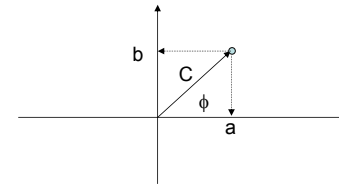


Class overview:

1. Brief review of physical optics, wave propagation, interference, diffraction, and polarization
2. Introduction to Integrated optics and integrated electrical circuits
3. Guide-wave optics: 2D and 3D optical waveguide, optical fiber, mode dispersion, group velocity and group velocity dispersion.
4. Mode-coupling theory, Mach-zehnder interferometer, Directional coupler, taps and WDM coupler.
5. Electro-optics, index tensor, electro-optic effect in crystal, electro-optic coefficient
6. Electro-optical modulators
7. Passive and active optical waveguide devices, Fiber Optical amplifiers and semiconductor optical amplifiers, Photonic switches and all optical switches
8. Opto-electronic integrated circuits (OEIC)

## Complex numbers

$$c = a + ib \text{ or } Ce^{i\phi} \quad C: \text{amplitude} \quad \phi: \text{angle}$$



$$c_1 \pm c_2 = (a_1 \pm a_2) + i(b_1 \pm b_2)$$

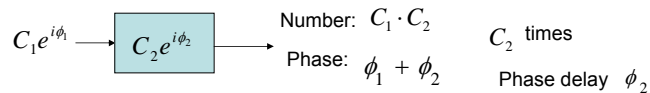
Real numbers do not have phases

Complex numbers have phases

$Ce^{i\phi}$   $\phi$  is the phase of the complex number

## Physical meaning of multiplication of two complex numbers

$$c_1 \cdot c_2 = C_1 e^{i\phi_1} \cdot C_2 e^{i\phi_2} = C_1 \cdot C_2 e^{i(\phi_1 + \phi_2)}$$



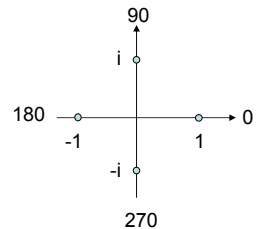
Why  $i \cdot i = -1$ ?

$$i = 1 \cdot e^{i\frac{\pi}{2}}$$

$$i \cdot i = 1 \cdot 1 \cdot e^{i(\frac{\pi}{2} + \frac{\pi}{2})} = e^{i(\pi)}$$

$$i^3 = -i \quad i^4 = 1$$

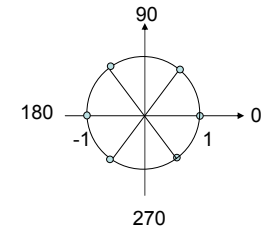
$$\sqrt{-1} = \pm i \quad \sqrt[4]{-1} = \pm i; \pm 1$$



$$\sqrt[n]{1} = ?$$

$$\sqrt[n]{1} = e^{i(\frac{2\pi}{n})}, n = 0, 1 \dots 5$$

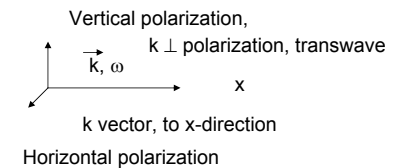
$$\sqrt[12]{2} = \sqrt[12]{2} e^{i(\frac{2\pi}{12}n)}, n = 0, 1 \dots 11$$



## Optical wave

$$Ae^{i(kx - \omega t + \phi)}$$

Initial phase  
Phase delay by time  
Phase delay by position  
Amplitude,  $I = |A|^2$ , Optical intensity



Optical wave propagation in air (free-space)

$$Ae^{i(kx - \omega t + \phi)}$$

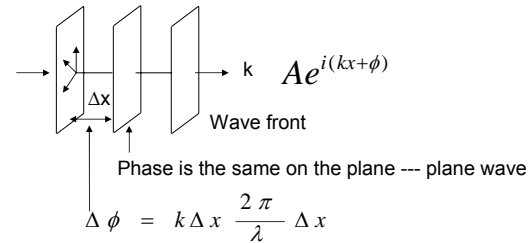
$$C_1 e^{i\phi_1} \rightarrow e^{i\phi_2} \rightarrow Ae^{i(kx - \omega t + \phi)}$$

Usually, ignore  $t$  term. Because look at system at the same time. Snap shoot.

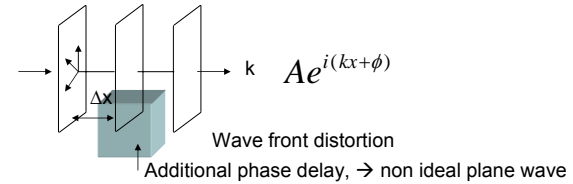
$$Ae^{i(kx + \phi)}$$

$$\text{Phase delay } \phi_{\text{delay}} = k \cdot x = \frac{2\pi}{\lambda} x$$

Optical wave propagation in air (free-space)



Phase distortion in atmosphere



Optical wave to different directions

$$Ae^{i(kx_1 + \phi)}$$

$$Ae^{i(kx_2 + \phi)}$$

Phase delay

$$\phi_{\text{delay}} = k \Delta x = \frac{2\pi}{\lambda} (x_1 - x_2)$$

Refractive index  $n$

In vacuum,

$$k = \frac{2\pi}{\lambda} \quad V = c = \frac{\lambda}{T} = \lambda f \quad c: \text{ speed of light}$$

In media, like glass,

$$k = \frac{2\pi}{\lambda/n} = \frac{2\pi}{\lambda} n \quad V = c/n = \frac{\lambda/n}{T} = \lambda f/n$$

Optical wave to different directions

$$Ae^{i(kx_1 + \phi)}$$

$$Ae^{i(kx_2 + \phi)}$$

Phase delay

$$\phi_{\text{delay}} = k \Delta x = \frac{2\pi}{\lambda} (x_1 - x_2)$$

In media with refractive index  $n$

$$Ae^{i(knx_1 + \phi)}$$

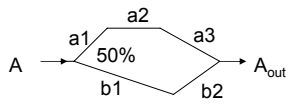
$$Ae^{i(knx_2 + \phi)}$$

Phase delay

$$\phi_{\text{delay}} = kn \Delta x = \frac{2\pi}{\lambda} n (x_1 - x_2)$$

$nx_1, nx_2$ , optical path

Interference of two optical waves



Upper arm :  $\frac{1}{2} Ae^{ikn(a_1+a_2+a_3)}$

Lower arm :  $\frac{1}{2} Ae^{ikn(b_1+b_2)}$

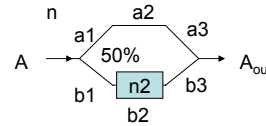
$A_{out} = \frac{1}{2} Ae^{ikn(a_1+a_2+a_3)} + \frac{1}{2} Ae^{ikn(b_1+b_2)}$

