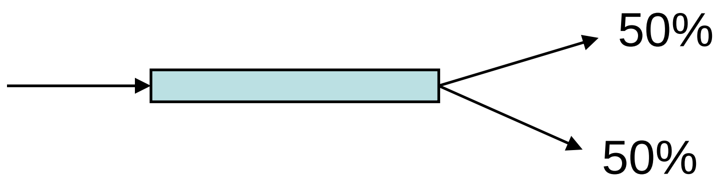
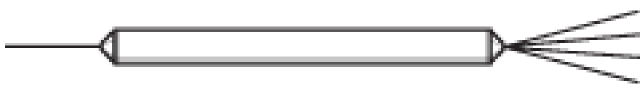
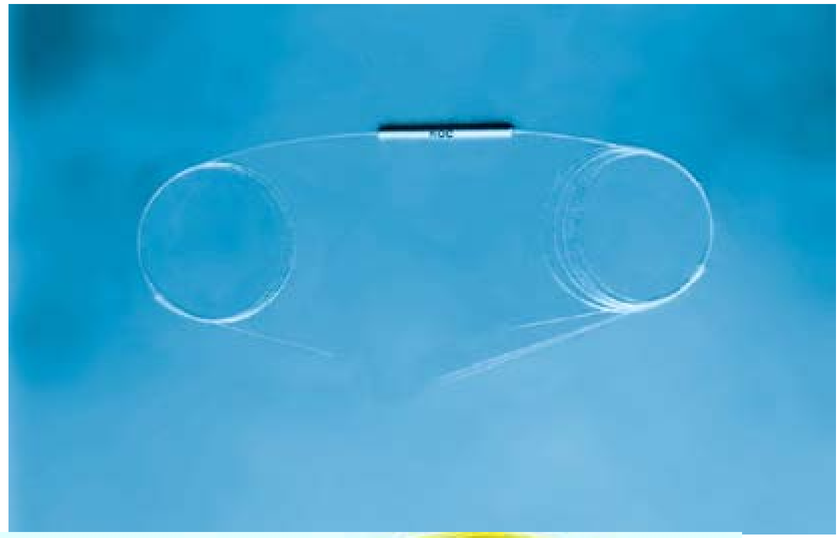
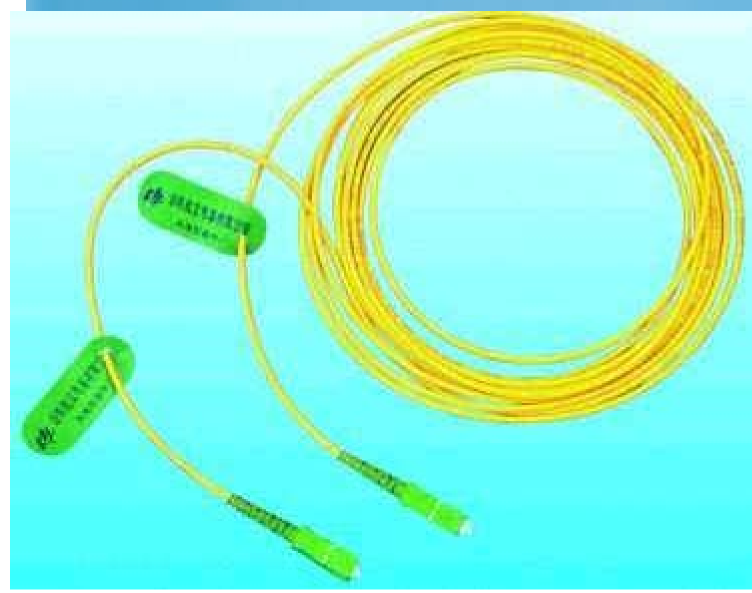
3-dB Couplers/splitters

