eq. (6.5) evaluates to $\Phi = \int \vec{B}\cdot d\vec{s} = \pm BA$ for a loop of area A and a uniform magnetic field with magnitude $B = |\mathbf{B}|$. Since we know the frequency of the field is $f = 300$ MHz, we can express B as $B = B_0\cos(\omega t + \alpha_0)$ with $\omega = 2\pi \times 300 \times 10^6$ rad/s and α_0 an arbitrary reference phase. From Eq. (6.6),

$$V_{\text{emf}} = -N\frac{d\Phi}{dt} = -A\frac{d}{dt}[B_0\cos(\omega t + \alpha_0)] = AB_0\omega\sin(\omega t + \alpha_0).$$

V_{emf} is maximum when $\sin(\omega t + \alpha_0) = 1$. Hence,

$$30 \times 10^{-3} = AB_0\omega = 0.02 \times B_0 \times 6\pi \times 10^8,$$

which yields $B_0 = 0.8$ (nT).

6.11 The loop shown in P6.11 moves away from a wire carrying a current $I_1 = 10$ A at a constant velocity $\mathbf{u} = \hat{\mathbf{y}}7.5$ (m/s). If $R = 10\ \Omega$ and the direction of I_2 is as defined in the figure, find I_2 as a function of y_0, the distance between the wire and the loop. Ignore the internal resistance of the loop.

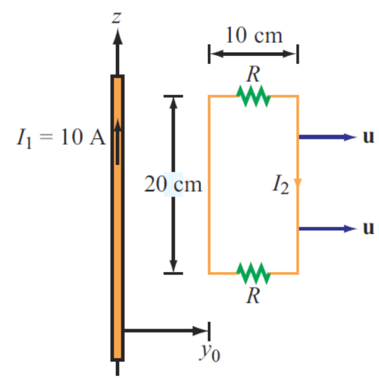

Figure P6.11 Moving loop of Problem 6.11.

Solution: Assume that the wire carrying current I_1 is in the same plane as the loop. The two identical resistors are in series, so $I_2 = V_{\text{emf}}/2R$, where the induced voltage is due to motion of the loop and is given by Eq. (6.26):

$$V_{\text{emf}} = V_{\text{emf}}^{\text{m}} = \oint_C \left(\vec{u} \times \vec{B}\right) \cdot d\vec{l}.$$

The magnetic field $\vec{B}$ is created by the wire carrying I_1. Choosing $\hat{\mathbf{z}}$ to coincide with the direction of I_1, Eq. (5.30) gives the external magnetic field of a long wire to be

$$\vec{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r}.$$

For positive values of y_0 in the y-z plane, $\hat{\mathbf{y}} = \hat{\mathbf{r}}$, so

$$\vec{u} \times \vec{B} = \hat{\mathbf{y}}|\vec{u}| \times \vec{B} = \hat{\mathbf{r}}|\vec{u}| \times \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r} = \hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r}.$$

Integrating around the four sides of the loop with $d\vec{l} = \hat{\mathbf{z}}\, dz$ and the limits of integration chosen in accordance with the assumed direction of I_2, and recognizing that only the two sides without the resistors contribute to $V_{\text{emf}}^{\text{m}}$, we have

$$V_{\text{emf}}^{\text{m}} = \int_0^{0.2} \left(\hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r}\right)\bigg|_{r=y_0} \cdot (\hat{\mathbf{z}}\, dz) + \int_{0.2}^{0} \left(\hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r}\right)\bigg|_{r=y_0+0.1} \cdot (\hat{\mathbf{z}}\, dz)$$

$$= \frac{4\pi \times 10^{-7} \times 10 \times 7.5 \times 0.2}{2\pi} \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1}\right)$$

$$= 3 \times 10^{-6} \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1}\right) \quad \text{(V)},$$

and therefore

$$I_2 = \frac{V_{\text{emf}}^{\text{m}}}{2R} = 150 \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1}\right) \quad \text{(nA)}.$$

6.17 In wet soil, characterized by $\sigma = 10^{-2}$ (S/m), $\mu_r = 1$, and $\varepsilon_r = 36$, at what frequency is the conduction current density equal in magnitude to the displacement current density?

Solution: For sinusoidal wave variation, the phasor electric field is

$$E = E_0 e^{j\omega t}.$$

$$J_c = \sigma E = \sigma E_0 e^{j\omega t}$$

$$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = j\omega\varepsilon E_0 e^{j\omega t}$$

$$\left|\frac{J_c}{J_d}\right| = 1 = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi\varepsilon f}$$

or

$$f = \frac{\sigma}{2\pi\varepsilon} = \frac{10^{-2}}{2\pi \times 36 \times 8.85 \times 10^{-12}} = 5 \times 10^6 = 5 \text{ MHz}.$$

6.26 The electric field radiated by a short dipole antenna is given in spherical coordinates by

$$\mathbf{E}(R,\theta;t) = \hat{\boldsymbol{\theta}} \frac{2 \times 10^{-2}}{R} \sin\theta \, \cos(6\pi \times 10^8 t - 2\pi R) \quad \text{(V/m)}.$$

Find $\mathbf{H}(R,\theta;t)$.