Phase delay between the two arms:

$kn(b_1 + b_2 - a_1 - a_2 - a_3)$

If phase delay is  $2m\pi$ , then:  $\frac{1}{2} Ae^{ikn(a_1+a_2+a_3)} = \frac{1}{2} Ae^{ikn(b_1+b_2)}$

If phase delay is  $2m\pi + \pi$ , then:  $\frac{1}{2} Ae^{ikn(a_1+a_2+a_3)} = -\frac{1}{2} Ae^{ikn(b_1+b_2)}$



Upper arm :  $\frac{1}{2} Ae^{ikn(a_1+a_2+a_3)}$

Lower arm :  $\frac{1}{2} Ae^{ikn(b_1+b_2)}$

$b_1 + b_2 + b_3 = a_1 + a_2 + a_3$

Phase delay between the two arms:

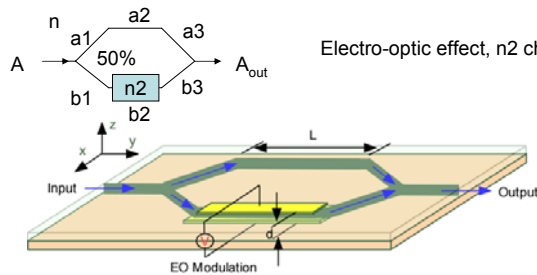
$k(n_2 b_2 - n a_2)$

If phase delay is  $2m\pi$ , then:  $\frac{1}{2} Ae^{ikn(a_1+a_2+a_3)} = \frac{1}{2} Ae^{ikn(b_1+b_2)}$

If phase delay is  $2m\pi + \pi$ , then:  $\frac{1}{2} Ae^{ikn(a_1+a_2+a_3)} = -\frac{1}{2} Ae^{ikn(b_1+b_2)}$

Electro-optic effect,  $n_2$  changes with E -field

Electro-optic modulator based on Mach-Zehnder interferometer



Electro-optic effect,  $n_2$  changes with E -field

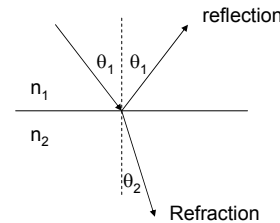
Phase delay between the two arms:

$k(n_2 b_2 - n a_2)$

If phase delay is  $2m\pi$ , then:  $\frac{1}{2} Ae^{ikn(a_1+a_2+a_3)} = \frac{1}{2} Ae^{ikn(b_1+b_2)}$

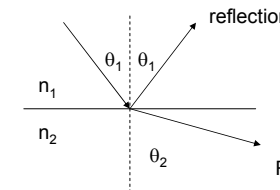
If phase delay is  $2m\pi + \pi$ , then:  $\frac{1}{2} Ae^{ikn(a_1+a_2+a_3)} = -\frac{1}{2} Ae^{ikn(b_1+b_2)}$

Reflection, refraction of light



$n_1 \sin \theta_1 = n_2 \sin \theta_2$

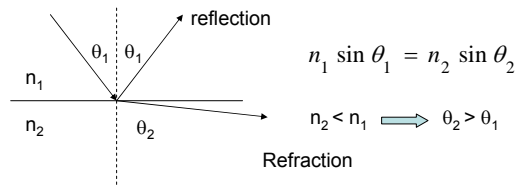
$n_2 > n_1 \Rightarrow \theta_2 < \theta_1$



$n_1 \sin \theta_1 = n_2 \sin \theta_2$

$n_2 < n_1 \Rightarrow \theta_2 > \theta_1$

Total internal reflection (TIR)

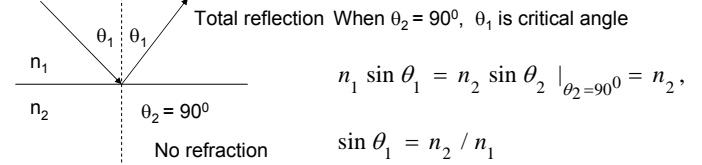


When  $\theta_2 = 90^\circ$ ,  $\theta_1$  is critical angle

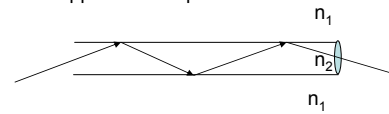
$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \Big|_{\theta_2=90^\circ} = n_2,$$

$$\sin \theta_1 = n_2 / n_1$$

Total internal reflection (TIR)



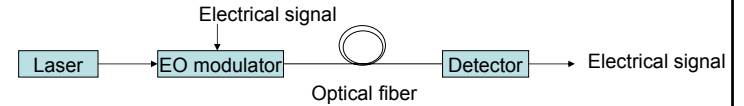
Application – optical fiber



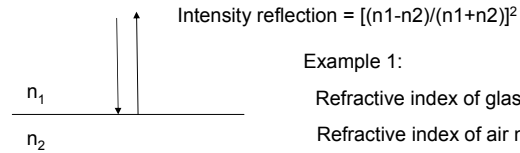
Low loss, TIR

Flexible

Communication system



Reflection percentage



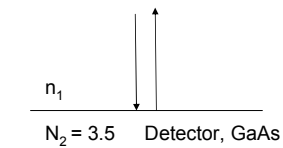
Example 1:

Refractive index of glass  $n = 1.5$ ;

Refractive index of air  $n = 1$ ;

The reflection from glass surface is 4%;

Example 2: detector is usually made from Gallium Arsenide (GaAs),  $n = 3.5$ ;

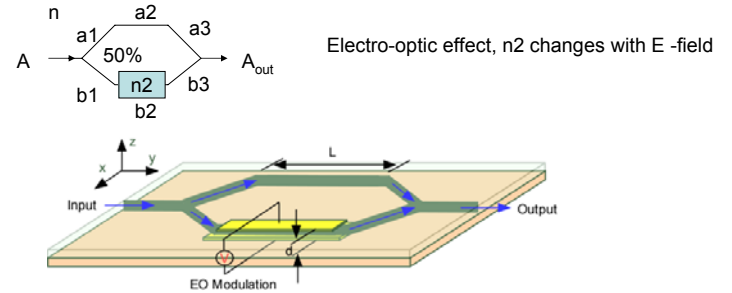


$$\text{Intensity reflection} = [(n_1-n_2)/(n_1+n_2)]^2$$

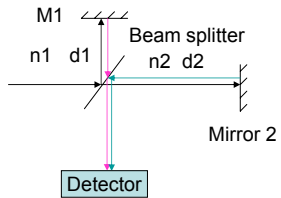
The reflection from GaAs surface is 31%;

