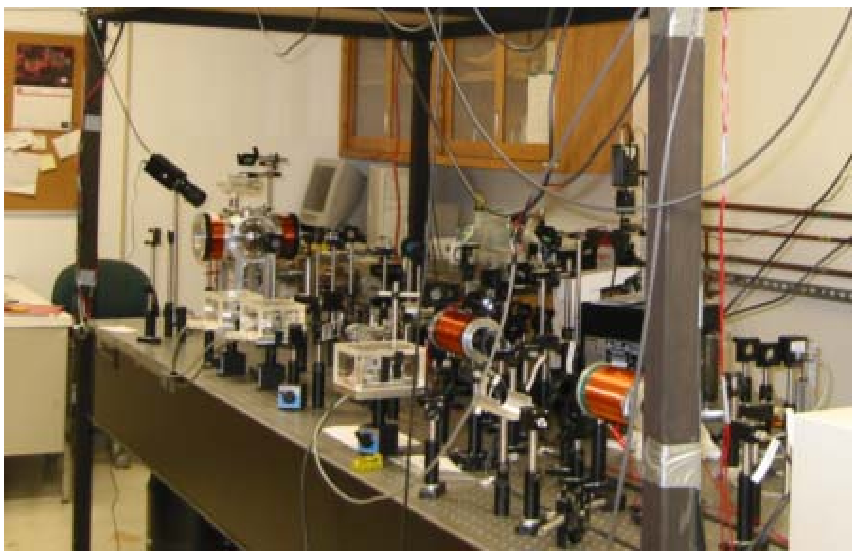
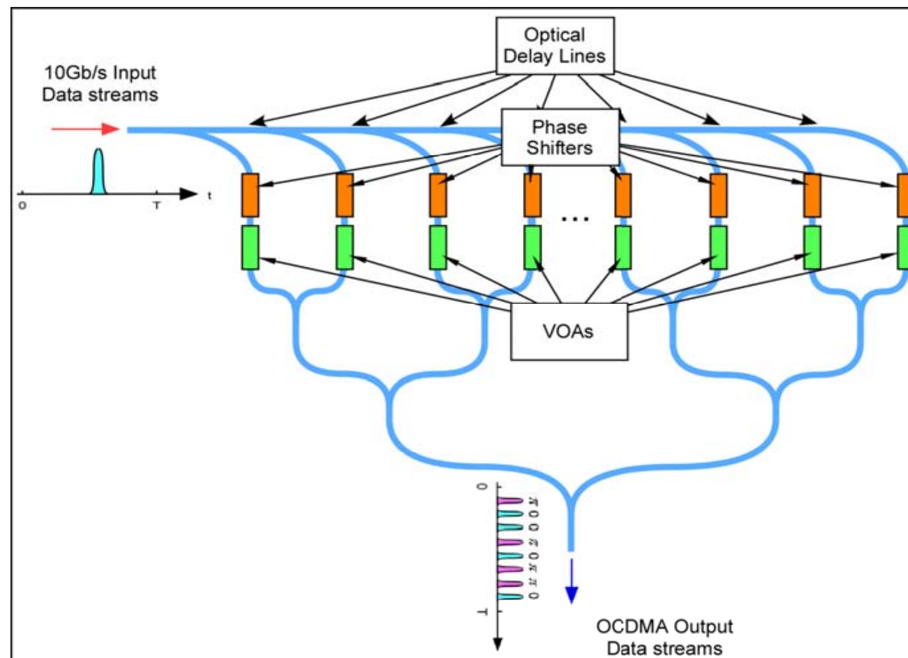
Discrete optics:

- Mirrors, lens, mechanical mounts
- bulky
- labor intensive alignment
- ray optics
- environment sensitive

Integrated optics:

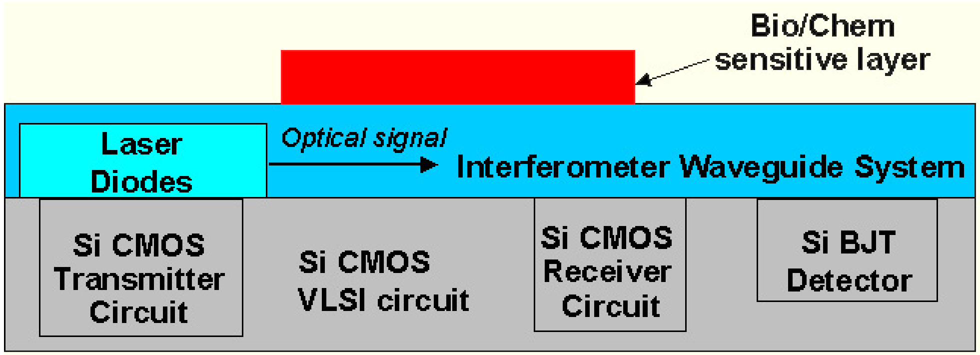
- Waveguide
- Bendable, portable
- Free-of-alignment
- wave optics
- robust
- more functionalities





Applications of Integrated optics:

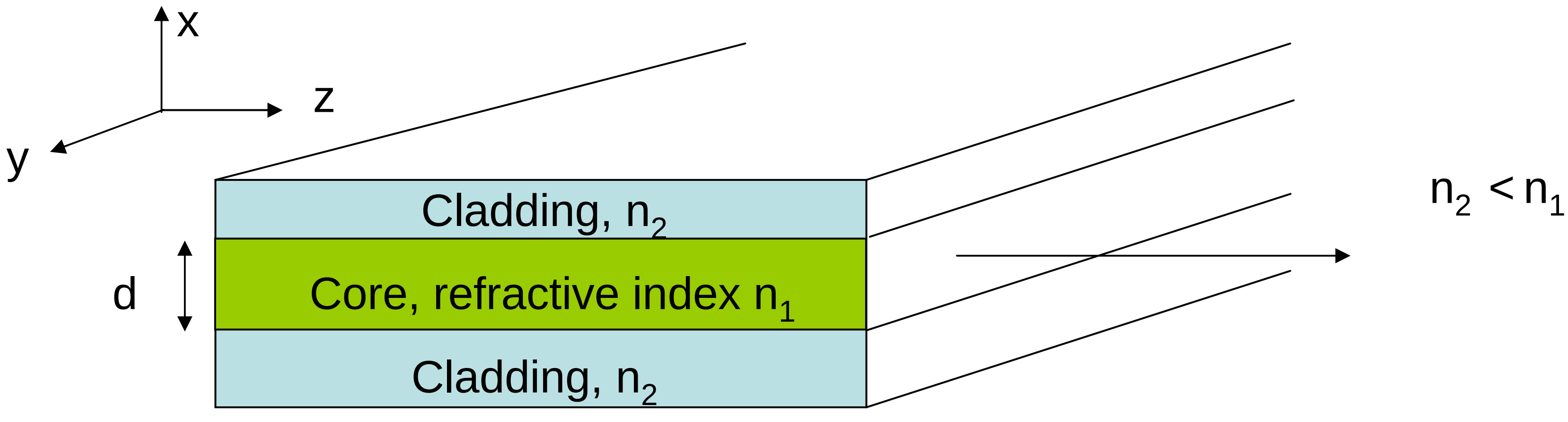
- Transmitters and receivers, transceivers
- All optical signal processing
- Ultra-high speed communications (100Gbit/s), optical packet switching
- RF spectrum analyzer
- Smart sensors

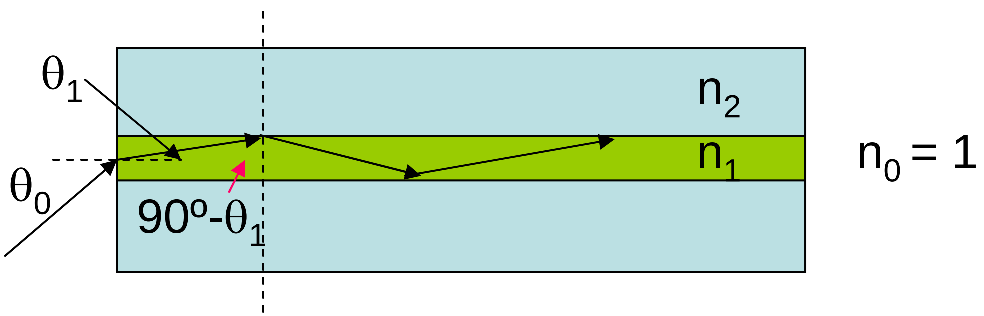


OEIC, bio/sensor

Optical transceivers

2-D Optical waveguide





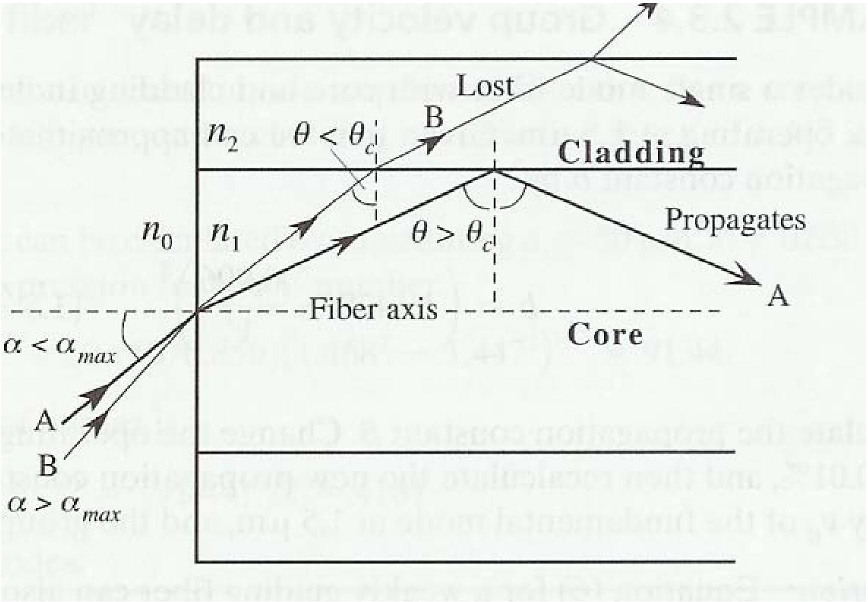
$n0 * \sin(\theta_0) = n1 * \sin(\theta_1)$ Numerical aperture (NA)

Critical angle

$n1 * \sin(90^o - \theta_1) = n2 * \sin(90^o)$

$\cos(\theta_1) = n2/n1$

2-D Optical waveguide



$$n_0 \sin\alpha_{\max} = n_1 \sin(90^0 - \theta_c),$$

$$\sin\theta_c = \frac{n_2}{n_1},$$

$$\sin\alpha_{\max} = \frac{(n_1^2 - n_2^2)^{1/2}}{n_0},$$

$2\alpha_{\max}$: total acceptance angle

$$NA = n_0 \sin\alpha_{\max} = (n_1^2 - n_2^2)^{1/2},$$

Example 2:

Calculate the acceptance angle of a core layer with index of n1= 1.468, and cladding layer of n_0 = 1.447 for wavelength of 1.3μm and 1.55 μm.