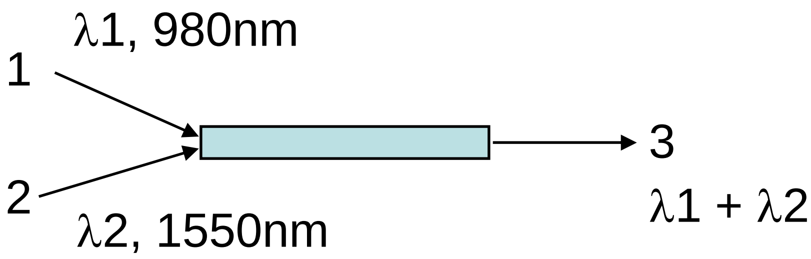


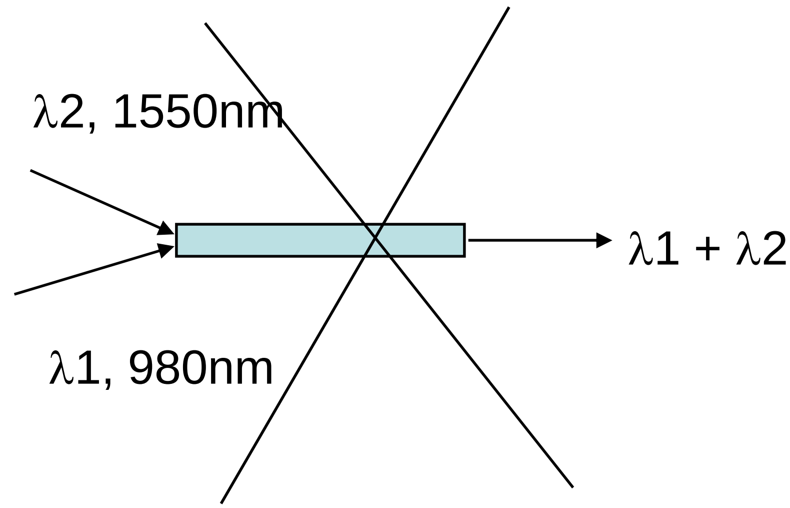






WDM Couplers/splitters





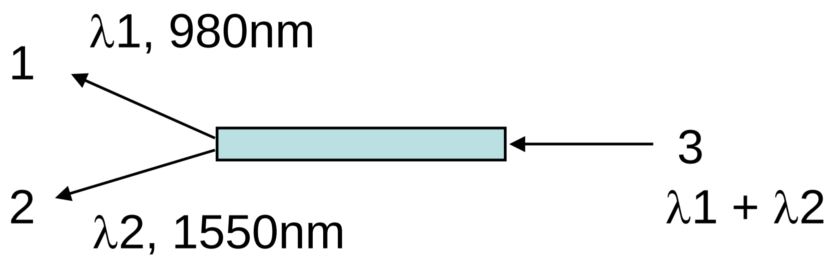
λ1, 980nm

	Insertion loss
Port 1 to 3	0.2dB
Port 2 to 3	20dB

λ2, 1550nm

	Insertion loss
Port 1 to 3	20dB
Port 2 to 3	0.3dB

WDM Couplers/splitters



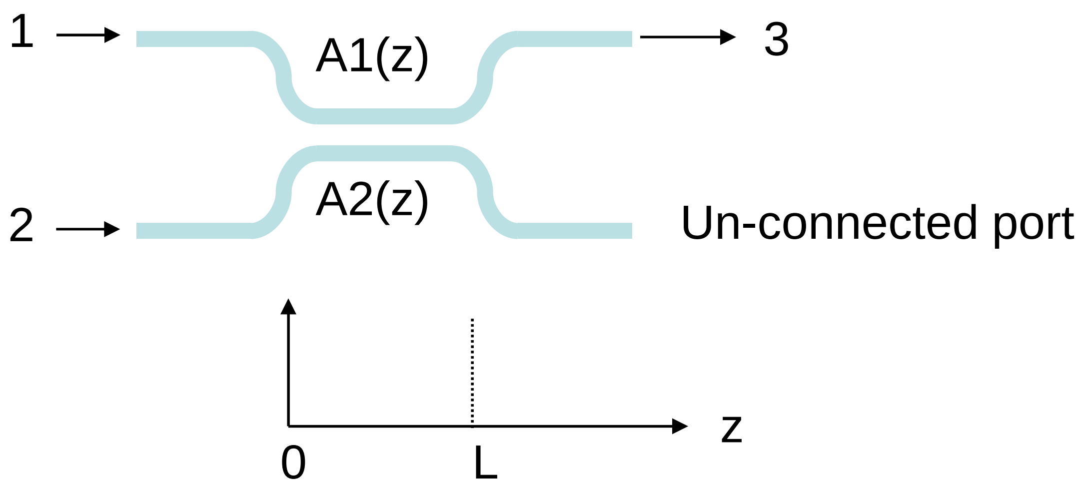
λ1, 980nm

	Insertion loss
Port 3 to 1	0.2dB
Port 3 to 2	20dB

λ2, 1550nm

	Insertion loss
Port 3 to 1	20dB
Port 3 to 2	0.3dB

Couplers/splitters, working principle – optical mode coupling



$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)$$