Interference of two optical beams, applications

1, Mach-Zehnder interferometer



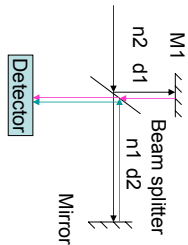
## 2, Michelson interferometer



Phase difference:

$$k(n_2 2d_2 - n_1 2d_1)$$

Measuring small moving or displacement

If the detected light changes from bright to dark,  
Then the distance moved is half wavelength/2Michelson interferometer measuring speed of light in ether,  
speed of light not depends on direction

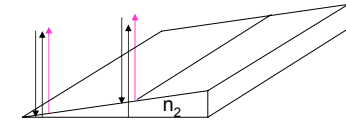
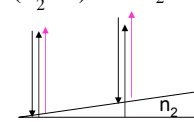
Phase difference:

$$k(n_1 2b_2 - n_2 2d_1)$$

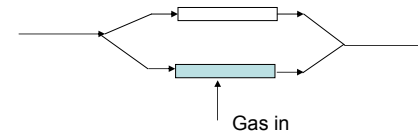
## 3, Measuring surface flatness

$$k(n_2 \Delta d) \quad k(n_2 \Delta d')$$

Accuracy: 0.1 wavelength = 0.1\*500nm = 50nm



## 4, Measuring refractive index of liquid or gas



## Equal difference series

$$a_1, a_2 \dots a_n \dots$$

$$a_2 - a_1 = b$$

$$a_3 - a_2 = b$$

$$+ a_n - a_{n-1} = b$$

$$= a_n - a_1 = (n-1)b$$

$$S_n = a_1 + a_2 + \dots a_n$$

$$a_2 + a_{n-1} = a_1 + b + a_{n-1} = a_1 + a_n$$

$$+ S_n = a_n + a_{n-1} + \dots a_1$$

$$a_3 + a_{n-2} = a_1 + 2b + a_{n-2} = a_1 + a_n$$

$$= 2S_n = n(a_n + a_1)$$

$$S_n = \frac{n}{2}(a_n + a_1) = \frac{n}{2}(a_n - a_1) + na_1 = \frac{n}{2}(n-1)b + na_1$$

## Power series

$$a_1, a_2 \dots a_n \dots$$

$$a_2 = ba_1$$

$$a_3 = ba_2$$

...

$$\times a_n = ba_{n-1}$$

$$a_n = b^{n-1} a_1$$

$$S_n = a_1 + a_2 + \dots a_n$$

$$- bS_n = ba_1 + ba_2 + \dots ba_n = a_2 + a_3 + \dots a_{n+1}$$

$$= (1-b)S_n = a_1 - a_{n+1}$$

$$S_n = \frac{a_1 - a_{n+1}}{1-b} = \frac{a_1 - b^n a_1}{1-b}$$

Power series

$$a_1, a_2, \dots, a_n, \dots$$

$$S = a_1 + a_2 + \dots + a_n + \dots$$

$$= \lim_{n \rightarrow \infty} \frac{a_1(1-b^n)}{1-b} = \frac{a_1}{1-b} \quad \text{When } |b| < 1$$

$b$  can be anything, number, variable, complex number or function

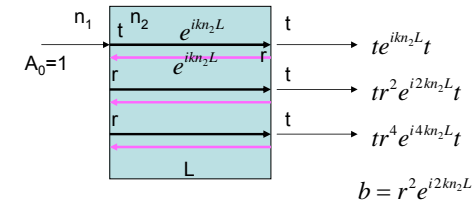
Optical cavity, multiple reflections

$$S_r = t_1 r + t_2 + \dots + t_n + \dots$$

$$= \frac{t_1 r}{1-r^2}$$

$$S_t = t_1 + t_2 + \dots + t_n + \dots = \frac{t_1}{1-r^2}$$

Optical cavity, wavelength dependence, resonant



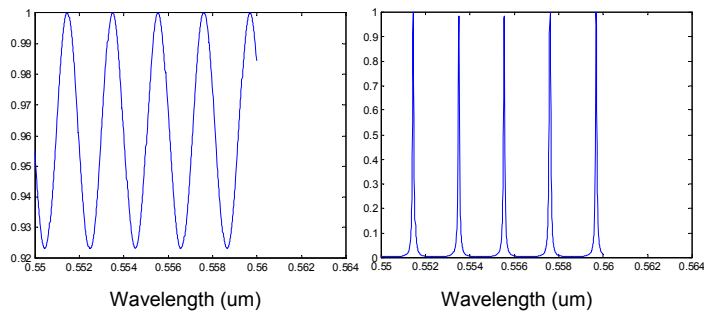
$$A_t = A_1 + A_2 + \dots = \frac{t^2 e^{i2kn_2 L}}{1 - r^2 e^{i2kn_2 L}}$$

$$I_t = |S_t|^2 = \left| \frac{t^2 e^{i2kn_2 L}}{1 - r^2 e^{i2kn_2 L}} \right|^2 = \left| \frac{t^2}{1 - \text{Re} e^{i2kn_2 L}} \right|^2 \quad \text{R, intensity reflection}$$

Optical cavity, wavelength dependence, resonant

$$I_t = |S_t|^2 = \left| \frac{t^2 e^{i2kn_2 L}}{1 - r^2 e^{i2kn_2 L}} \right|^2 = \left| \frac{t^2}{1 - \text{Re} e^{i2kn_2 L}} \right|^2$$

Example:  $n_2 = 1.5$ ,  $R = 0.04$ ,  $L = 0.05 \text{ mm}$ ,  $\lambda = 0.55 \mu\text{m}$  R, intensity reflection



Optical cavity, wavelength dependence, resonant

$$I_t = |S_t|^2 = \left| \frac{t^2 e^{i2kn_2 L}}{1 - r^2 e^{i2kn_2 L}} \right|^2 = \left| \frac{t^2}{1 - \text{Re} e^{i2kn_2 L}} \right|^2$$

Resonant condition

$$2kn_2 L = (2m + 1)\pi \quad \text{Destructively Interference}$$

$$2kn_2 L = 2m\pi \quad \text{Constructively Interference}$$

$$\frac{2\pi}{\lambda} n_2 L = m\pi \quad 2n_2 L = m\lambda \quad \text{Resonant condition, } m = 1, 2, 3, \dots$$