Solution: Converting to phasor form, the electric field is given by

$$\tilde{\vec{E}}(R,\theta) = \hat{\boldsymbol{\theta}} E_\theta = \hat{\boldsymbol{\theta}} \frac{2 \times 10^{-2}}{R} \sin\theta \exp -j2\pi R \qquad \text{(V/m)},$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned}
\tilde{\vec{H}}(R,\theta) &= \frac{1}{-j\omega\mu} \nabla \times \tilde{\vec{E}} = \frac{1}{-j\omega\mu} \left[\hat{\mathbf{R}} \frac{1}{R\sin\theta} \frac{\partial E_\theta}{\partial \phi} + \hat{\boldsymbol{\phi}} \frac{1}{R} \frac{\partial}{\partial R}(R E_\theta) \right] \\
&= \frac{1}{-j\omega\mu} \hat{\boldsymbol{\phi}} \frac{2 \times 10^{-2}}{R} \sin\theta \frac{\partial}{\partial R} (\exp - j2\pi R) \\
&= \hat{\boldsymbol{\phi}} \frac{2\pi}{6\pi \times 10^8 \times 4\pi \times 10^{-7}} \frac{2 \times 10^{-2}}{R} \sin\theta \exp -j2\pi R \\
&= \hat{\boldsymbol{\phi}} \frac{53}{R} \sin\theta \, \exp -j2\pi R \qquad (\mu\text{A/m}).
\end{aligned}$$

Converting back to instantaneous value, this is

$$\vec{H}(R,\theta;t) = \hat{\boldsymbol{\phi}} \frac{53}{R} \sin\theta \cos\left(6\pi \times 10^8 t - 2\pi R\right) \qquad (\mu\text{A/m}).$$

6.28 In free space, the magnetic field is given by

$$\mathbf{H} = \hat{\boldsymbol{\phi}}\,\frac{36}{r}\cos(6\times 10^9 t - kz) \qquad \text{(mA/m)}.$$

(a) Determine k.

(b) Determine $\mathbf{E}$.

(c) Determine $\mathbf{J}_d$.

Solution:

(a) From the given expression, $\omega = 6\times 10^9$ (rad/s), and since the medium is fr space,

$$k = \frac{\omega}{c} = \frac{6\times 10^9}{3\times 10^8} = 20 \qquad \text{(rad/m)}.$$

(b) Convert $\mathbf{H}$ to phasor:

$$\widetilde{\mathbf{H}} = \hat{\boldsymbol{\phi}}\,\frac{36}{r}\,e^{-jkz} \qquad \text{(mA/m)}$$

$$\begin{aligned}
\widetilde{\mathbf{E}} &= \frac{1}{j\omega\varepsilon_0}\,\nabla\times\widetilde{\mathbf{H}} \\
&= \frac{1}{j\omega\varepsilon_0}\left[-\hat{\mathbf{r}}\,\frac{\partial H_\phi}{\partial z} + \hat{\mathbf{z}}\,\frac{1}{r}\,\frac{\partial}{\partial r}(rH_\phi)\right] \\
&= \frac{1}{j\omega\varepsilon_0}\left[-\hat{\mathbf{r}}\,\frac{\partial}{\partial z}\left(\frac{36}{r}\,e^{-jkz}\right) + \frac{\hat{\mathbf{z}}}{r}\,\frac{\partial}{\partial r}(36e^{-jkz})\right] \\
&= \frac{1}{j\omega\varepsilon_0}\left[\hat{\mathbf{r}}\,\frac{j36k}{r}\,e^{-jkz}\right] \\
&= \hat{\mathbf{r}}\,\frac{36k}{\omega\varepsilon_0 r}\,e^{-jkz} = \hat{\mathbf{r}}\,\frac{36\times 377}{r}\,e^{-jkz}\times 10^{-3} = \hat{\mathbf{r}}\,\frac{13.6}{r}\,e^{-j20z} \qquad \text{(V/m)}.
\end{aligned}$$

$$\begin{aligned}
\mathbf{E} &= \mathfrak{Re}[\widetilde{E}e^{j\omega t}] \\
&= \hat{\mathbf{r}}\,\frac{13.6}{r}\cos(6\times 10^9 t - 20z) \qquad \text{(V/m)}.
\end{aligned}$$

(c)

$$\begin{aligned}
\mathbf{J}_d &= \varepsilon_0\,\frac{\partial \mathbf{E}}{\partial t} \\
&= \hat{\mathbf{r}}\,\frac{13.6}{r}\,\varepsilon_0\frac{\partial}{\partial t}(\cos(6\times 10^9 t - 20z)) \\
&= -\hat{\mathbf{r}}\,\frac{13.6\varepsilon_0\times 6\times 10^9}{r}\sin(6\times 10^9 t - 20z) \qquad \text{(A/m}^2\text{)} \\
&= -\hat{\mathbf{r}}\,\frac{0.72}{r}\sin(6\times 10^9 t - 20z) \qquad \text{(A/m}^2\text{)}.
\end{aligned}$$