$$\frac{dA_2(z)}{dz} = i\kappa A_1(z)$$

Without coupling

$$\frac{dA_1(z)}{dz} = 0 \qquad A_1(z) = A_1(0)$$

$$\frac{dA_2(z)}{dz} = 0 \qquad A_2(z) = A_2(0)$$

Mode-coupling equations, phase matched case

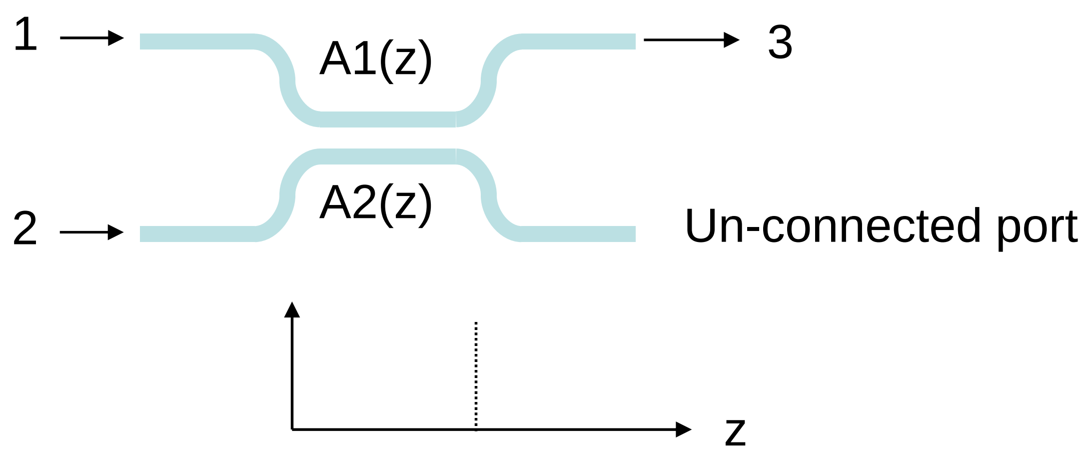
Couplers/splitters, working principle – optical mode coupling

$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)$$

$$\frac{dA_2(z)}{dz} = i\kappa A_1(z)$$

$$\frac{d\,|A_1(z)|^2}{dz} = A_1^*(z)\frac{dA_1(z)}{dz} + A_1(z)\frac{dA_1^*(z)}{dz}$$
$$= A_1^*(z)i\kappa A_2(z) + A_1(z)(-i\kappa A_2^*(z))$$