$m=1$ , half- $\lambda$  cavity

$m=2$ ,  $\lambda$  cavity

## Optical cavity, Free-spectral range (FSR)

$$2n_2L = m\lambda \quad \text{Resonant condition, } m = 1, 2, 3, \dots$$

$$\frac{2n_2L}{\lambda_m} = m$$

$$- \frac{2n_2L}{\lambda_{m+1}} = m+1$$

$$\frac{2n_2L}{\lambda_{m+1}} - \frac{2n_2L}{\lambda_m} = 1 \quad \frac{2n_2L(\lambda_m - \lambda_{m+1})}{\lambda_{m+1}\lambda_m} = 1$$

$$\frac{2n_2L\Delta\lambda}{\lambda^2} = 1 \quad \Delta\lambda = \frac{\lambda^2}{2n_2L} \quad FSR = \frac{\lambda^2}{2n_2L}$$

$$\text{Example: } n_2 = 1.5, R=0.04, L = 1\text{mm}, \lambda=0.55\mu\text{m}$$

## Optical cavity, Free-spectral range (FSR)

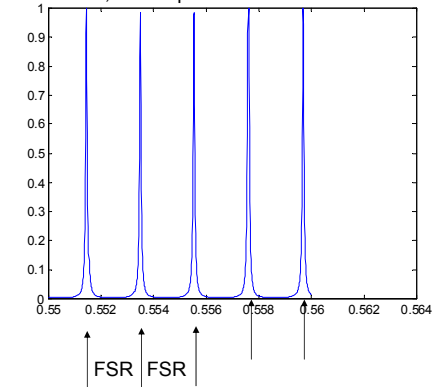
$$FSR = \frac{\lambda^2}{2n_2L} \quad L \text{ increase, FSR decrease, FSR not dependent on } R$$

$$\text{Example: } n_2 = 1.5, R=0.04, L = 0.05\text{mm}, \lambda=0.55\mu\text{m}$$

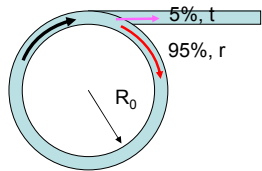
$$FSR = \frac{\lambda^2}{2n_2L} = 0.02\mu\text{m}$$

$$L = 0.5\text{mm}$$

$$FSR = \frac{\lambda^2}{2n_2L} = 0.002\mu\text{m}$$



## Ring cavity, Ring resonator



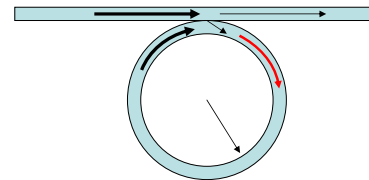
$$L = 2\pi R_0$$

$$I_t = |S_t|^2 = \left| \frac{t^2 e^{ik_2L}}{1 - r^2 e^{i2k_2L}} \right|^2 = \left| \frac{t^2}{1 - R e^{i2k_2L}} \right|^2$$