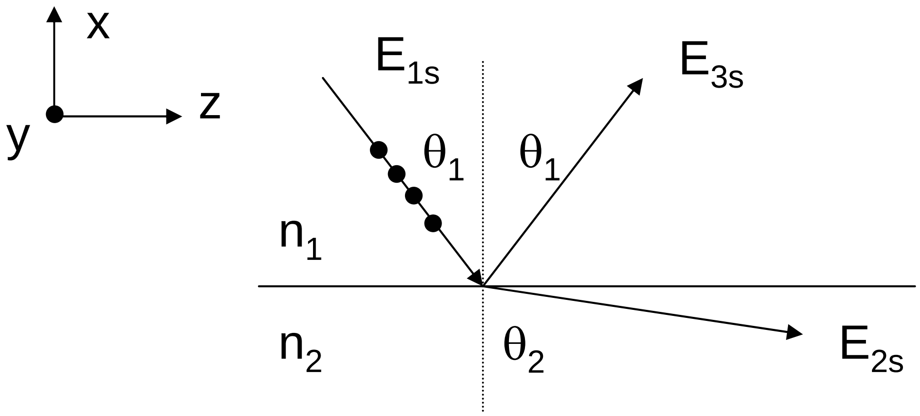
Solution:

$$\sin\alpha_{\max} = \frac{(n_1^2 - n_2^2)^{1/2}}{n_0},$$

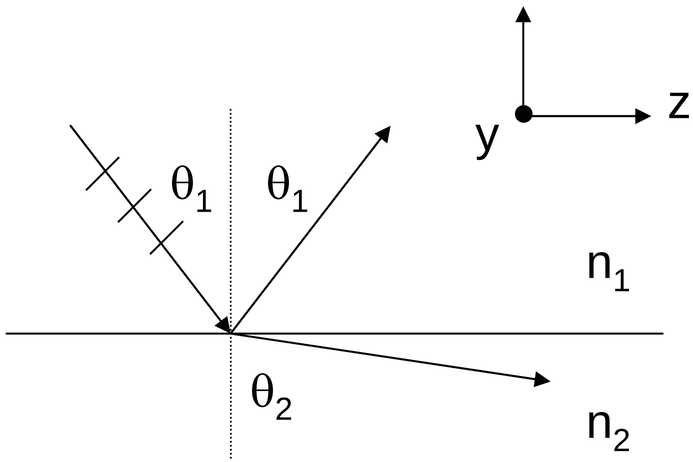
$$\alpha_{\max} = \sin^{-1}\frac{(n_1^2 - n_2^2)^{1/2}}{n_0} = 9.7^0,$$

acceptance angle: $2\alpha_{\max} = 19.4^0,$ Wavelength independent:

Fresnel equations



$n_2 < n_1$



s-polarized beam (*senkrecht:* perpendicular)

Trans-electric beam (TE)

$$E_{2s} = t_s E_{1s} \qquad E_{3s} = r_s E_{1s}$$

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$t_s = 1 + r_s$$

p-polarized beam (parallel)

Trans-magnetic beam (TM)

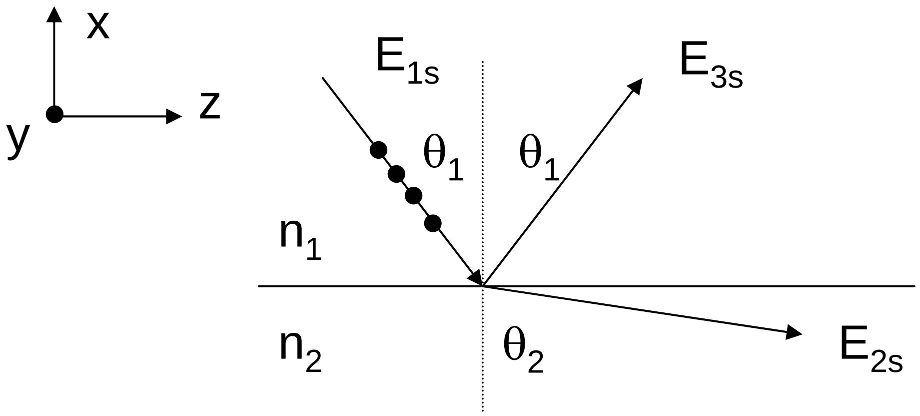
$$E_{2p} = t_p E_{1p} \qquad E_{3p} = r_p E_{1p}$$

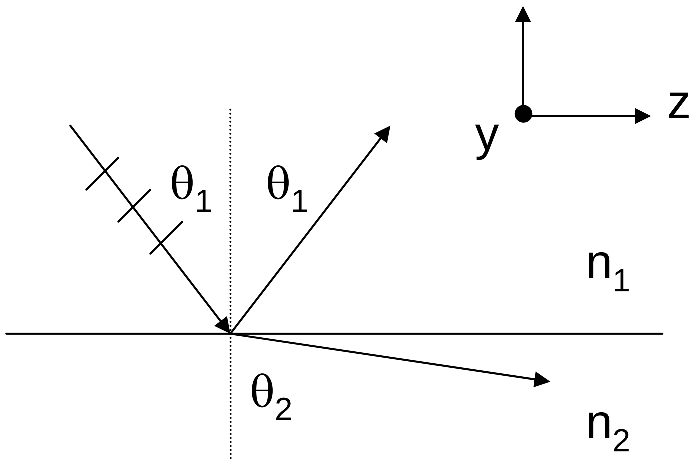
$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$t_p = \frac{n_1}{n_2}(1 + r_p)$$

Fresnel equations

$n_2 < n_1$





$$R_s = \left|\frac{E_{3s}}{E_{1s}}\right|^2 \qquad T_s = \frac{n_2}{n_1}\left|\frac{E_{2s}}{E_{1s}}\right|^2$$

$$R_p = \left|\frac{E_{3p}}{E_{1p}}\right|^2 \qquad T_p = \frac{n_2}{n_1}\left|\frac{E_{2p}}{E_{1p}}\right|^2$$

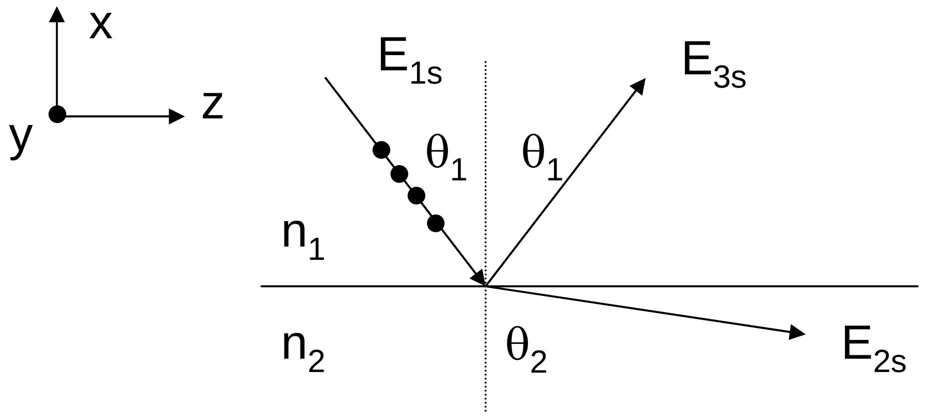
Poynting vector, energy flow rate

$$\vec{S} = \vec{E} \times \vec{H} \qquad \nabla \times \vec{E} = i\omega\mu H \qquad iknE\hat{a} = i\omega\mu\vec{H}$$

$$\vec{S} = n|E|^2$$

Phase shift of reflection

$n_2 < n_1$



$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 \cos\theta_2 = (n_2^2 - n_1^2 \sin^2\theta_1)^{1/2}$$

when $n_2^2 > n_1^2 \sin^2\theta_1$ i.e. $\sin\theta_1 < \dfrac{n_2}{n_1} = \sin\theta_c$

$n_1 \cos\theta_1 - n_2 \cos\theta_2 > 0$ because $(n_1 \cos\theta_1)^2 - (n_2 \cos\theta_2)^2 = n_1^2 - n_2^2 > 0$

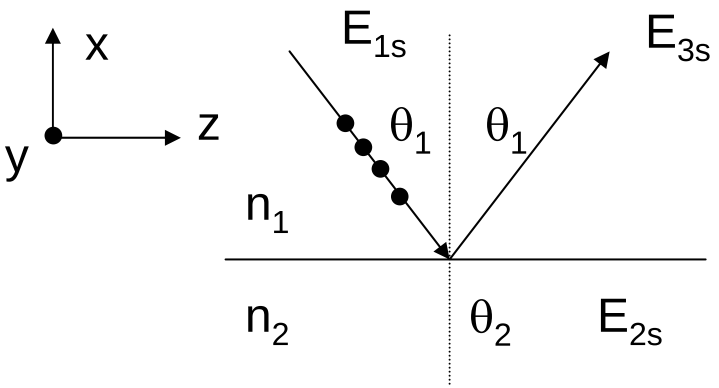
In this case, $r_s > 0$ is a real number

The reflection is not associated with phase shift, or phase shift is 0

Phase shift of reflection

$n_2 < n_1$

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

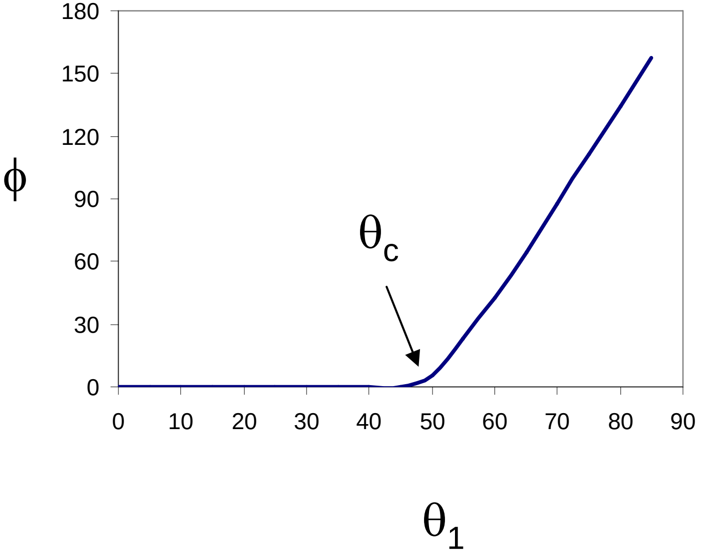


$$n_2 \cos\theta_2 = (n_2^2 - n_1^2 \sin^2\theta_1)^{1/2}$$

when $n_2^2 < n_1^2 \sin^2\theta_1$ i.e. $\sin\theta_1 > \frac{n_2}{n_1} = \sin\theta_c$