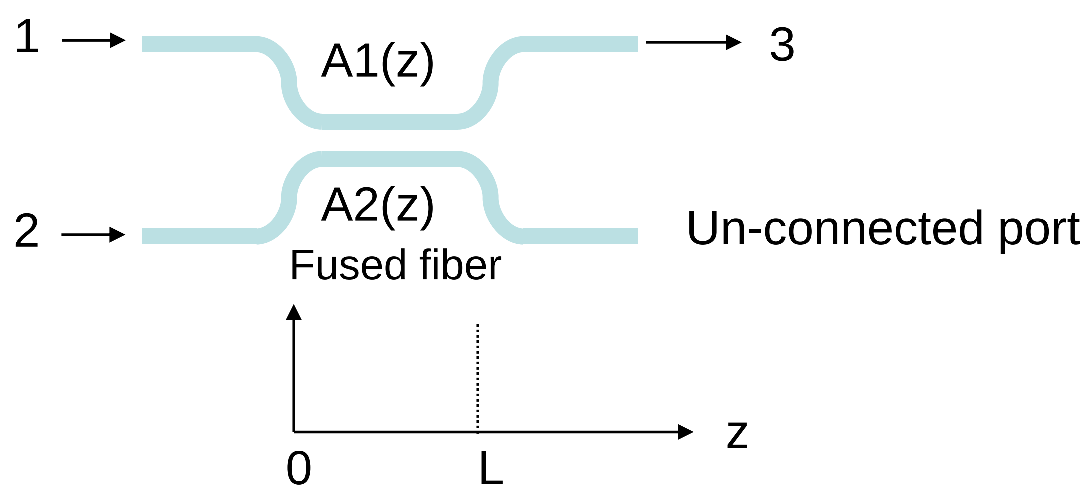
$$\frac{d\,|A_2(z)|^2}{dz} = A_2^*(z)\frac{dA_2(z)}{dz} + A_2(z)\frac{dA_2^*(z)}{dz}$$
$$= A_2^*(z)(i\kappa A_1(z) + A_2(z)(-i\kappa A_1^*(z))$$



$$\frac{d}{dz}\left(\left|A_1(z)\right|^2 + \left|A_2(z)\right|^2\right) = 0$$

Energy conservation

WDM Couplers/splitters, working principle – optical mode coupling



$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)$$

$$\frac{dA_2(z)}{dz} = i\kappa A_1(z)$$

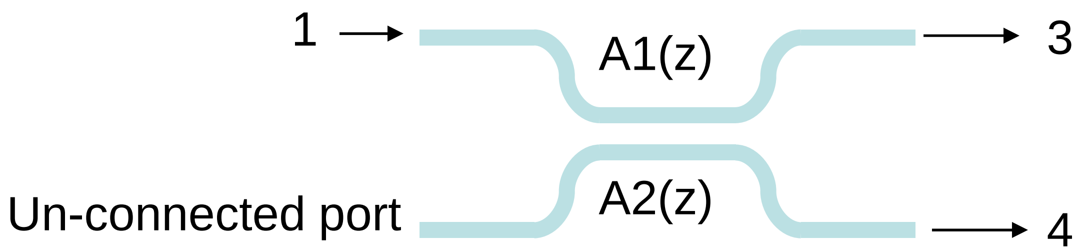
$$\frac{d^2 A_1(z)}{dz^2} = i\kappa \frac{d}{dz} A_2(z) = -\kappa^2 A_1(z)$$