$$\frac{2\pi}{\lambda} n_2 L = m\pi \quad 2n_2 L = m\lambda$$

$$FSR = \frac{\lambda^2}{2n_2L}$$

## Ring cavity, Ring resonator filter

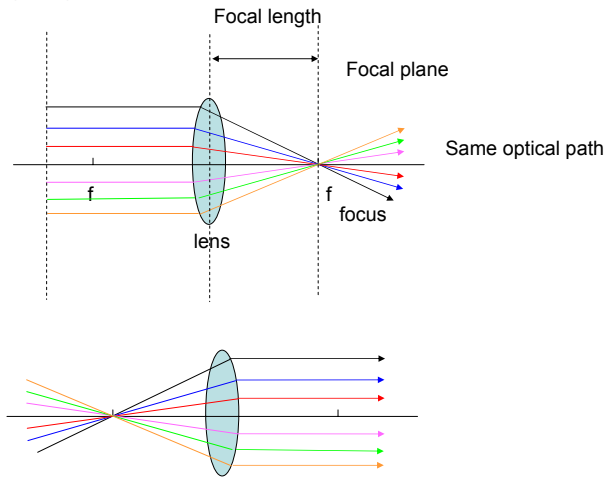


$$L = 2\pi R_0$$

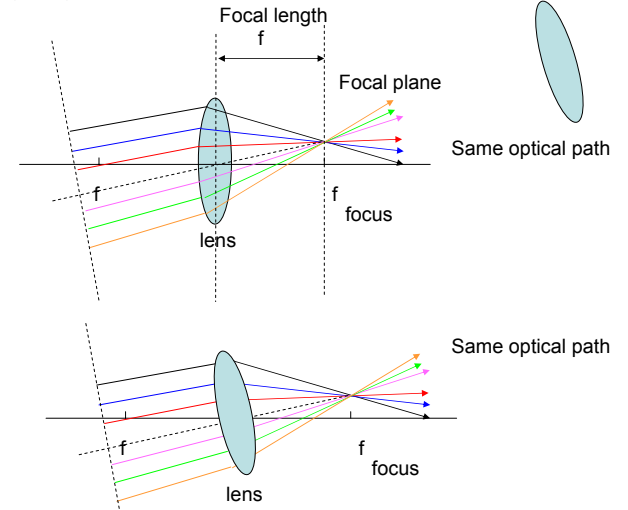
$$\frac{2\pi}{\lambda} n_2 L = m\pi$$

$$FSR = \frac{\lambda^2}{2n_2L} \quad 2n_2L = m\lambda$$

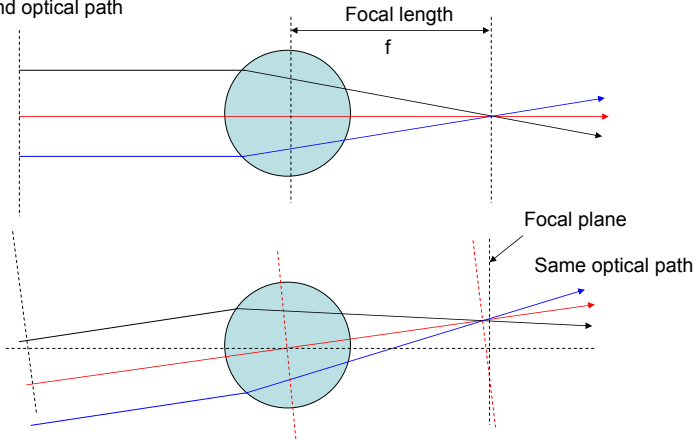
Lens and optical path



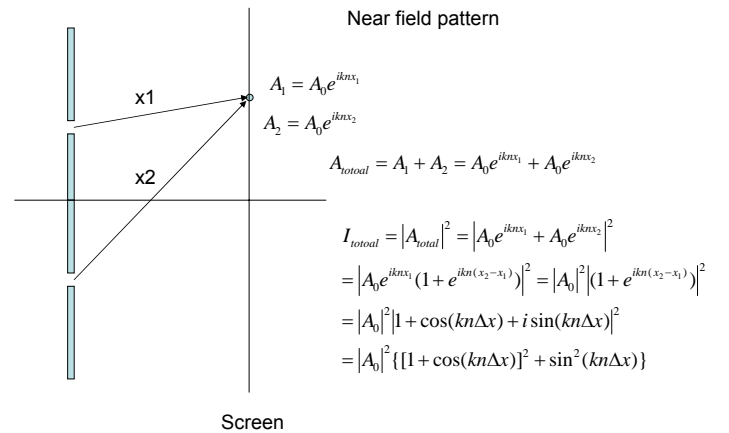
Lens and optical path



Lens and optical path



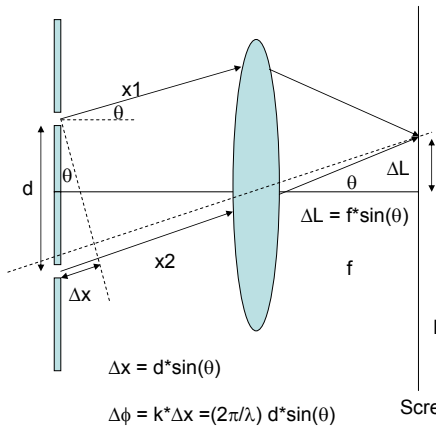
Interference of multiple Waves, gratings





Diffraction of Waves

Far field pattern

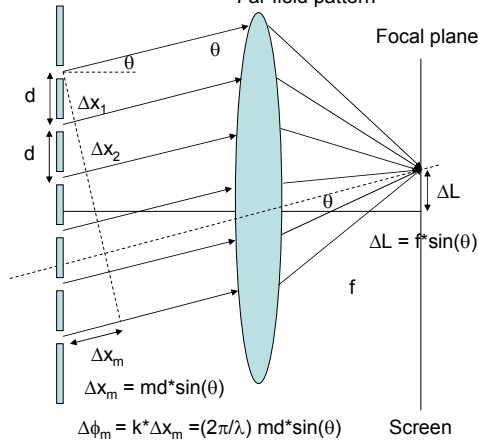


When  $\Delta\phi = 2m\pi$ , bright  
 When  $\Delta\phi = 2m\pi + \pi$ , dark  
 $2m\pi = (2\pi/\lambda) d \sin(\theta)$   
 $m\lambda = d \sin(\theta)$   
 $m\lambda = d \Delta L / f$   
 $\Delta L = m \lambda * f / d$ , bright spots

Focal plane  
 Screen

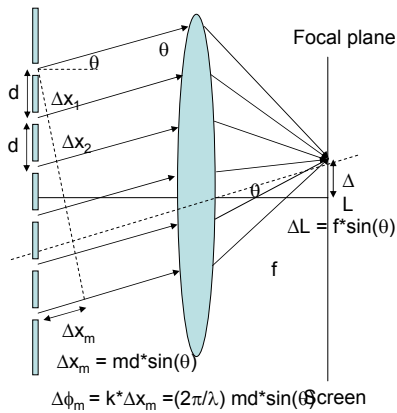
Multiple slots, grating

Far field pattern



When  $\Delta\phi = 2m\pi$ , bright  
 When  $\Delta\phi = 2m\pi + \pi$ , dark  
 $2m\pi = (2\pi/\lambda) d \sin(\theta)$   
 $m\lambda = d \sin(\theta)$   
 $\tan(\theta) = \Delta L / f$   
 $\theta \sim \tan(\theta) \sim \sin(\theta)$ , when  $\theta$  is small  
 $m\lambda = d \Delta L / f$   
 $\Delta L = m \lambda * f / d$ , bright spots  
 Bright spot still at small position

Multiple slots, grating



$$A_{total} = A_0 + A_1 + A_2 + \dots + A_m$$

$$= A_0 e^{ikx_0} + A_0 e^{ikx_1} + A_0 e^{ikx_2} + \dots + A_0 e^{ikx_m}$$

$$= A_0 e^{ikx_0} (1 + e^{ik\Delta x_1} + e^{ik\Delta x_2} + \dots + e^{ik\Delta x_m})$$

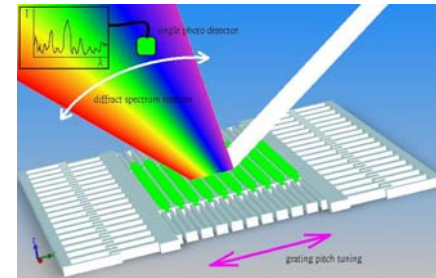
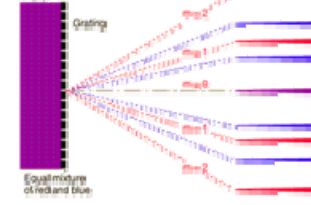
$$= A_0 e^{ikx_0} (1 + e^{ikd \sin \theta} + e^{ik2d \sin \theta} + \dots + e^{ikmd \sin \theta})$$

$$= A_0 e^{ikx_0} \frac{1 - e^{ik(m+1)d \sin \theta}}{1 - e^{ikd \sin \theta}}$$

$$I_{total} = |A_{total}|^2 = \left| A_0 e^{ikx_0} \frac{1 - e^{ik(m+1)d \sin \theta}}{1 - e^{ikd \sin \theta}} \right|^2$$

$$= |A_0|^2 \frac{[1 - \cos(kmd \sin \theta)]^2 + \sin^2(kmd \sin \theta)}{[1 - \cos(kd \sin \theta)]^2 + \sin^2(kd \sin \theta)}$$

Multiple reflections



Optical wave, photon

Photon energy:  $E = h\nu$ ,  $\nu$  is the frequency of the photon,  $c = f\lambda = \nu\lambda$ ,

$$E = hc / \lambda = \frac{2\pi}{\lambda} \hbar c = \hbar k c = pc,$$

$$p = \hbar k, \quad \text{momentum of the photon,}$$

$$E = mc^2 = m c c = pc,$$

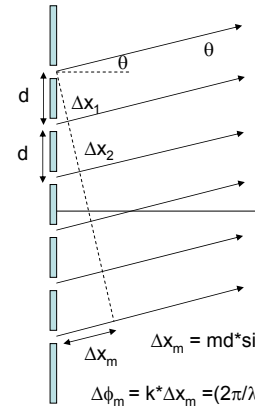
For photon:

$$E = pc = h\nu,$$

$$p = \hbar k,$$

$$k = \frac{2\pi}{\lambda},$$

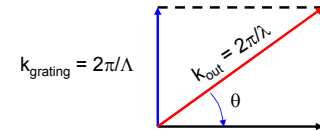
More on grating, Grating period, Phase matching condition