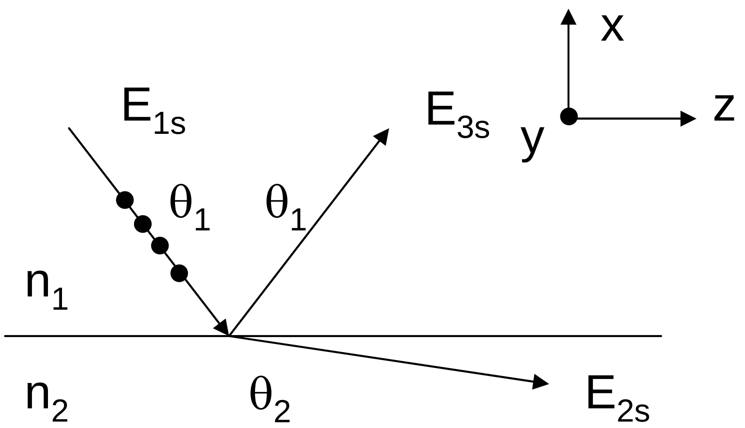
$$r_s = \frac{n_1 \cos\theta_1 - i(n_1^2 \sin^2\theta_1 - n_2^2)^{1/2}}{n_1 \cos\theta_1 + i(n_1^2 \sin^2\theta_1 - n_2^2)^{1/2}}$$

$$\tan\frac{\phi}{2} = \frac{(\sin^2\theta_1 - n_2^2/n_1^2)^{1/2}}{\cos\theta_1} = \frac{(\sin^2\theta_1 - \sin^2\theta_c)^{1/2}}{\cos\theta_1}$$



Evanescent wave

$n_2 < n_1$



$$E_{2s} = E_{2s}(0)e^{i\vec{k}_2\cdot\vec{r}} = E_{2s}(0)e^{i(-k_{2x}x+k_{2z}z)}$$

Momentum conservation

$$k_{2z} = k_{1z} \qquad \frac{2\pi}{\lambda}n_1\sin\theta_1 = \frac{2\pi}{\lambda}n_2\sin\theta_2$$

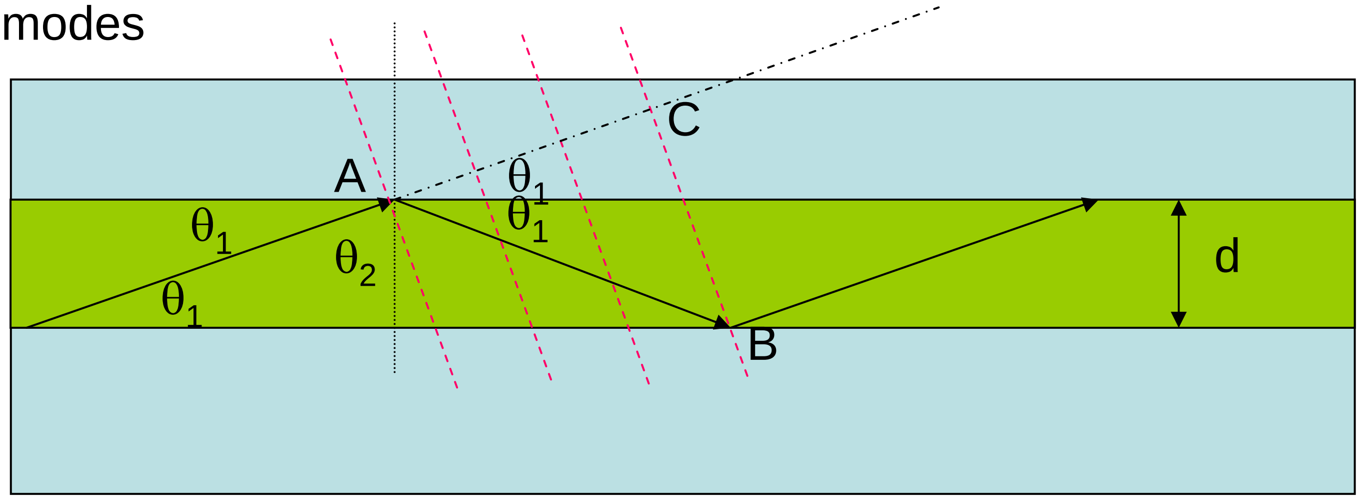
$$k_{2x} = \left(k_2^2 - k_{2z}^2\right)^{1/2} = \frac{2\pi}{\lambda}\left(n_2^2 - n_1^2\sin^2\theta_1\right)^{1/2} = i\,\frac{2\pi n_2}{\lambda}\left(\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1\right)^{1/2} = i\alpha_2$$

$$E_{2s} = E_{2s}(0)e^{-\alpha_2 x}e^{ik_{2z}z} = E_{2s}(0)e^{-x/d}e^{ik_{2z}z}$$

Attenuated wave, penetration depth: d

$$d = \alpha_2^{-1} = \left[\frac{2\pi n_2}{\lambda}\left(\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1\right)^{1/2}\right]^{-1}$$

Optical modes



k*n1*AC – k*n1*AB = 2mπ

AC = AB*cos(2θ_1) AB = d/sin(θ_1)

$$kn_1 \frac{d}{\sin\theta_1}\cos(2\theta_1) - kn_1\frac{d}{\sin\theta_1} = 2m\pi$$

$$kn_1 d\left(\frac{\cos^2\theta_1 - \sin^2\theta_1}{\sin\theta_1} - \frac{d}{\sin\theta_1}\right) = 2m\pi$$

$$kn_1 d\left(\frac{\cos^2\theta_1 - \sin^2\theta_1 - 1}{\sin\theta_1}\right) = 2m\pi$$

$$kn_1 d\left(\frac{1-\sin^2\theta_1 - \sin^2\theta_1 - 1}{\sin\theta_1}\right) = 2m\pi$$

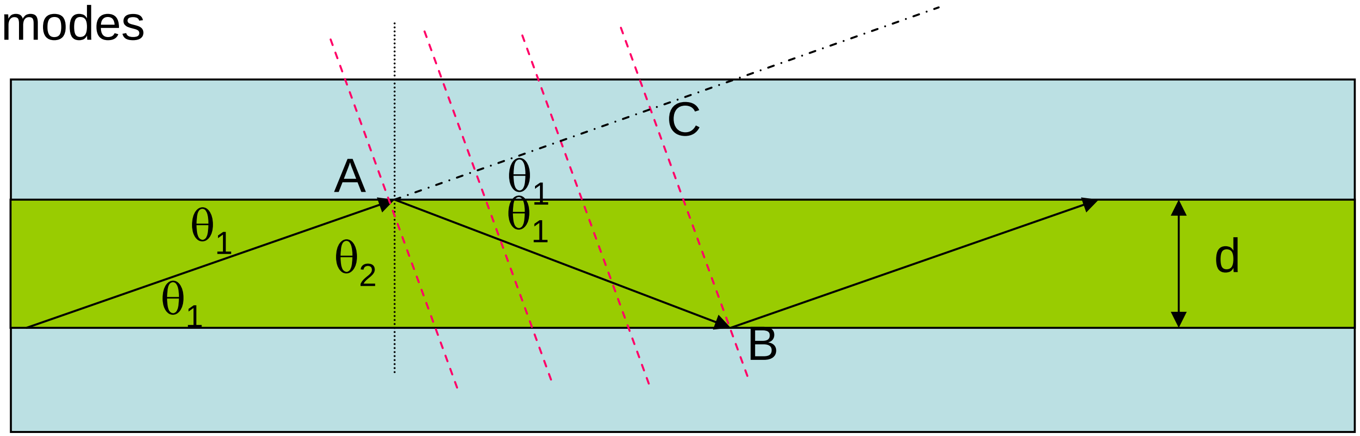
$$kn_1 d\left(\frac{2\sin^2\theta_1}{\sin\theta_1}\right) = 2m\pi \qquad kn_1 2d\sin\theta_1 = 2m\pi$$

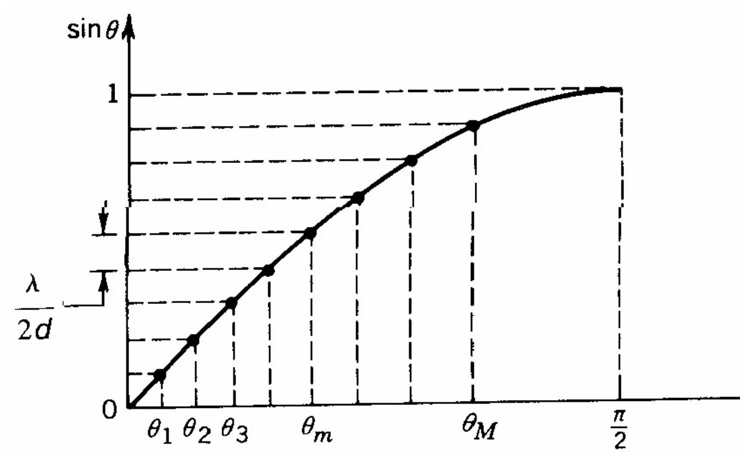
$$n_1\sin\theta_{1,m} = \sin\theta_{0,m} = m\frac{\lambda}{2d}$$

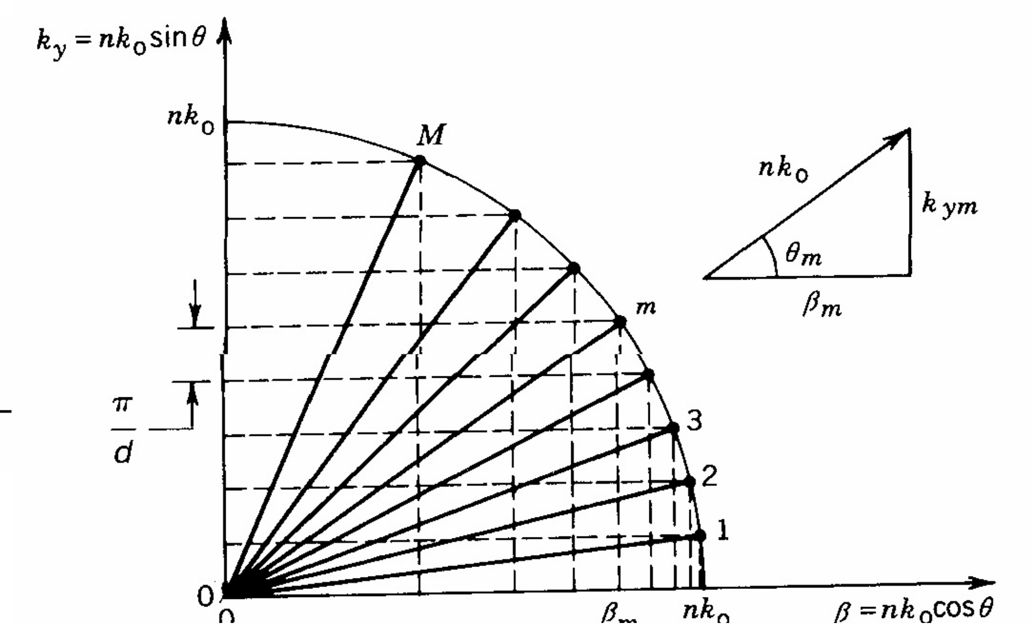
$$\beta_m = n_1 k_0\cos\theta_{1,m} = n_1 k_0\left[(1-\sin^2\theta_{1,m})\right]^{1/2}$$