$$A_1(z) = A\cos(\kappa z) + B\sin(\kappa z)$$

$$A_2(z) = C\cos(\kappa z) + D\sin(\kappa z)$$

A, B, C, D are determined by the initial conditions

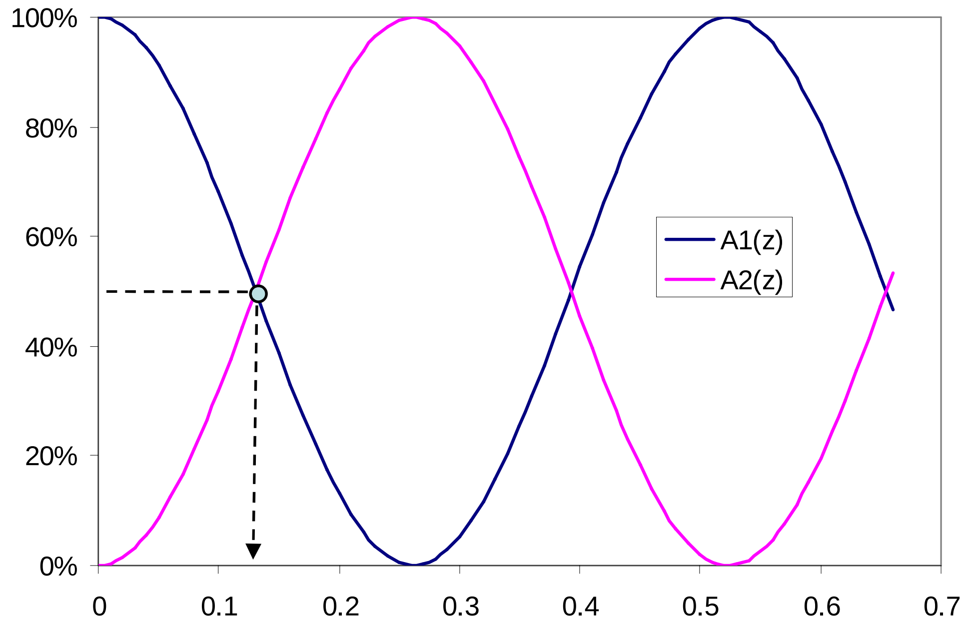
Splitter



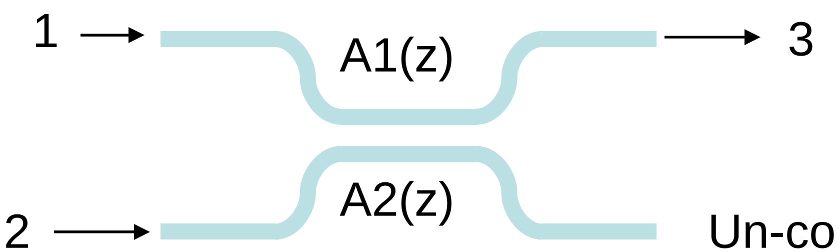
For A1(0) = A1, A2(0) =0

$$A_1(z) = A_1(0)\cos(\kappa z)$$

$$A_2(z) = iA_1(0)\sin(\kappa z)$$



2 to 1 combiner



Un-connected port

Initially, A1(0), A2(0)
A1(0) = A2(0)

$$A_1(z) = A_1(0)\cos(\kappa z) + B\sin(\kappa z)$$

$$\frac{dA_1(z)}{dz} = i\kappa A_2(z) \longrightarrow A_2(z) = iA_1(0)\sin(\kappa z) - iB\cos(\kappa z)\Big|_{z=0}$$