$k$  vector,  $k = (2\pi/\lambda)$ , along the beam propagation direction

$d$  is called grating period, often use symbol:  $\Lambda$ ,  
Grating vector  $= 2\pi / \Lambda$ ,

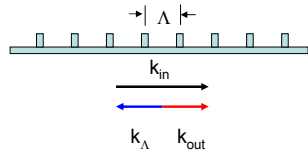
Phase-matching condition



$$k_{out} \sin(\theta) = k_{grating} = 2\pi/\Lambda$$

$$d \sin(\theta) = \lambda$$

Distributed feedback grating (DFB grating)

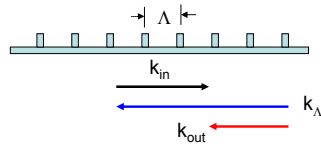


$$k_{out} = k_{in} - k_{\Lambda}$$

Phase-matching condition

$$2\pi n_{out} / \lambda_{out} = 2\pi n_{in} / \lambda_{in} - 2\pi / \Lambda$$

Effective reflection



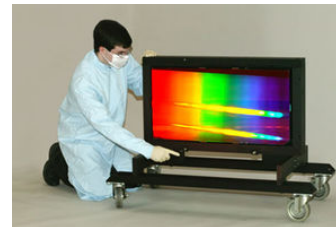
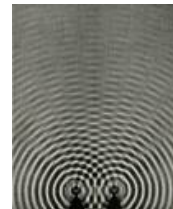
$$k_{out} = k_{in} - k_{\Lambda} = -k_{in}$$

Phase-matching condition

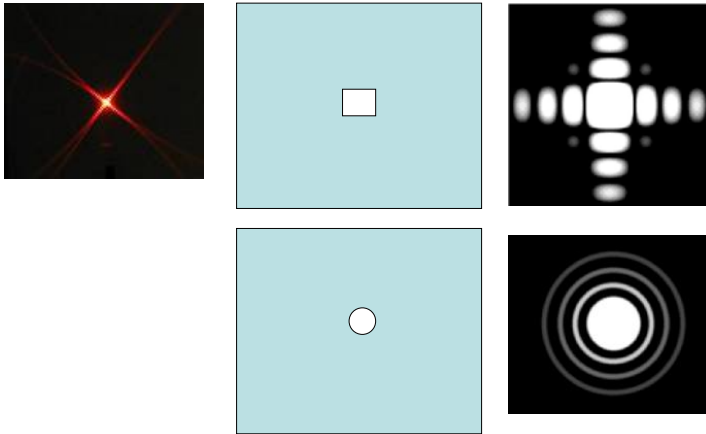
$$2\pi / \Lambda = 2 * 2\pi / \lambda$$

$$\Lambda = \lambda / 2$$

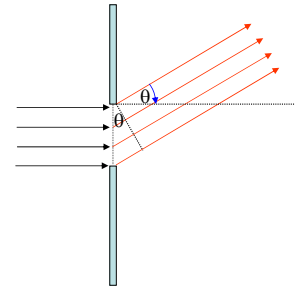
Diffraction: Multiple beam interference



## Diffraction:

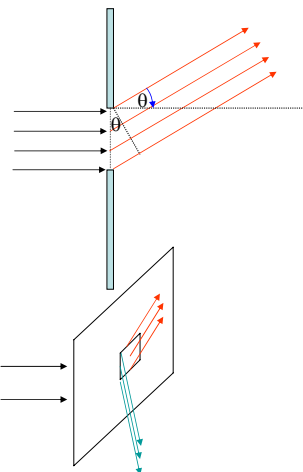


## Single slot diffraction:



$$\begin{aligned}
 A_{total} &= A_0 + A_1 + A_2 + \dots + A_n \\
 &= A_0 e^{ikx_0} + A_0 e^{ikx_1} + A_0 e^{ikx_2} + \dots + A_0 e^{ikx_n} \\
 &= \int_0^d A_0 e^{ikz \sin(\theta)} \frac{dz}{d} = \frac{A_0 (1 - e^{ikd \sin(\theta)})}{ikd \sin(\theta)} \\
 I_{total} &= |A_{total}|^2 = \left| \frac{A_0 (1 - e^{ikd \sin(\theta)})}{kd \sin(\theta)} \right|^2 \\
 &= I_0 \left| \frac{e^{ikd \sin \theta / 2} (e^{-ikd \sin \theta / 2} - e^{ikd \sin \theta / 2})}{kd \sin(\theta)} \right|^2 \\
 &= I_0 \left| \frac{2i \sin(kd \sin \theta / 2)}{kd \sin(\theta)} \right|^2 = I_0 \left| \frac{\sin(kd \sin \theta / 2)}{kd \sin \theta / 2} \right|^2 \\
 &= I_0 \frac{\sin^2(\alpha)}{\alpha^2}, \\
 \alpha &= kd \sin \theta / 2 = \frac{\pi}{\lambda} d \sin \theta
 \end{aligned}$$

## Single slot diffraction:



$$I_{total} = I_0 \frac{\sin^2(\alpha)}{\alpha^2},$$

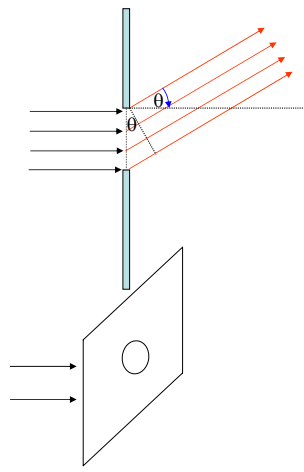
$$\alpha = kd \sin \theta / 2 = \frac{\pi}{\lambda} d \sin \theta$$

$$\text{When } \alpha = \frac{\pi}{\lambda} d \sin \theta = m\pi + \frac{\pi}{2} \quad \text{bright}$$

$$\text{When } \alpha = \frac{\pi}{\lambda} d \sin \theta = m\pi \quad \text{dark}$$



