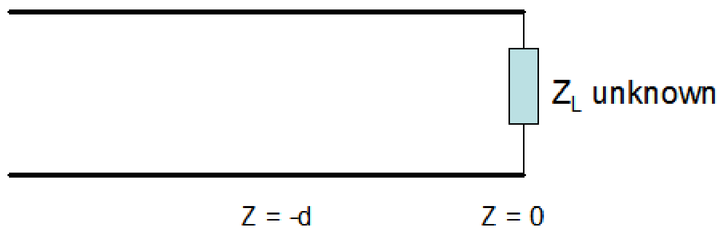
A 50Ω lossless transmission line is terminated with an unknown load Z_L (see figure below). The phase velocity is 2.0×10^{10}cm/s at the frequency of 10GHz. The minimum voltages on the transmission line are zeros. The first voltage minimum is located at z = -d from load, d = 0.2cm. Find out the impedance of the load.



Solution:

The formula for voltage minimum is:

$$\left|\tilde{V}\right|_{\min} = \left|V_0^+\right|(1-|\Gamma|) = 0,$$

Therefore, $|\Gamma| = 1$.

$\lambda f = v$, where λ is the wavelength, f = 10GHz is the frequency and v is the phase velocity. f = 10GHz and $v = 2.0\times10^{10}$cm/s are given. Therefore, $\lambda = 1.0cm$.

The first voltage minimum occurs at

$2\beta(-d) + \theta_r = -\pi$, d = 0.2cm is given.

so, $2\frac{2\pi}{\lambda}(-d) + \theta_r = -\pi$, => $\theta_r = -\pi + 0.8\pi = -0.2\pi$,

$\Gamma = 1\angle -0.2\pi = 0.809 - j0.5878$,

$$Z_L = Z_0\frac{1+\Gamma}{1-\Gamma} = 50\frac{1+0.809-j0.5878}{1-(0.809-j0.5878)} = -j154\Omega$$

Graphic solution is on the next page.

graphic Solution:

$z_L = -j3.1 \qquad Z_L = Z_0 \cdot (-j3.1) = -j155\ \Omega$