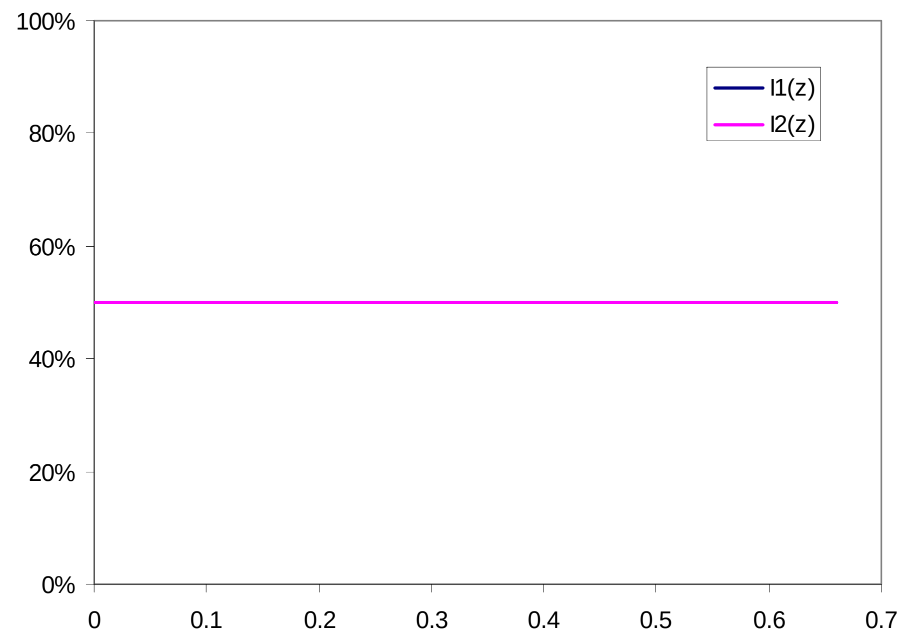
$$B = iA_2(0)$$

$$A_2(z) = A_2(0)\cos(\kappa z) + D\sin(\kappa z)$$

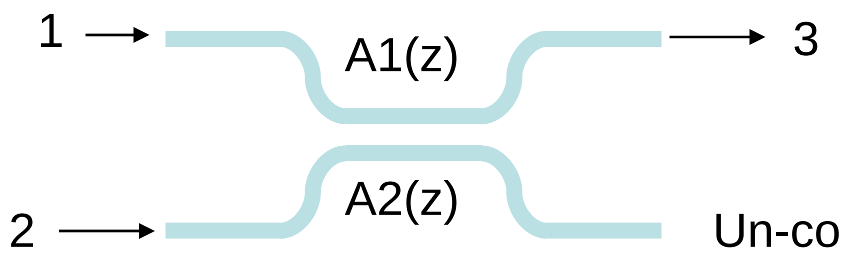
Similarly

$$D = iA_1(0)$$

$$A_1(z) = A_1(0)\cos(\kappa z) + iA_2(0)\sin(\kappa z)$$



2 to 1 combiner



Un-connected port

Initially, A1(0), A2(0)
A1(0) not eq. A2(0)

$$A_1(z) = A_1(0)\cos(\kappa z) + B\sin(\kappa z)$$

$$\frac{dA_1(z)}{dz} = i\kappa A_2(z) \longrightarrow A_2(z) = iA_1(0)\sin(\kappa z) - iB\cos(\kappa z)\Big|_{z=0}$$

