## 1: grating



$$
\Delta \phi_{\mathrm{m}}=\mathrm{k}^{*} \Delta \mathrm{x}_{\mathrm{m}}=(2 \pi / \lambda) \mathrm{md}^{*} \sin (\theta)
$$

For the grating, $\mathrm{d}=1.0 \mu \mathrm{~m}, \mathrm{f}=$ $20 \mathrm{~cm} . \mathrm{m}=20$.
(1) What's the first maximum angle and $\Delta \mathrm{L}$ for the wavelength $\lambda=0.5 \mu \mathrm{~m}$ (green)?
$\mathrm{d}^{*} \sin \left(\theta_{\mathrm{m}}\right)=\mathrm{m} \lambda$
$\theta_{1} \sim \sin \left(\theta_{1}\right)=\lambda / d$
$\theta_{1} \sim 0.5 \mathrm{rad}$
$\Delta L=f \theta_{1}=10 \mathrm{~cm}$
(2) What's the first maximum angle and $\Delta \mathrm{L}$ for the wavelength $\lambda=0.6 \mu \mathrm{~m}$ (red)?
$\mathrm{d}^{*} \sin \left(\theta_{\mathrm{m}}\right)=\mathrm{m} \lambda$
$\theta_{1} \sim \sin \left(\theta_{1}\right)=\lambda / d$
$\theta_{1} \sim 0.6 \mathrm{rad}$
$\Delta \mathrm{L}=\mathrm{f} \theta_{1}=12 \mathrm{~cm}$

## 1: grating



$$
\Delta \phi_{\mathrm{m}}=\mathrm{k}^{*} \Delta \mathrm{x}_{\mathrm{m}}=(2 \pi / \lambda) \mathrm{md}^{*} \sin (\theta)
$$

For the grating, $\mathrm{d}=1.0 \mu \mathrm{~m}, \mathrm{f}=$ $20 \mathrm{~cm} . \mathrm{m}=20$.
(3) What's you conclusion from the two lights? That's why CDs show beautiful colors.
The maximum of different wavelengths appear at different locations. Gratings are dispersive optical elements.
(4) for the wavelength $\lambda=0.5 \mu \mathrm{~m}, \mathrm{~m}=10$, what's the first dark angle and $\Delta \mathrm{L}$ ?

Dark positions: $k m{ }^{*} \sin \left(\theta_{m}\right)=2 n \pi+\pi$

$$
\begin{aligned}
2 \mathrm{md}^{*} \sin \left(\theta_{1}\right) & =\lambda \\
\theta_{1} \sim \sin \left(\theta_{1}\right) & =\lambda /(\mathrm{md})
\end{aligned}
$$

For $m=10, \theta_{1} \sim 0.05 \mathrm{rad}$

$$
\Delta \mathrm{L}=\mathrm{f} \theta_{1}=1 \mathrm{~cm}
$$

For $m=100, \theta_{1} \sim 0.005 \mathrm{rad}$

$$
\Delta \mathrm{L}=\mathrm{f} \theta_{1}=0.1 \mathrm{~cm}
$$

## $\underline{2}$ : reflective grating



For the grating, $\mathrm{d}=1.0 \mu \mathrm{~m}$, incident angle is $10^{\circ}$.
(1) What's the first maximum angle $\lambda=0.5 \mu \mathrm{~m}$ (green)?
$d^{*} \sin \left(\theta_{m}\right)-d \sin (\phi)=m \lambda$
$\theta_{1} \sim \sin \left(\theta_{1}\right)=\lambda / d+\sin (\phi)$
$\theta_{1} \sim 0.5+0.17=0.67 \mathrm{rad}$
$\Delta L=f \theta_{1}=13.4 \mathrm{~cm}$

3: photon
$\mathrm{h}=6.67 \times 10^{-34} . \mathrm{c}=3 \times 10^{8}(\mathrm{~m} / \mathrm{s})$
(1) For the light with wavelength $\lambda=0.6 \mu \mathrm{~m}$ (red), what's the energy of a single photon? What's the frequency of the light? What's the momentum of the photon? What's the k vector of the light?

$$
\begin{aligned}
& v=c / \lambda=5 \times 10^{14} \mathrm{~Hz} \\
& E=h v=6.67 \times 10^{-34} \times 5 \times 10^{14}=3.3 \times 10^{-19}=2 \mathrm{eV} \\
& k=2 \pi / \lambda=1.1 \times 10^{7} \mathrm{~m}^{-1} \\
& p=E / c=1.1 \times 10^{-27} \mathrm{Ns}
\end{aligned}
$$

(2) For the wavelength $\lambda=0.5 \mu \mathrm{~m}$, what's the energy of a single photon? What's the frequency of the light? What's the momentum of the photon? What's the k vector of the light?

$$
\begin{aligned}
& v=c / \lambda=6 \times 10^{14} \mathrm{~Hz} \\
& E=h v=6.67 \times 10^{-34} \times 6 \times 10^{14}=4.0 \times 10^{-19}=2.5 \mathrm{eV} \\
& k=2 \pi / \lambda=1.3 \times 10^{7} \mathrm{~m}^{-1} \\
& p=E / c=1.3 \times 10^{-27} \mathrm{Ns}
\end{aligned}
$$

## 4: phase matching



## 5: phase matching



For the left grating, if the incident angle is $\phi=10^{\circ}, \mathrm{d}$ $=1 \mu \mathrm{~m}$,
(1) For $\lambda=1.5 \mu \mathrm{~m}$, determine the output angle using the phase matching condition.
(2) For $\lambda=0.6 \mu \mathrm{~m}$, determine the output angle using the phase matching condition.

$\mathrm{k}_{\text {out }}{ }^{*} \sin (\theta)-\mathrm{k}_{\text {in }}{ }^{*} \sin (\phi)=\mathrm{k}_{\text {grating }}=2 \pi / \mathrm{d}$
$\sin (\theta)=\sin (\phi)+\lambda / d$
for the wavelength $\lambda=0.6 \mu \mathrm{~m}, \theta=0.87 \mathrm{rad}$
for the wavelength $\lambda=1.5 \mu \mathrm{~m}$, no diffration

## 6: DFB grating



For the DFB grating grating,
(1) For $\lambda=1.5 \mu \mathrm{~m}$, determine the grating period for effective reflection.
(2) For $\lambda=0.6 \mu \mathrm{~m}$, determine the grating period for effective reflection.
(1) $\mathrm{k}_{\Lambda}=2 \mathrm{k}_{\text {in }}, \Lambda=\lambda / 2=0.75 \mu \mathrm{~m}$
(2) $\mathrm{k}_{\Lambda}=2 \mathrm{k}_{\text {in }}, \Lambda=\lambda / 2=0.3 \mu \mathrm{~m}$