$$= \left[(n_1 k_0)^2 - (k_0 m\frac{\lambda}{2d})^2\right]^{1/2}$$

Optical modes





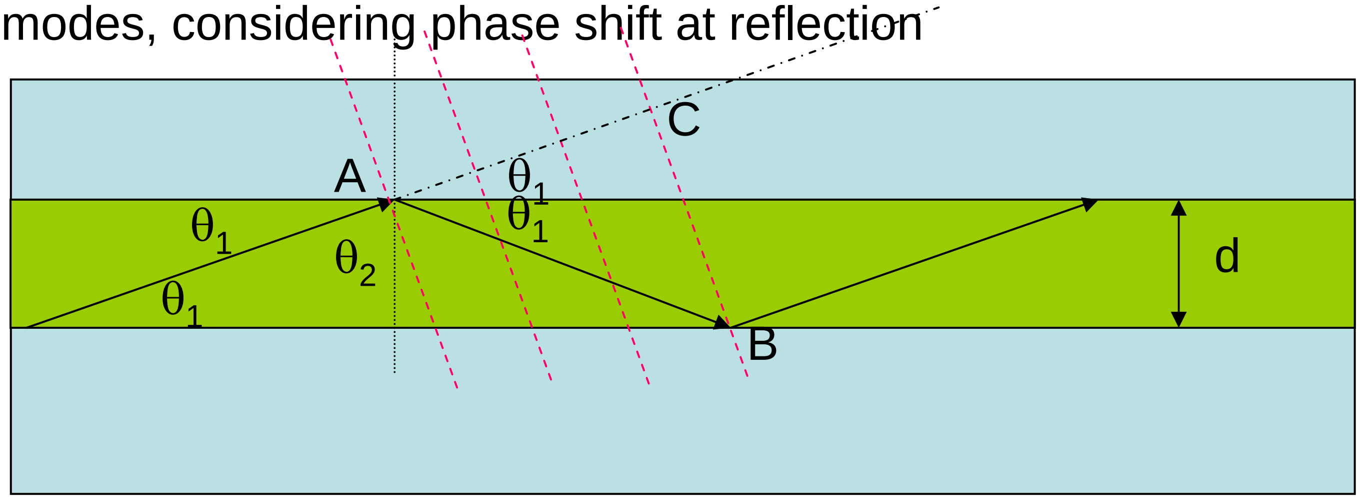


Propagation constant

$$\beta_m^2 = (n_1 k_0)^2 - (m k_0 \frac{\lambda}{2d})^2$$

Effective index: $\beta_m = k_0 n_{eff}$

Optical modes, considering phase shift at reflection



k*n1*AC – k*n1*AB + 2*ϕ = 2mπ

$AC = AB^*\cos(2\theta_1)$ $AB = d/\sin(\theta_1)$

$$kn_1 \frac{d}{\sin\theta_1}\cos(2\theta_1) - kn_1 \frac{d}{\sin\theta_1} + 2\phi = 2m\pi$$

$$kn_1 d\left(\frac{\cos^2\theta_1 - \sin^2\theta_1}{\sin\theta_1} - \frac{d}{\sin\theta_1}\right) + 2\phi = 2m\pi$$

$$kn_1 d\left(\frac{\cos^2\theta_1 - \sin^2\theta_1 - 1}{\sin\theta_1}\right) + 2\phi = 2m\pi$$

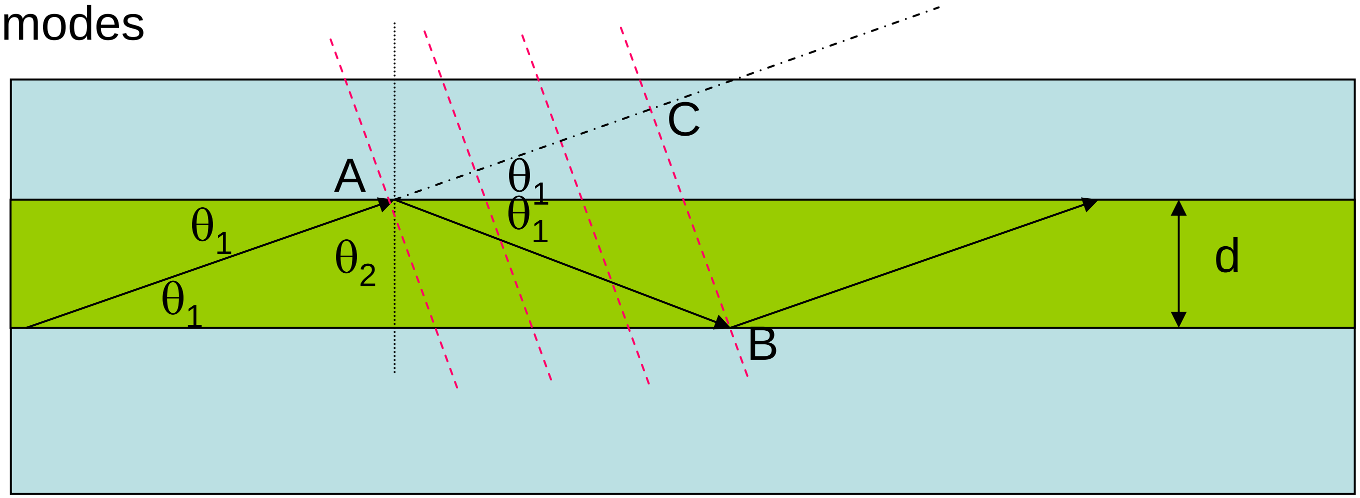
$$kn_1 d\left(\frac{1 - \sin^2\theta_1 - \sin^2\theta_1 - 1}{\sin\theta_1}\right) + 2\phi = 2m\pi$$

$$kn_1 d\left(\frac{2\sin^2\theta_1}{\sin\theta_1}\right) + 2\phi = 2m\pi$$

$$\tan(kn_1 \frac{d}{2}\sin\theta_1 - m\frac{\pi}{2}) = \tan\frac{\phi}{2}$$

$$\tan(kn_1 \frac{d}{2}\sin\theta_1 - m\frac{\pi}{2}) = \frac{(\sin^2\theta_2 - \sin^2\theta_c)^{1/2}}{\cos\theta_2}$$

Optical modes



$$kn_1 2d\sin\theta_1 + 2\phi = 2m\pi \qquad \sin\theta_1 = \cos\theta_2 \geq \sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$m \leq [\frac{2\pi d}{\lambda}(n_1^2 - n_2^2)^{1/2} - \phi]/\pi, \qquad V = \frac{2\pi d/2}{\lambda}(n_1^2 - n_2^2)^{1/2},$$

V number, normalized thickness, or normalized frequency

$$m \leq [2V - \phi]/\pi,$$

Cut-off wavelength λ_c: $V(\lambda_c) = \frac{\pi}{2}$,

Optical modes

Example: estimate the number of modes

- waveguide thickness 100μm, free-space wavelength 1μm,

$n_1 = 1.490,\ \ n_2 = 1.470,$

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = 76.4, \qquad m \le [\frac{2\pi d}{\lambda}(n_1^2 - n_2^2)^{1/2} - \phi]/\pi,$$

$m \le [2V - \phi]/\pi = 48.7,$ 49 modes

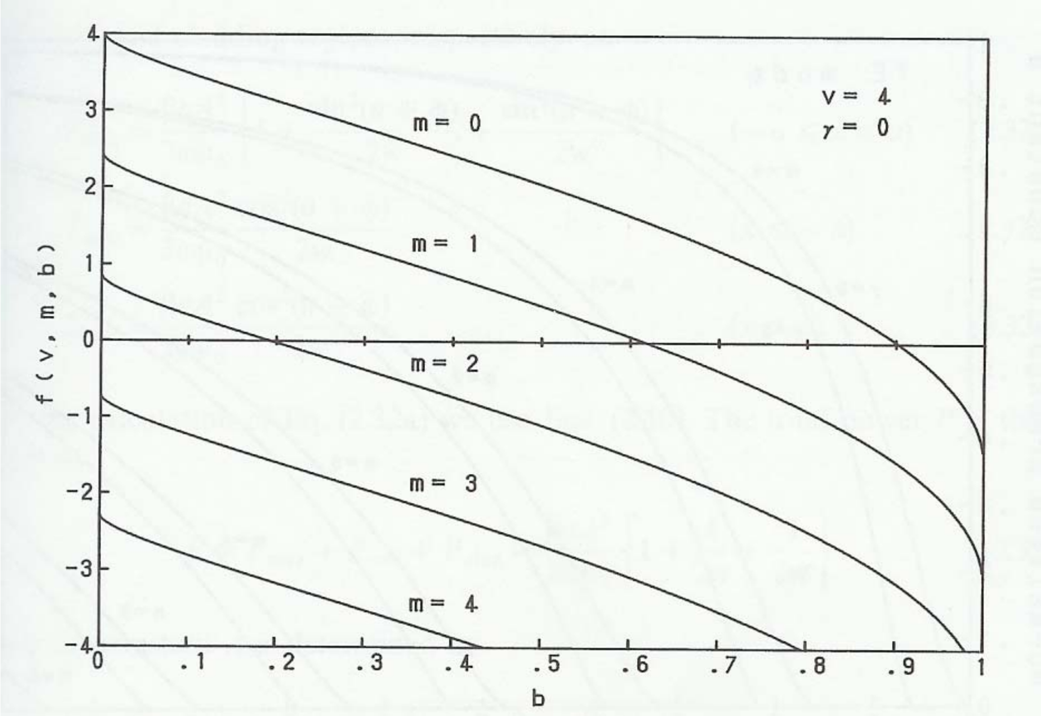
Normalized waveguide equation:

$$b = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2} = \frac{n_1^2 \cos^2\theta_1 - n_2^2}{n_1^2 - n_2^2}, \quad n_{eff} = n_1 \cos\theta_m,$$

b: normalized propagation constant

$$\tan(kn_1 \frac{d}{2}\sin\theta_1 - m\frac{\pi}{2}) = \frac{(\cos^2\theta_1 - \sin^2\theta_c)^{1/2}}{\sin\theta_1}$$