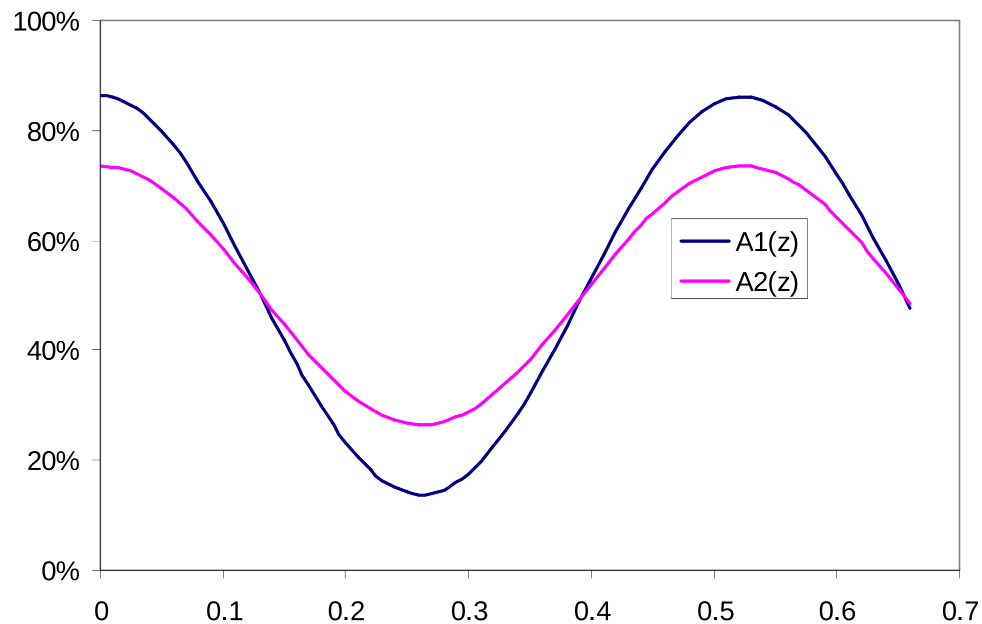
$$B = iA_2(0)$$

$$A_2(z) = A_2(0)\cos(\kappa z) + D\sin(\kappa z)$$

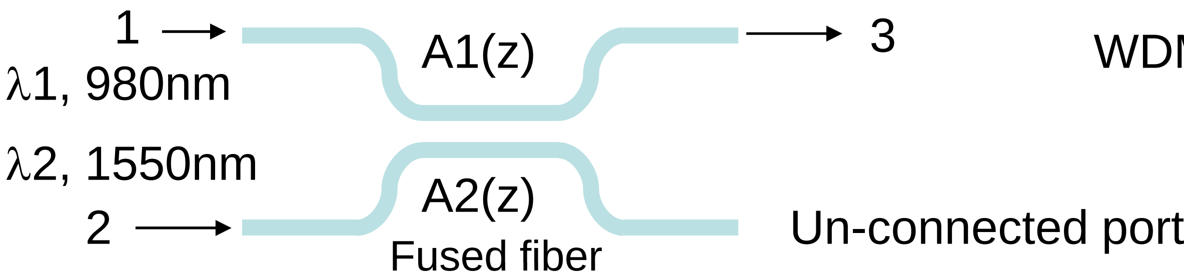
Similarly

$$D = iA_1(0)$$

$$A_1(z) = A_1(0)\cos(\kappa z) + iA_2(0)\sin(\kappa z)$$



WDM coupler



No coupling between two different wavelengths

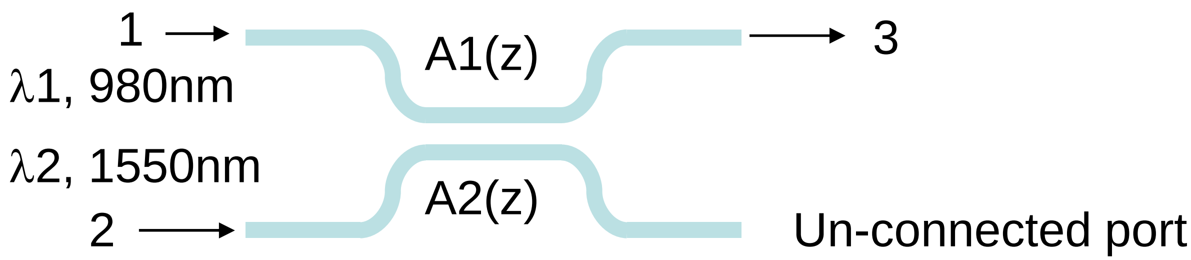
For λ1, 980nm Initially, A1(0, λ1), A2(0, λ1) =0

$$A_1(z) = A_1(0)\cos(\kappa_1 z) + B\sin(\kappa_1 z) \qquad \frac{dA_1(z)}{dz} = i\kappa A_2(z) \longrightarrow A_2(z) = iA_1(0)\sin(\kappa z) - iB\cos(\kappa z)\Big|_{z=0}$$

$$A_1(z) = A_1(0)\cos(\kappa_1 z)$$

$$A_2(z) = D\sin(\kappa z) \qquad \frac{dA_2(z)}{dz} = i\kappa A_1(z) \longrightarrow A_1(z) = iD\sin(\kappa z)\Big|_{z=0}$$

$$A_2(z) = iA_1(0)\sin(\kappa_1 z)$$



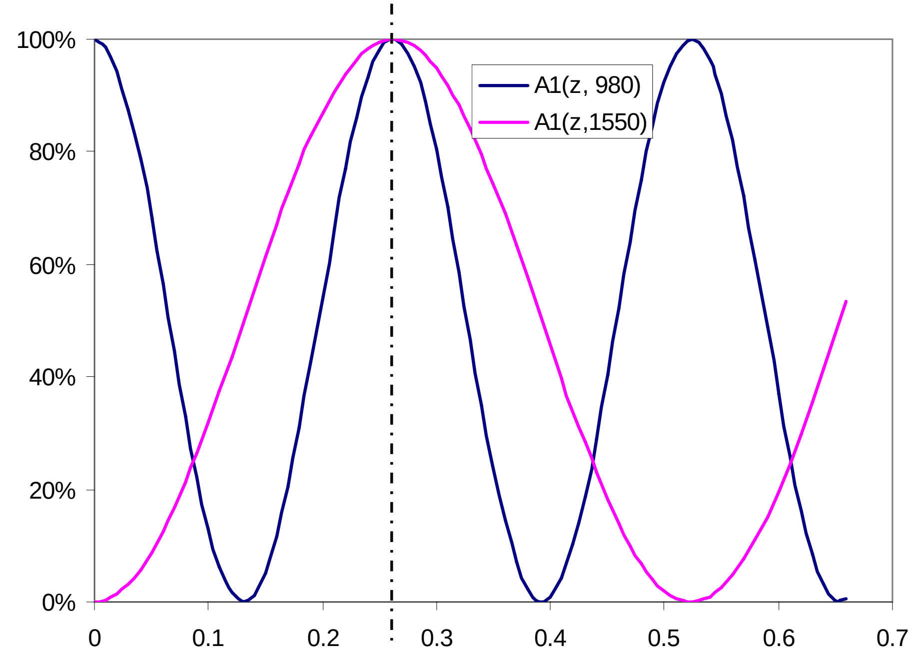
For λ2, 1550nm   Initially, A1(0, λ2)=0, A2(0, λ2)