## Single slot diffraction:



$$I_{total} = I_0 \frac{\sin^2(\alpha)}{\alpha^2},$$

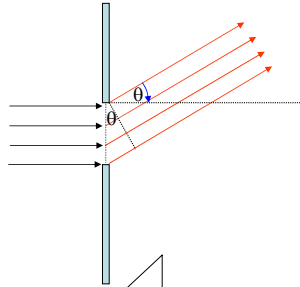
$$\alpha = kd \sin \theta / 2 = \frac{\pi}{\lambda} d \sin \theta$$

$$\text{When } \alpha = \frac{\pi}{\lambda} d \sin \theta = m\pi + \frac{\pi}{2} \quad \text{bright}$$

$$\text{When } \alpha = \frac{\pi}{\lambda} d \sin \theta = m\pi \quad \text{dark}$$



Angular width :



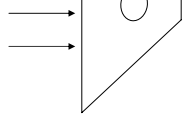
$$I_{total} = I_0 \frac{\sin^2(\alpha)}{\alpha^2},$$

$$\alpha = kd \sin \theta / 2 = \frac{\pi}{\lambda} d \sin \theta$$

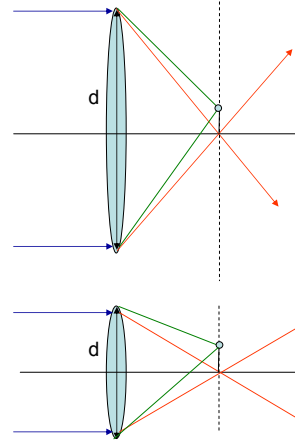
first dark:  $\alpha = \frac{\pi}{\lambda} d \sin \theta = m\pi$

$$\sin \theta = \frac{\lambda}{d} \quad \theta \cong \frac{\lambda}{d}$$

Half angular width



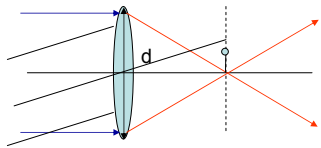
Diffraction of a lens



• Focus size

● Focus size

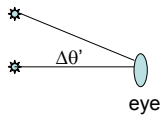
Resolution of optical instrument



$$\theta \cong \frac{\lambda}{d}$$

● Focus size  $f \cdot \theta \cong f \frac{\lambda}{d}$

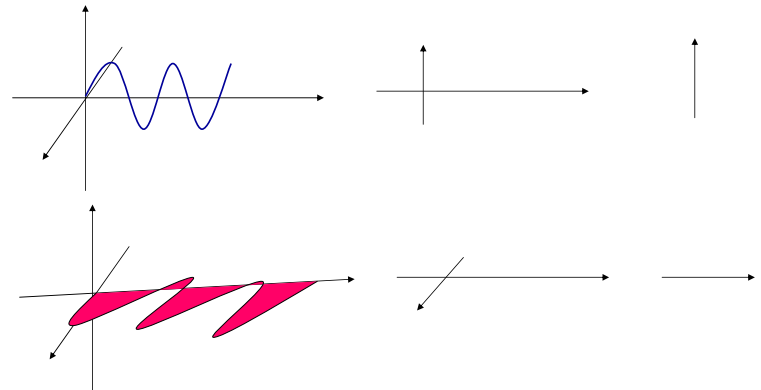
$$\Delta \theta' > 1.22 \frac{\lambda}{d}$$



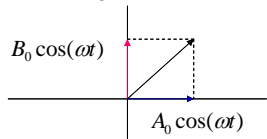
$$\Delta \theta' > 1.22 \frac{\lambda}{d}$$

d = 16mm.

Polarization of light: Linear polarization



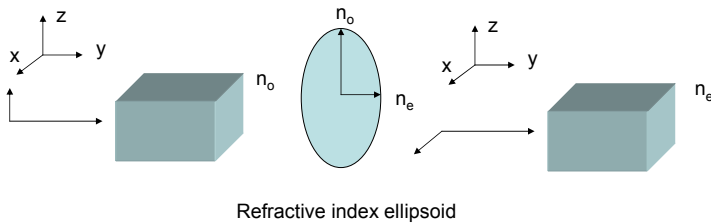
Polarization of light: linear polarization



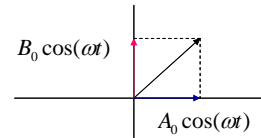
Any linear polarization can be decomposed into two primary polarization with the same phase

Why decompose into two primary polarization directions?

In crystal, the refractive index is different along different polarizations

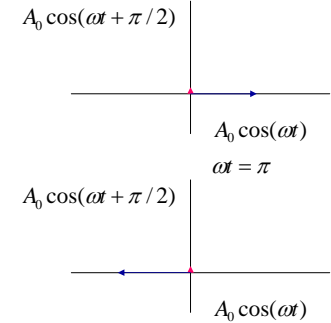
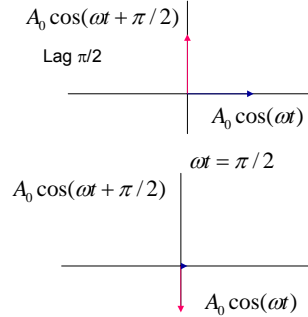


