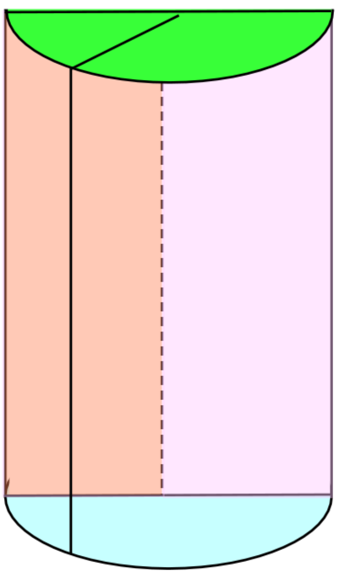
Quiz #4, 11/06/2017

Name:________________________________Signature:_____________________

3. For a vector filed $\vec{A} = 3r^2\hat{r} + 3r\phi\hat{\phi} - 2\hat{z}$, verify the divergence theorem

$\oiint_s \vec{A}\cdot d\vec{s} = \iiint_v (\nabla\cdot\vec{A})dv$, on a section of a cylinder bounded by

$r = 1,\ -\frac{\pi}{2} \le \phi \le \frac{\pi}{2},\ \ 1 \le z \le 3.$



Solution:

(1) $\oiint_s \vec{A}\cdot d\vec{s} = \iint_{top} \vec{A}\cdot d\vec{s} + \iint_{bottom} \vec{A}\cdot d\vec{s} + \iint_{outter} \vec{A}\cdot d\vec{s} + \iint_{left} \vec{A}\cdot d\vec{s} \iint_{right} \vec{A}\cdot d\vec{s}$

$\iint_{top} \vec{A}\cdot d\vec{s} = \iint_{top} -2rdrd\phi\,|_{z=3} = -\iint_{bottom} \vec{A}\cdot d\vec{s}$

$\iint_{outter} \vec{A}\cdot d\vec{s} = \iint_{outter} 3r^2 rd\phi dz\,|_{r=1} = \int_1^3 dz \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3d\phi = 6\pi$

$\iint_{left} \vec{A}\cdot d\vec{s} = -\iint_{left} 3r\phi drdz\,|_{\phi=-\frac{\pi}{2}} = \frac{3\pi}{2}\int_1^3 dz \int_0^1 rdr = \frac{3\pi}{2}$

$\iint_{right} \vec{A}\cdot d\vec{s} = \iint_{right} 3r\phi drdz\,|_{\phi=\frac{\pi}{2}} = \frac{3\pi}{2}\int_1^3 dz \int_0^1 rdr = \frac{3\pi}{2}$

$\oiint_s \vec{A}\cdot d\vec{s} = 9\pi$

(2) $\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(r3r^2) + \frac{1}{r}\frac{\partial}{\partial \phi}(3r\phi) + \frac{\partial}{\partial z}(-2) = 9r + 3$

$$\iiint_v (\nabla \cdot \vec{A}) dv = \int_1^3 dz \int_0^1 dr \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r(9r+3) d\phi$$

$$= 2\pi \int_0^1 r(9r+3) dr = 9\pi$$