$$V = \frac{2\pi d/2}{\lambda}(n_1^2 - n_2^2)^{1/2},$$



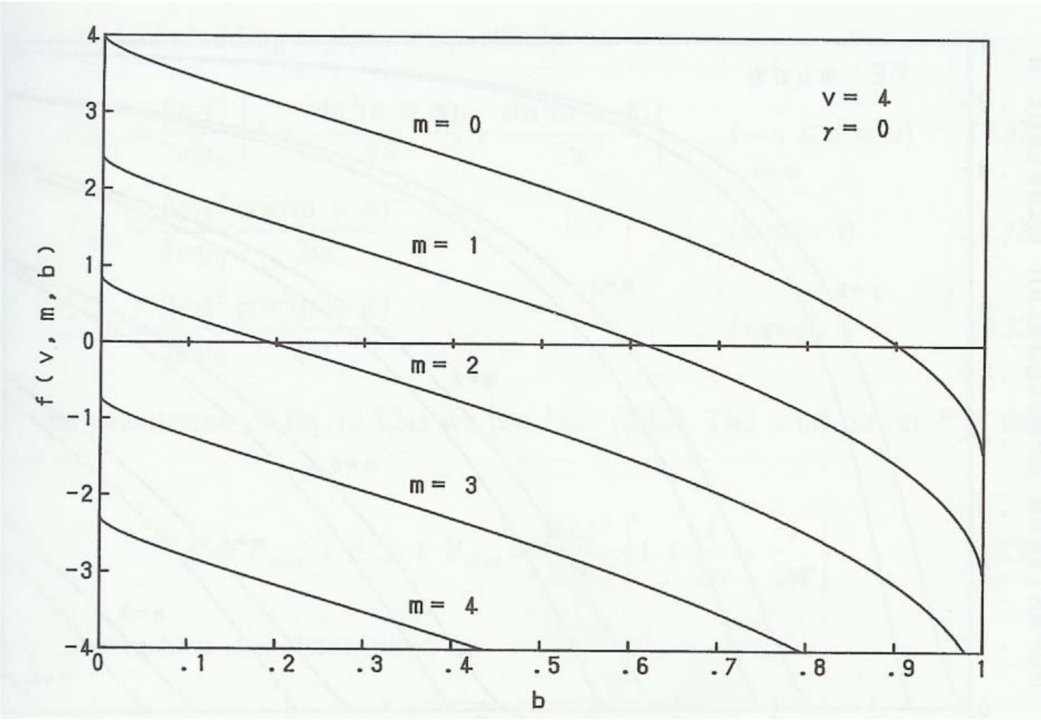
$$V\sqrt{(1-b)} - m\frac{\pi}{2} = \tan^{-1}\sqrt{\frac{b}{1-b}},$$

$$f(V,m,b) = V\sqrt{(1-b)} - m\frac{\pi}{2} - \tan^{-1}\sqrt{\frac{b}{1-b}} = 0,$$

Discussion:

$$b = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2} = \frac{n_1^2 \cos^2\theta_1 - n_2^2}{n_1^2 - n_2^2}, \quad n_{eff} = n_1 \cos\theta_m,$$

$$(n_1^2 - n_2^2)b = n_1^2 \cos^2\theta_1 - n_2^2,$$



Attenuated wave, penetration depth: D

$$D = \alpha_2^{-1} = \left[\frac{2\pi n_2}{\lambda}\left(\frac{n_1^2}{n_2^2}\sin^2\theta - 1\right)^{1/2}\right]^{-1}$$

$$= \left[\frac{2\pi}{\lambda}\left(n_1^2 - n_2^2)b\right)^{1/2}\right]^{-1}$$

Discussions:

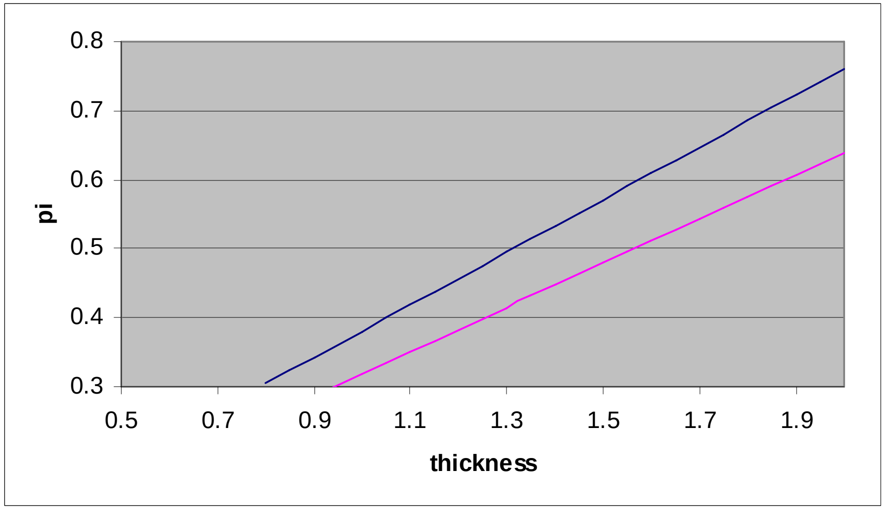
- mode numbers v.s. index difference and wavelength
- effective index difference of higher and lower order modes
- mode profiles dependence on index difference and wavelength

Example 1:

Calculate the thickness of a core layer with index of n1= 1.468, and cladding layer of n0 = 1.447 for wavelength of 1.3μm.

Solution:

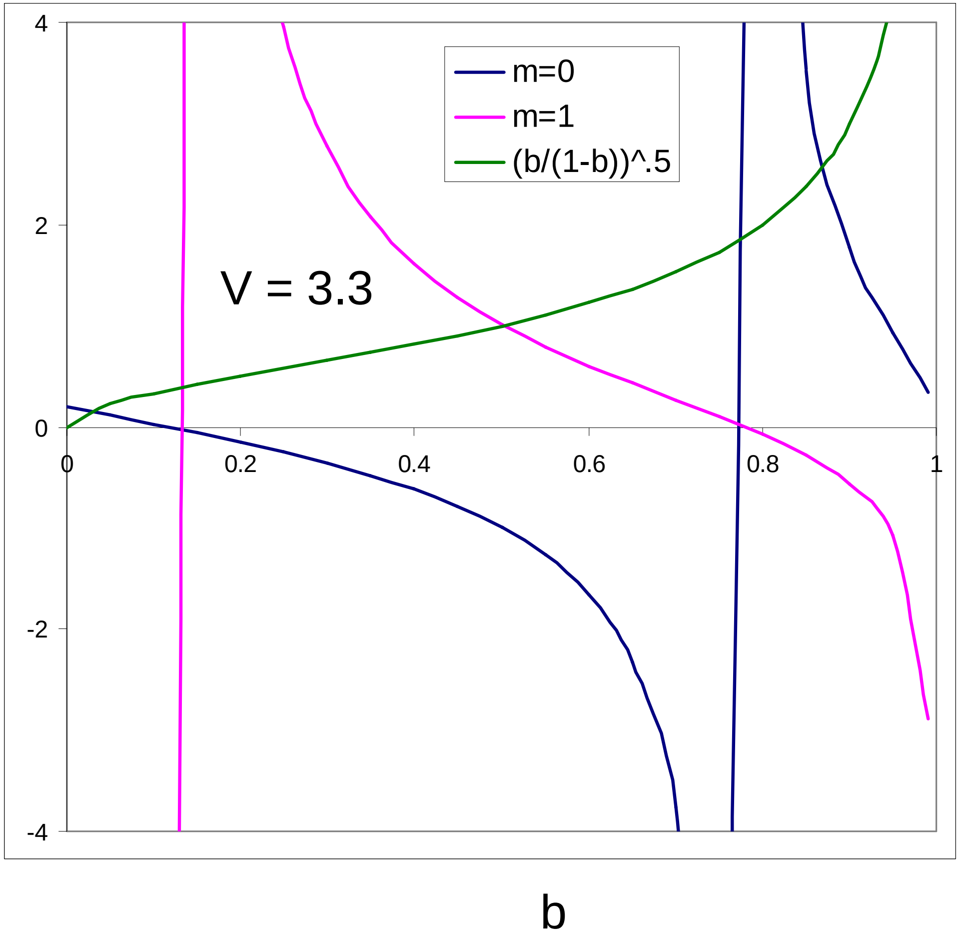
$m \leq [2V - \phi]/\pi,$ For single mode: $m = 1,$ $\phi = 0,$ $V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} \geq \frac{\pi}{2},$



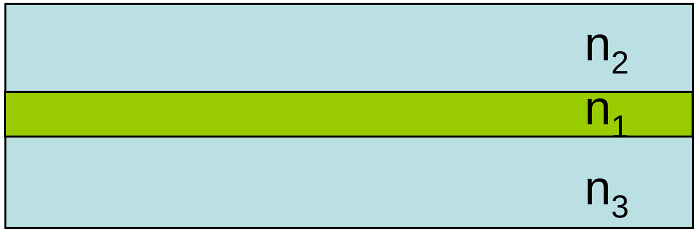
Normalized waveguide equation:

$$\tan(kn_1 \frac{d}{2} \sin\theta_m - m\frac{\pi}{2}) = \frac{(\cos^2\theta_m - \sin^2\theta_c)^{1/2}}{\sin\theta_m} \qquad n_{eff} = n_1 \cos\theta_m,$$

$$\tan(V\sqrt{(1-b)} - m\frac{\pi}{2}) = \sqrt{\frac{b}{1-b}},$$



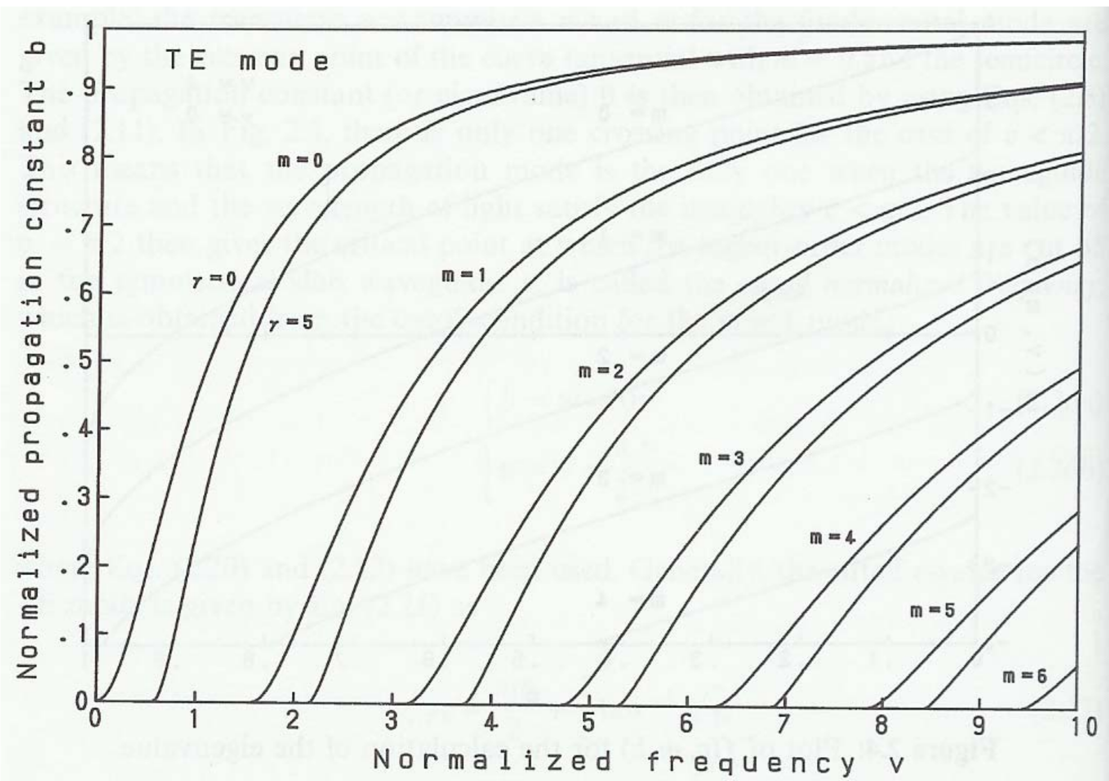
Asymmetric waveguide



$n_0 = 1$

$$\gamma = \frac{n_3^2 - n_2^2}{n_1^2 - n_3^2}$$

$$V\sqrt{(1-b)} - m\frac{\pi}{2} = \tan^{-1}\left(\sqrt{\frac{b}{1-b}}\right) + \tan^{-1}\left(\sqrt{\frac{b+\gamma}{1-b}}\right),$$



