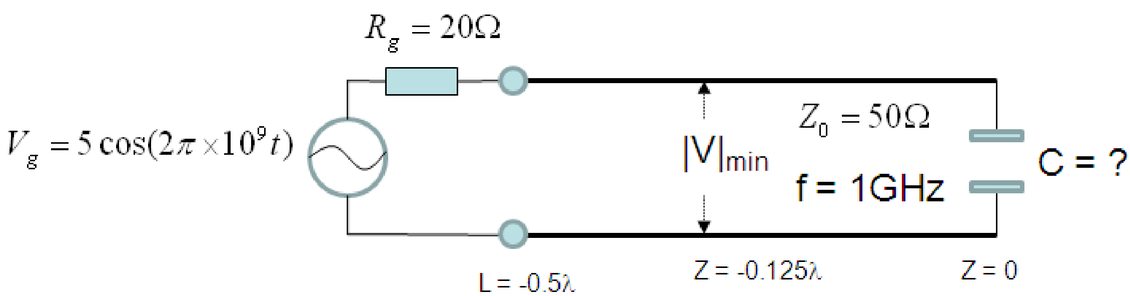
EE Problem 8

A lossless transmission line is terminated with an unknown capacitor C. The characteristic impedance Z_0 of the transmission line is 50 Ω. The length of transmission line is 0.5λ. The transmission line is connected to a signal source $V_g = 5\cos(2\pi \times 10^9 t)$ with a resistance of 20 Ω (see figure below). The first voltage minimum $|V|_{min}$ is located at z = -0.125λ from load.

(1) Calculate the voltage standing wave ratio (VSWR);

(2) Find out the capacitance of the capacitor

(3) Calculate the voltage maximum $|V|_{max}$ and its location



Solution:

(1) The voltage reflection coefficient Γ is: $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$,

Therefore:

$$|\Gamma| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \frac{\sqrt{\left(\frac{1}{j\omega C}\right)^2 + Z_0^2}}{\sqrt{\left(\frac{1}{j\omega C}\right)^2 + Z_0^2}} = 1,$$ Z_0 is real for a lossless transmission line.

The formula for voltage standing wave ratio (VSWR) is:

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \infty$$

(2) The first voltage minimum occurs at

$2\beta(-0.125\lambda) + \theta_r = -\pi$, θ_r is the angle of the reflection coefficient.

so, $2\frac{2\pi}{\lambda}(-0.125\lambda) + \theta_r = -\pi$, => $\theta_r = -0.5\pi$,

$$\Gamma = 1\angle -0.5\pi = 0 - j1,$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\frac{1-j1}{1+j1} = -j50\Omega$$

$$Z_L = \frac{1}{j\omega C} = -j50\Omega,\ C = \frac{1}{50\omega} = 20\,pF\ .$$

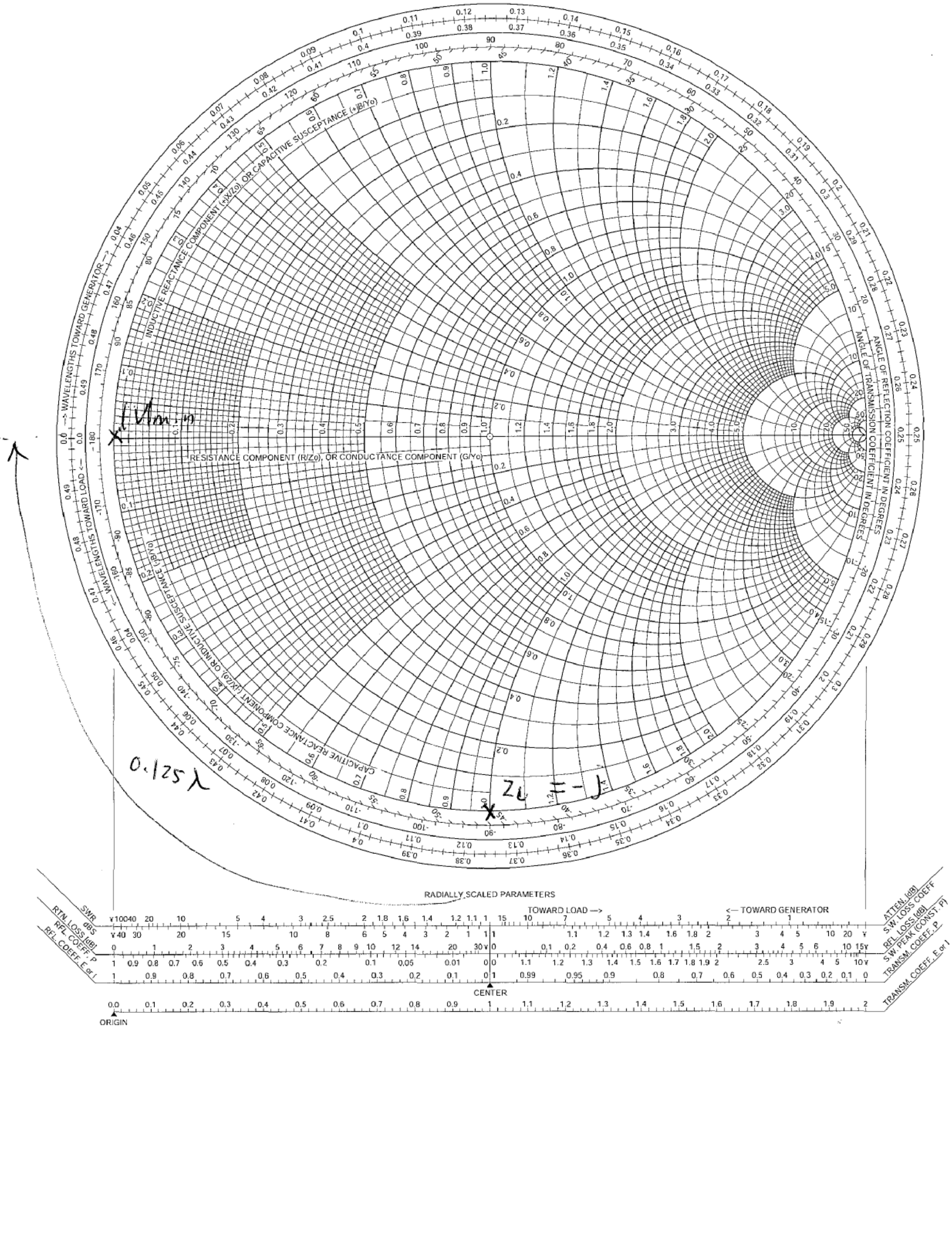
(3) The voltage phasor of the source is $\tilde{V}_g = 5(V)$, voltage at the beginning of the transmission line is:

$$V(z=-0.5\lambda) = \frac{Z_{in}}{Z_{in}+R_g}\tilde{V}_g = \frac{-j50}{-j50+20} = V_0^+\left(e^{j\beta 0.5\lambda} + \Gamma e^{-j\beta 0.5\lambda}\right) = V_0^+(-1+j)$$

$$|V_0^+| = \frac{250/\sqrt{(50)^2+(20)^2}}{\sqrt{2}}$$

$$|V|_{\max} = 2|V_0^+| = \frac{250\sqrt{2}}{\sqrt{(50)^2+(20)^2}}$$

Graphic solution:


WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER
ORIGIN
Vmin
0.125λ
ZL = -j