Problem 3.39 For the vector field $\mathbf{E} = \hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy$, verify the divergence theorem by computing:

- (a) the total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each and parallel to the Cartesian axes, and
- **(b)** the integral of $\nabla \cdot \mathbf{E}$ over the cube's volume.

Problem 3.40 For the vector field $\mathbf{E} = \hat{\mathbf{r}} 10e^{-r} - \hat{\mathbf{z}} 3z$, verify the divergence theorem for the cylindrical region enclosed by r = 2, z = 0, and z = 4.

Problem 3.41 A vector field $\mathbf{D} = \hat{\mathbf{r}}r^3$ exists in the region between two concentric cylindrical surfaces defined by r = 1 and r = 2, with both cylinders extending between z = 0 and z = 5. Verify the divergence theorem by evaluating:

- (a) $\oint_{S} \mathbf{D} \cdot d\mathbf{s}$, (b) $\int_{\mathcal{V}} \nabla \cdot \mathbf{D} \, d\mathcal{V}$.