Maxwell equations:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{H} = J + \frac{\partial\vec{D}}{\partial t}$$

$$\nabla\cdot\vec{D} = \rho \qquad \nabla\cdot\vec{B} = 0$$

Dielectric materials

$$\rho = 0 \qquad \vec{D} = \varepsilon\vec{E} \qquad \vec{J} = 0 \qquad \vec{B} = \mu\vec{H}$$

Maxwell equations in dielectric materials:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$$

$$\nabla\cdot\vec{D} = 0 \qquad \nabla\cdot\vec{B} = 0$$

phasor →

$$\nabla\times\vec{E} = -j\omega\vec{B} \qquad \nabla\times\vec{H} = j\omega\vec{D}$$

$$\nabla\cdot\vec{D} = 0 \qquad \nabla\cdot\vec{B} = 0$$

$$\nabla\times(\nabla\times\vec{E}) = -j\omega(\nabla\times\vec{B}) \longrightarrow \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = \omega^2\mu\varepsilon\vec{E}$$

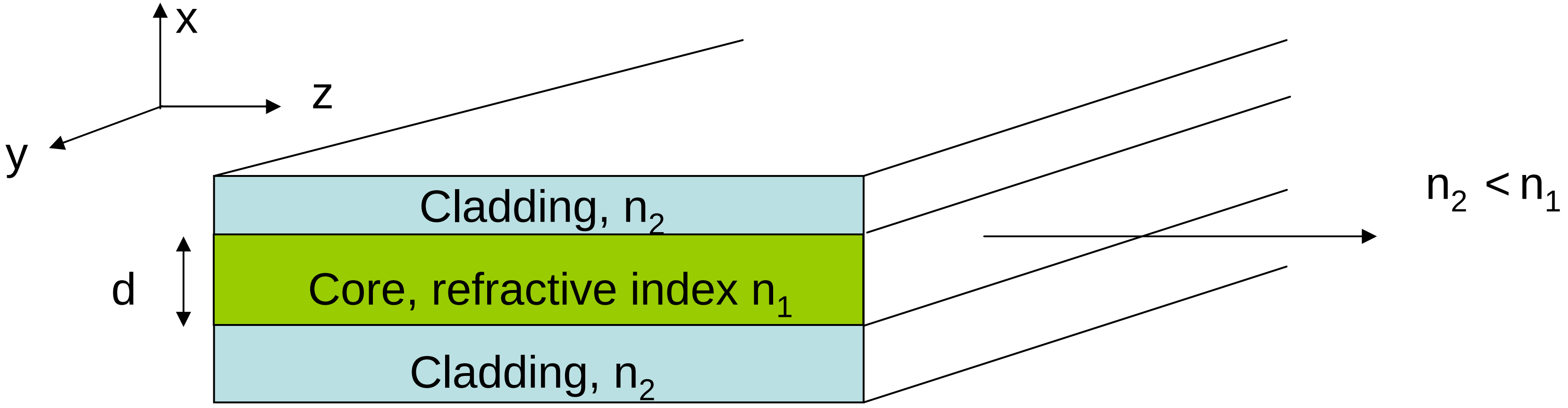
Helmholtz Equation:

$$\nabla^2\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0 \qquad \nabla^2\vec{E} + k^2\vec{E} = 0 \qquad k^2 = \omega^2\mu\varepsilon$$

Free-space solutions

$$\vec{E} = \hat{y}E_0e^{ikz} \qquad \vec{E} = \hat{x}E_0e^{ikz}$$

2-D Optical waveguide

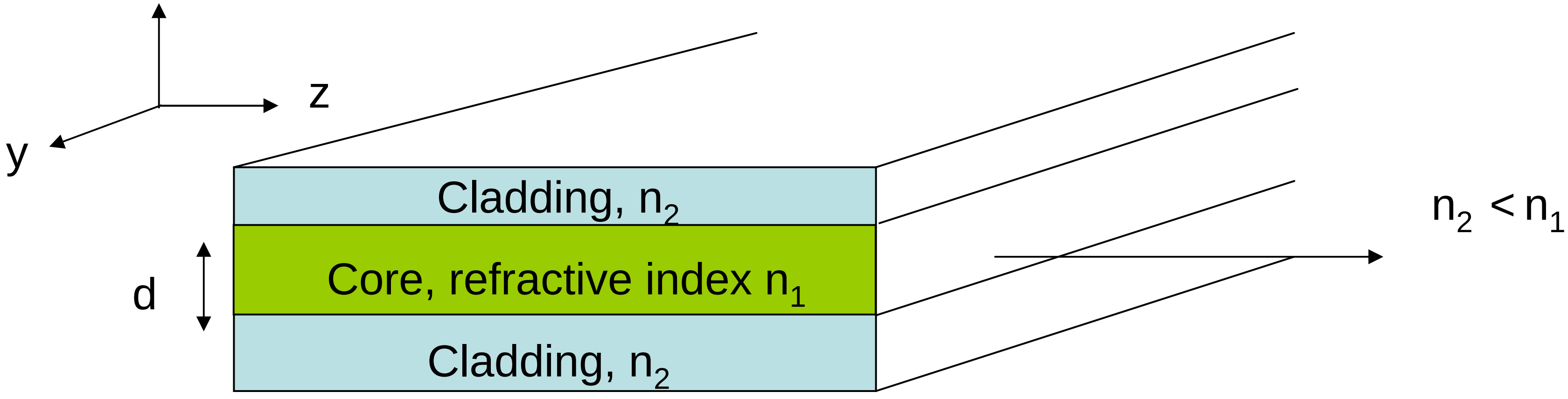


TE mode:

$$\vec{E} = \hat{y}E(x)$$

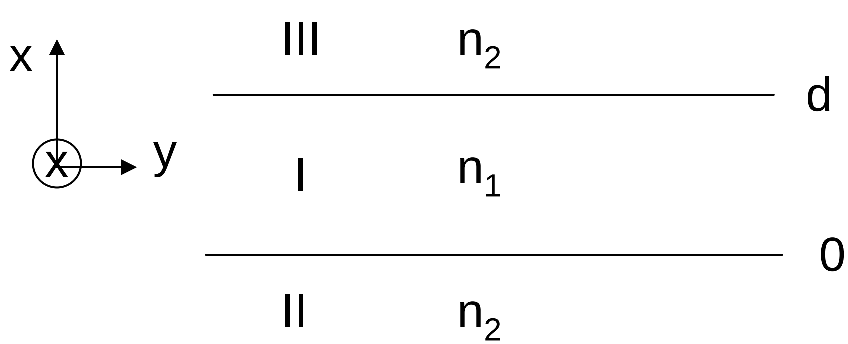
TM mode:

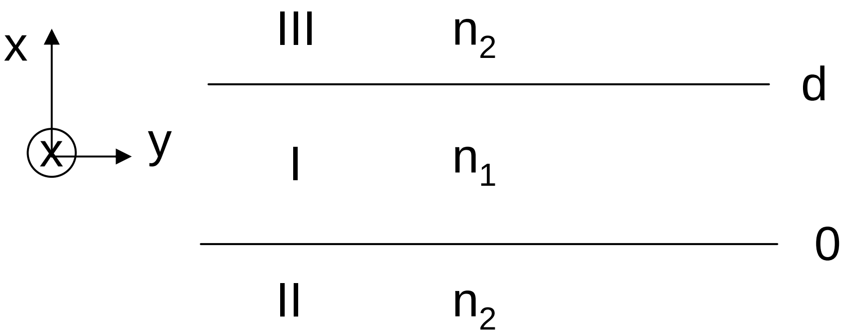
$$\vec{H} = \hat{y}H(x)$$



$$\nabla^2 \vec{E} + n^2 k_0^2 \vec{E} = 0 \qquad \vec{E} = \hat{y} E(x) e^{i\beta z}$$

$$\frac{d^2 E(x)}{dx^2} + (n^2 k_0^2 - \beta^2) E(x) = 0$$





$$\frac{d^2E(x)}{dx^2}+(n_2^2k_0^2-\beta^2)E(x)=0$$

$$\frac{d^2E(x)}{dx^2}+(n_1^2k_0^2-\beta^2)E(x)=0$$

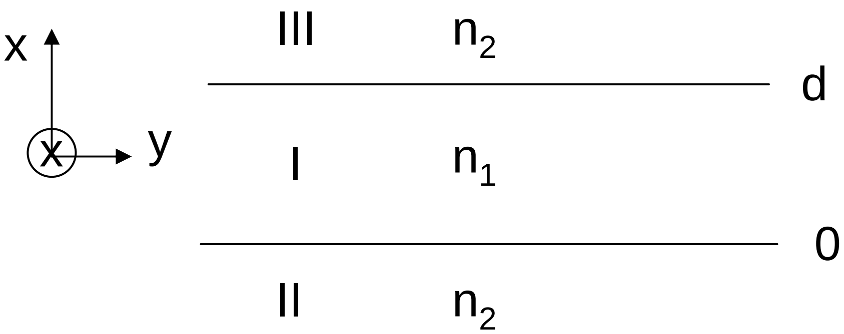
$$\frac{d^2E(x)}{dx^2}+(n_2^2k_0^2-\beta^2)E(x)=0$$

$$E_I(x)=A_I\cos\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)x\right]+B_I\sin\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)x\right]$$

$$E_{II}(x)=A_{II}\exp\left[\left(\sqrt{\beta^2-n_2^2k_0^2}\right)(x)\right]$$

$$E_{III}(x)=A_{III}\exp\left[-\left(\sqrt{\beta^2-n_2^2k_0^2}\right)(x-d)\right]$$