Polarization of light: circular polarization

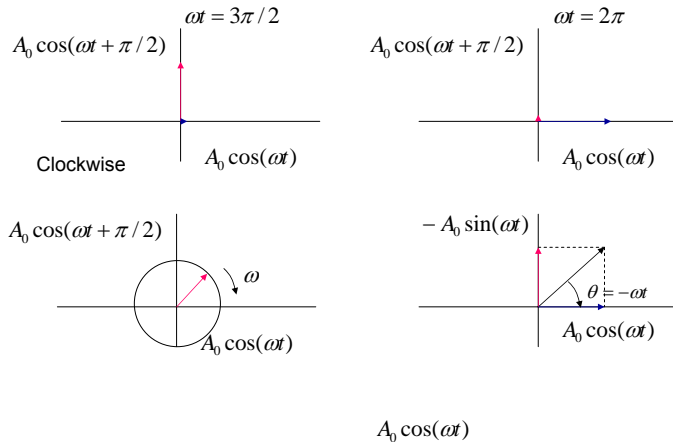


$$\theta = \tan^{-1}[B_0 \cos(\omega t) / A_0 \cos(\omega t)]$$

$$= \tan^{-1}[B_0 / A_0]$$

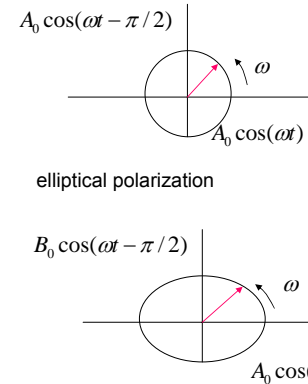


Polarization of light: circular polarization

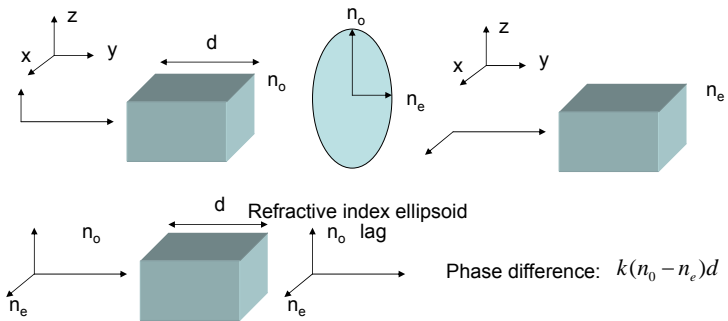


Polarization of light: circular polarization

Count-clockwise



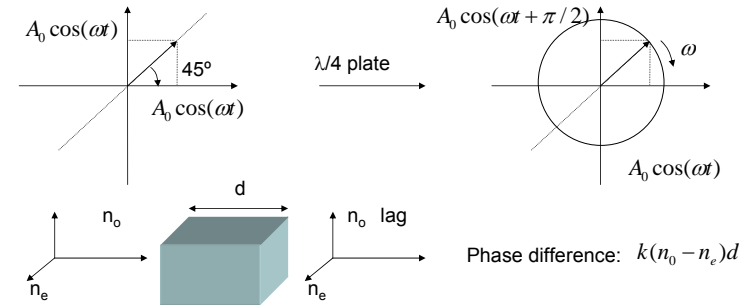
Delays along different directions:



when  $k(n_o - n_e)d = \pi/2$  Right (clockwise) circular polarization

$(n_o - n_e)d = \lambda/4$  Quarter-wavelength  $\lambda/4$  plate

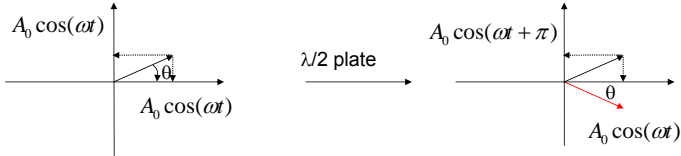
Right (clockwise) circular polarization



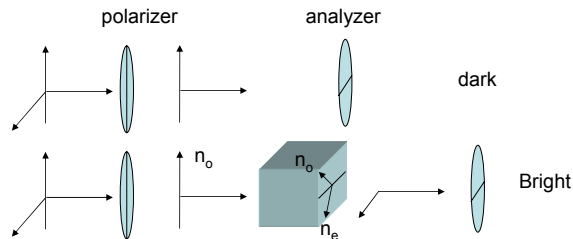
when  $k(n_o - n_e)d = \pi$  Change the direction of the polarization

$(n_o - n_e)d = \lambda/2$  half-wavelength  $\lambda/2$  plate

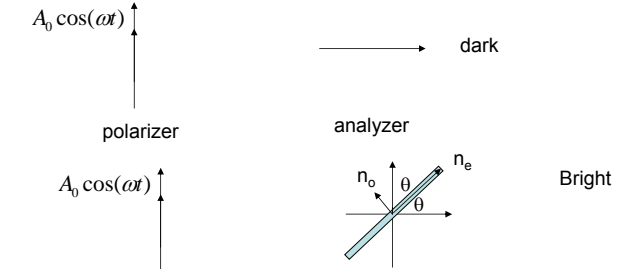
Change the direction of the polarization by  $2\theta$



EO modulator



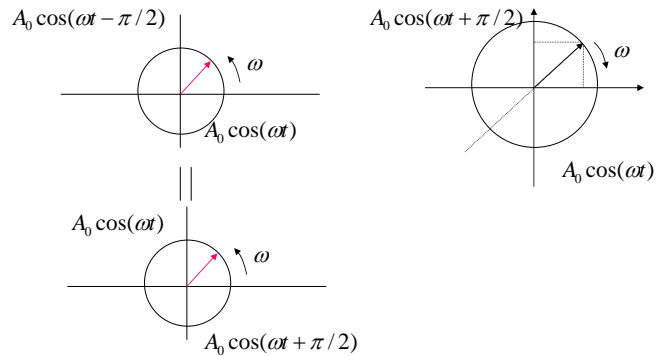
EO modulator



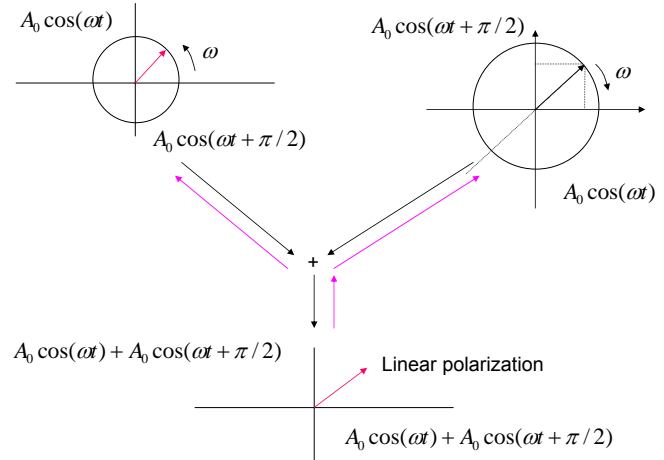
$k(n_o - n_e)d = \pi$

$$n_o \propto E = \frac{V}{t} \quad V_\pi$$

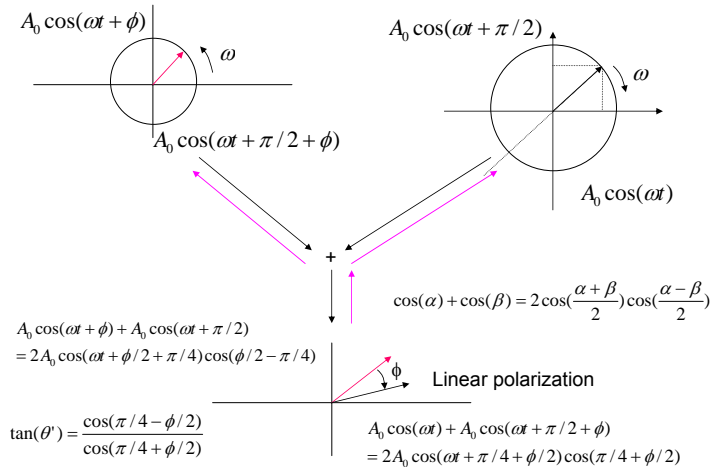
Circular polarizations and linear polarizations



Circular polarizations and linear polarizations



Circular polarizations and linear polarizations



Faraday rotation