Wave optics approach



$$\frac{d^2E(x)}{dx^2} + (n_2^2k_0^2 - \beta^2)E(x) = 0$$

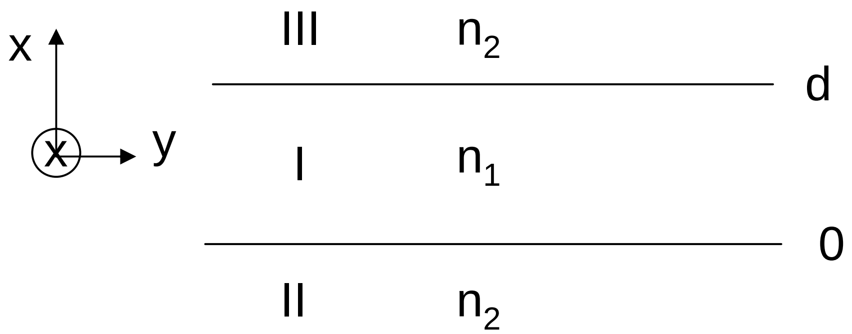
$$\frac{d^2E(x)}{dx^2} + (n_1^2k_0^2 - \beta^2)E(x) = 0$$

$$\frac{d^2E(x)}{dx^2} + (n_2^2k_0^2 - \beta^2)E(x) = 0$$

$$E_I(x) = A_I \cos\left[\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)x\right] + B_I \sin\left[\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)x\right]$$

$$E_{II}(x) = A_{II} \exp\left[\left(\sqrt{\beta^2 - n_2^2k_0^2}\right)(x + d/2)\right]$$

$$E_{III}(x) = A_{III} \exp\left[-\left(\sqrt{\beta^2 - n_2^2k_0^2}\right)(x - d/2)\right]$$



$$E_I(x) = E_{III}(x)\,|_{x=d}$$

$$\frac{d}{dx}E_I(x) = \frac{d}{dx}E_{III}(x)\,|_{x=d}$$

$$E_I(x) = E_{II}(x)\,|_{x=0}$$

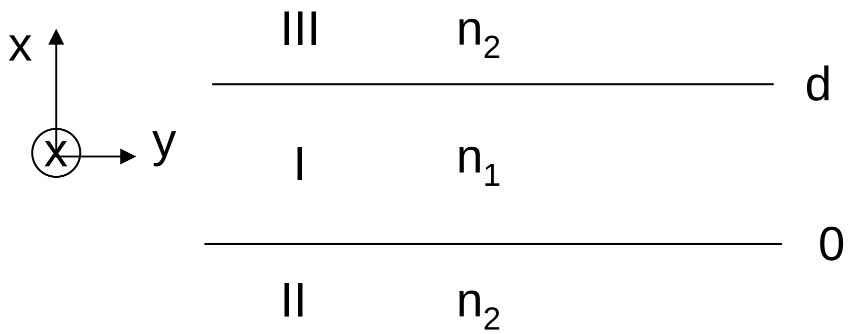
$$\frac{d}{dx}E_I(x) = \frac{d}{dx}E_{II}(x)\,|_{x=0}$$

$$E_I(x) = E_{II}(x)\,|_{x=0}$$

$$E_I(x) = A_I \cos\left[\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)(0)\right] + B_I \sin\left[\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)(0)\right]$$

$$= E_{II}(x) = A_{II} \exp\left[\left(\sqrt{\beta^2 - n_2^2k_0^2}\right)(0)\right]$$

$$A_{II} = A_I$$



$$E_I(x) = E_{III}(x)\,|_{x=d}$$

$$\frac{d}{dx}E_I(x) = \frac{d}{dx}E_{III}(x)\,|_{x=d}$$

$$E_I(x) = E_{II}(x)\,|_{x=0}$$

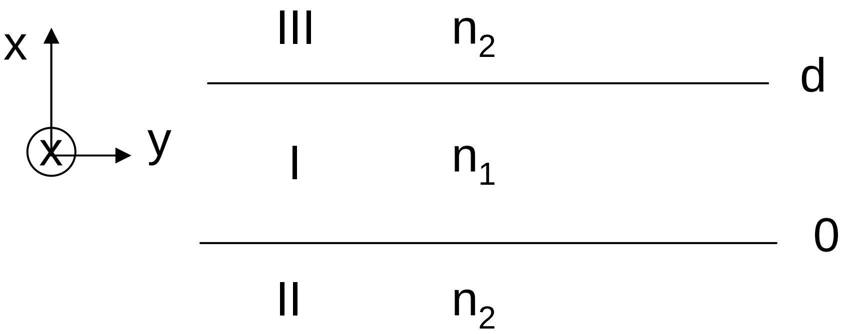
$$\frac{d}{dx}E_I(x) = \frac{d}{dx}E_{II}(x)\,|_{x=0}$$

$$\frac{d}{dx}E_I(x) = \frac{d}{dx}E_{II}(x)\,|_{x=0}$$

$$A_{II}\left(\sqrt{\beta^2 - n_2^2k_0^2}\right) = -\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)A_I \sin\left[\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)(0)\right] + B_I\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)\cos\left[\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)(0)\right]$$

$$B_I = A_{II}\,\frac{\left(\sqrt{\beta^2 - n_2^2k_0^2}\right)}{\left(\sqrt{n_1^2k_0^2 - \beta^2}\right)}$$

Wave optics approach



$$E_I(x) = E_{III}(x)\,|_{x=d}$$

$$\frac{d}{dx}E_I(x) = \frac{d}{dx}E_{III}(x)\,|_{x=d}$$

$$E_I(x) = E_{II}(x)\,|_{x=0}$$

$$\frac{d}{dx}E_I(x) = \frac{d}{dx}E_{II}(x)\,|_{x=0}$$

$$A_{III} = A_I \cos\left[\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)(d)\right] + B_I \sin\left[\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)(d)\right]$$

$$-A_{III}\left(\sqrt{\beta^2 - n_2^2 k_0^2}\right) = -\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)A_I \sin\left[\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)(d)\right] + B_I\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)\cos\left[\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)(d)\right]$$

$$\frac{\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)}{\left(\sqrt{\beta^2 - n_2^2 k_0^2}\right)} = \frac{A_I \cos\left[\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)(d)\right] + B_I \sin\left[\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)(d)\right]}{A_I \sin\left[\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)(d)\right] - B_I \cos\left[\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)(d)\right]}$$

$$B_I = A_{II}\,\frac{\left(\sqrt{\beta^2 - n_2^2 k_0^2}\right)}{\left(\sqrt{n_1^2 k_0^2 - \beta^2}\right)}$$

$$\frac{\left(\sqrt{n_1^2k_0^2-\beta^2}\right)}{\left(\sqrt{\beta^2-n_2^2k_0^2}\right)}=\frac{A_I\cos\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)(d)\right]+B_I\sin\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)(d)\right]}{A_I\sin\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)(d)\right]-B_I\cos\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)(d)\right]}$$

$$B_I=A_I\frac{\left(\sqrt{\beta^2-n_2^2k_0^2}\right)}{\left(\sqrt{n_1^2k_0^2-\beta^2}\right)}$$

$$\frac{\left(\sqrt{n_1^2k_0^2-\beta^2}\right)}{\left(\sqrt{\beta^2-n_2^2k_0^2}\right)}=\frac{\cos\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)(d)\right]+\frac{\left(\sqrt{\beta^2-n_2^2k_0^2}\right)}{\left(\sqrt{n_1^2k_0^2-\beta^2}\right)}\sin\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)(d)\right]}{\sin\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)(d)\right]-\frac{\left(\sqrt{\beta^2-n_2^2k_0^2}\right)}{\left(\sqrt{n_1^2k_0^2-\beta^2}\right)}\cos\left[\left(\sqrt{n_1^2k_0^2-\beta^2}\right)(d)\right]}$$

$$\frac{h}{q}=\frac{h+q\tan(hd)}{h\tan(hd)-q}\qquad h=\sqrt{n_1^2k_0^2-\beta^2}\qquad q=\sqrt{\beta^2-n_2^2k_0^2}$$

$$\tan(hd)=\frac{2hq}{h^2-q^2}=\frac{2q}{h(1-q^2/h^2)}$$

Graphic solution

Dispersion

$$A(z) = e^{i(\beta z - \omega t)} = e^{i((\beta_0 + \frac{d\beta}{d\omega}\Delta\omega)z - \omega t)} = e^{i((\beta_0 z - \omega t) + \frac{d\beta}{d\omega}|_{\omega_0} Z\Delta\omega)} = e^{i((\beta_0 z - \omega t) + \frac{Z}{v_g}\Delta\omega)}$$

$$v_g = \frac{d\omega}{d\beta}|_{\omega_0} = \left(\frac{d\beta}{d\omega}\right)^{-1}$$

$A(t)$ Time delay $A(t-\tau) = A(t - \frac{Z}{v})$

$$A(\omega) = A(\omega)e^{i\frac{Z}{v}\omega}$$

• Material dispersion

$$v_g = \frac{d\omega}{d\beta}\Big|_{\omega_0}, \quad \tau_g = \frac{L}{v_g}, \quad \frac{\Delta\tau_g}{L} = \Delta(\frac{d\beta}{d\omega}) = \frac{\lambda}{c}(\frac{d^2n}{d\lambda^2})\Delta\lambda,$$

$$D_m = \frac{\lambda}{c}(\frac{d^2n}{d\lambda^2}), \qquad \Delta\tau_g = D_m L\Delta\lambda,$$

Example --- material dispersion

Calculate the material dispersion effect for LED with line width of 100nm and a laser with a line width of 2nm for a fiber with dispersion coefficient of Dm = 22pskm-1nm-1 at 1310nm.

Solution:

$\Delta\tau = D_m\Delta\lambda L = 2.2ns,$ for the LED

$\Delta\tau = D_m\Delta\lambda L = 44\,ps,$ for the Laser

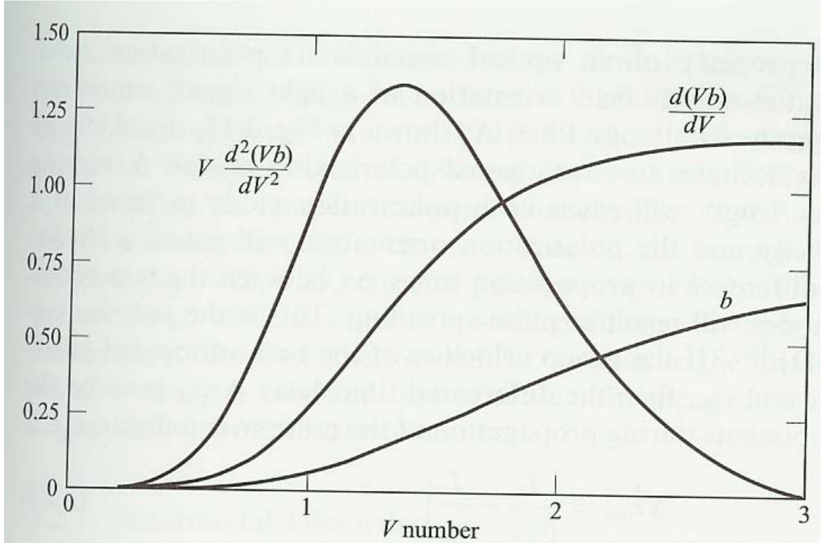
• Waveguide dispersion

$$v_g = \frac{d\omega}{d\beta}\Big|_{\omega_0}, \quad \tau_g = \frac{L}{v_g}, \qquad \frac{\Delta\tau_g}{L} = \Delta(\frac{d\beta}{d\omega}) = -\frac{n_2(n_1-n_2)\Delta\lambda}{c\lambda} V \frac{d^2(Vb)}{dV^2},$$

$$\frac{\Delta\tau_g}{L} = \Delta(\frac{d\beta}{d\omega}) \approx \frac{1.984 N_{g2}}{(2\pi a)^2 2cn_2^2}, \quad D_w = -\frac{n_2(n_1-n_2)}{c\lambda} V \frac{d^2(Vb)}{dV^2}, \qquad \Delta\tau_g = D_m L \Delta\lambda,$$

Example --- waveguide dispersion

n2 = 1.48, and delta n = 0.2 percent. Calculate Dw at 1310nm.



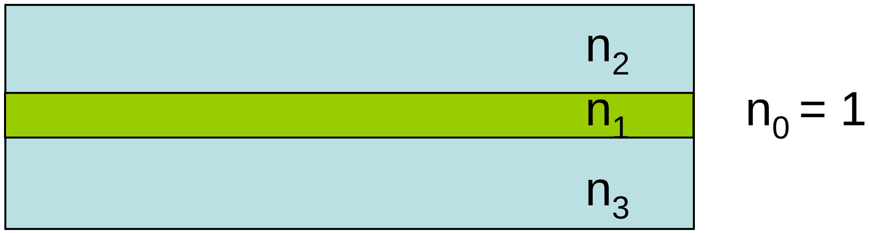
Solution:

$$b \approx (1.1428 - 0.996/V)^2, \text{ for V between } 1.5 - 2.5.$$

$$V\frac{d^2(Vb)}{dV^2} = 0.26,$$

$$D_w = -\frac{n_2(n_1-n_2)}{c\lambda} V \frac{d^2(Vb)}{dV^2} = -1.9\,ps/(nm\cdot km),$$

• <u>Waveguide mode dispersion</u>

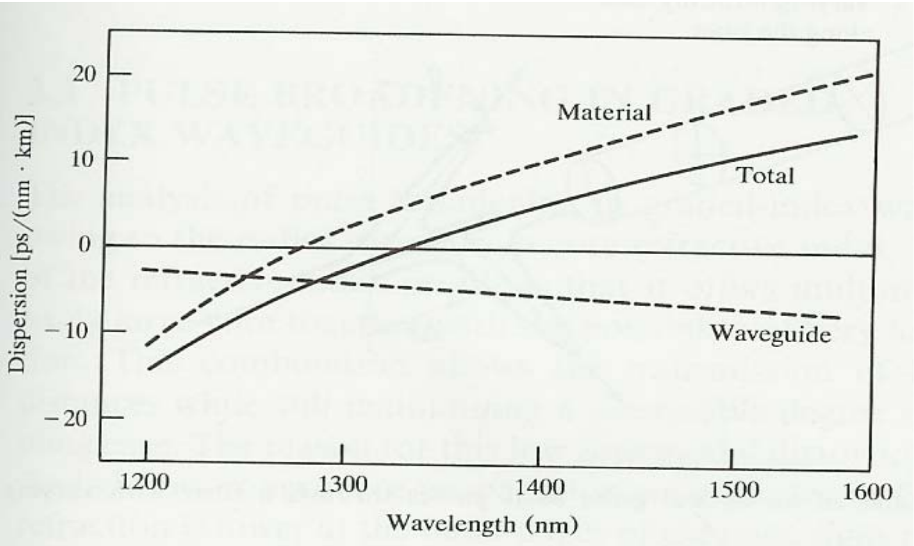


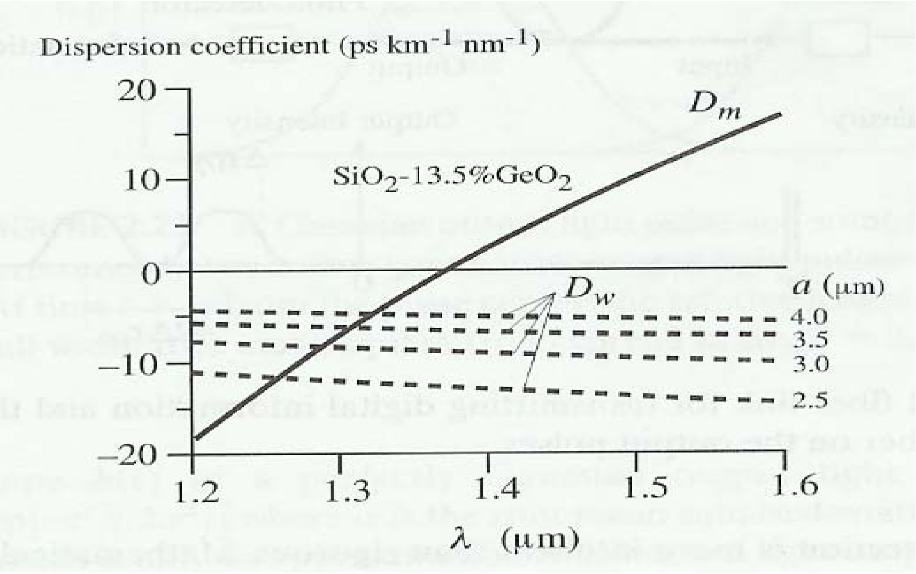
Higher order mode, $v_g = \frac{d\omega}{d\beta}\Big|_{\omega_0} \sim \frac{c}{n_2}$

Lower order mode, $v_g = \frac{d\omega}{d\beta}\Big|_{\omega_0} \sim \frac{c}{n_1}$

$$\frac{\Delta\tau_g}{L} = 1/(\frac{c}{n_2} - \frac{c}{n_1})$$

• chromatic dispersion (material plus waveduide dispersion)



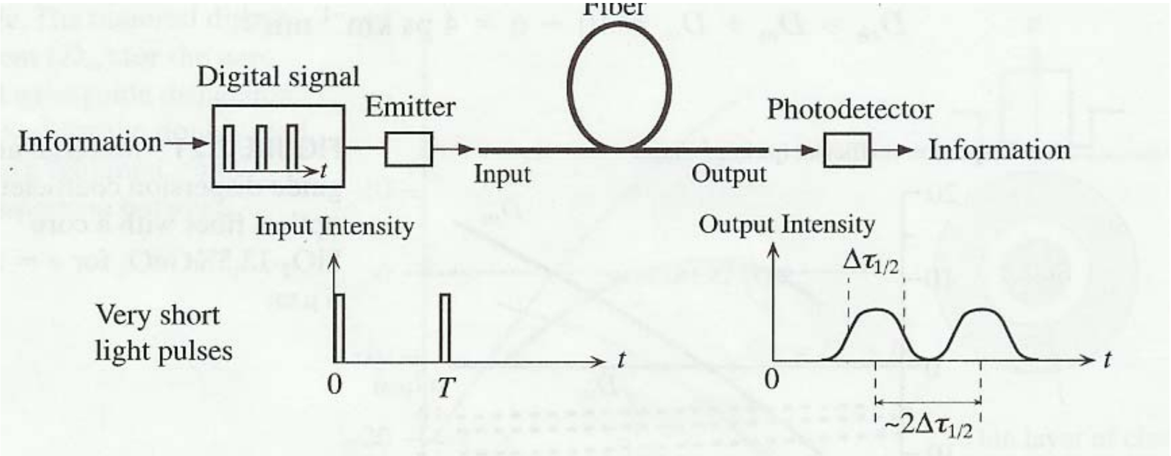


$$\frac{\Delta \tau_g}{L} = (D_m + D_w)\Delta\lambda,$$

• material dispersion is determined by the material composition of a fiber.

• waveguide dispersion is determined by the waveguide index profile of a fiber

• Dispersion induced limitations



• For RZ bit With no intersymbol interference

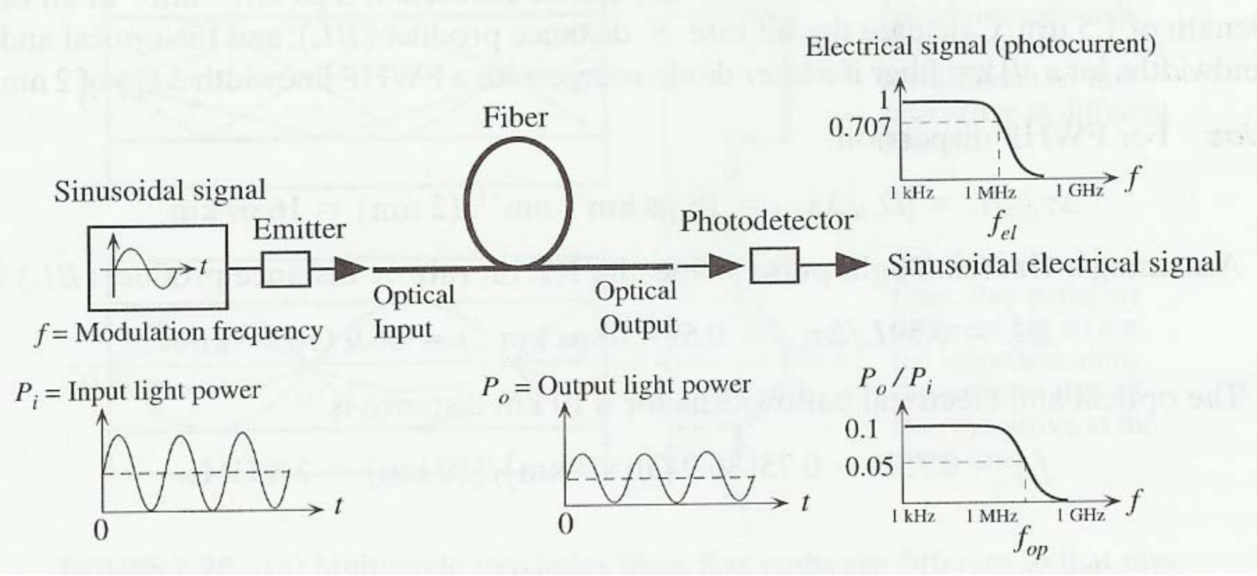
$$B \approx \frac{1}{2\Delta\tau_{1/2}},$$

• For NRZ bit With no intersymbol interference

$$B \approx \frac{1}{\Delta\tau_{1/2}},$$

Dispersion induced limitations

• Optical and Electrical Bandwidth



$$B \approx \frac{1}{2\Delta\tau_{1/2}}, \quad f_{3dB} \approx 0.7B,$$

• Bandwidth length product

$$BL \approx \frac{0.25}{D\Delta\lambda},$$

Dispersion induced limitations

Example --- bit rate and bandwidth

Calculate the bandwidth and length product for an optical fiber with chromatic dispersion coefficient 8pskm-1nm-1 and optical bandwidth for 10km of this kind of fiber and linewidth of 2nm.

Solution:

$$\Delta\tau_{1/2} / L = D\Delta\lambda = 16\,pskm^{-1}, \quad BL \approx \frac{0.25}{D\Delta\lambda} = 36.9Gbs^{-1}km,$$

$$f_{3dB} \approx 0.7B = 2.8GHz,$$

• Fiber limiting factor absorption or dispersion?

$$Loss \approx 0.25dB \cdot 10km = 2.5dB,$$