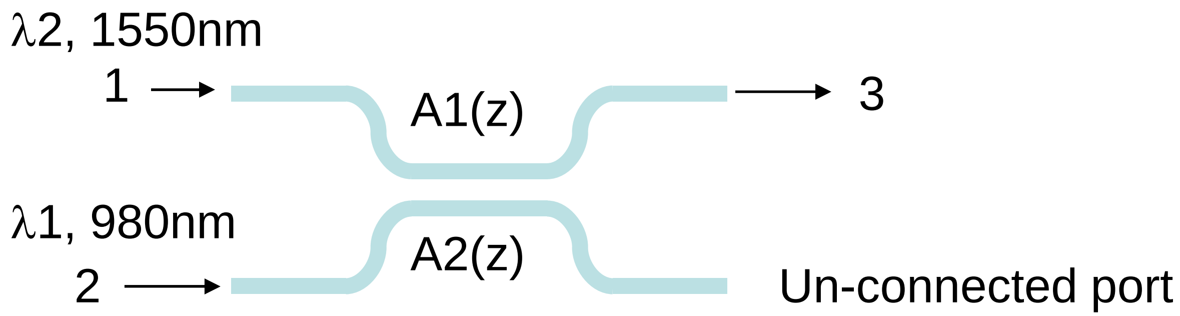
$$A_1(z) = B\sin(\kappa_2 z) \qquad \frac{dA_1(z)}{dz} = i\kappa A_2(z) \longrightarrow \qquad A_2(z) = iB\cos(\kappa z)$$

z = 0

$$A_1(z) = iA_2(0)\sin(\kappa_2 z)$$

$$A_2(z) = A_2(0)\cos(\kappa_2 z)$$



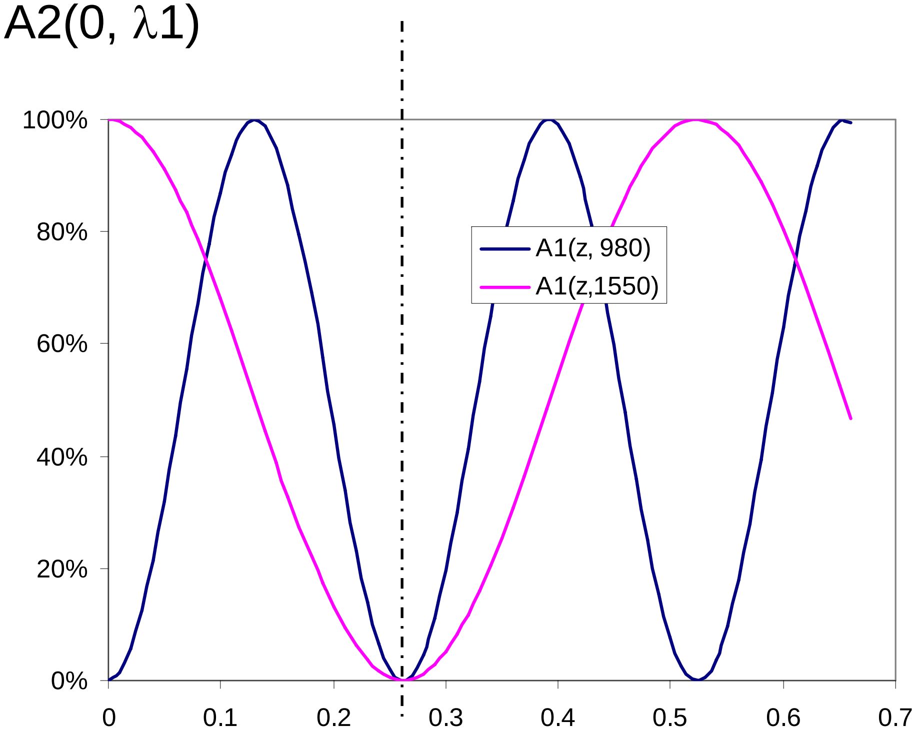


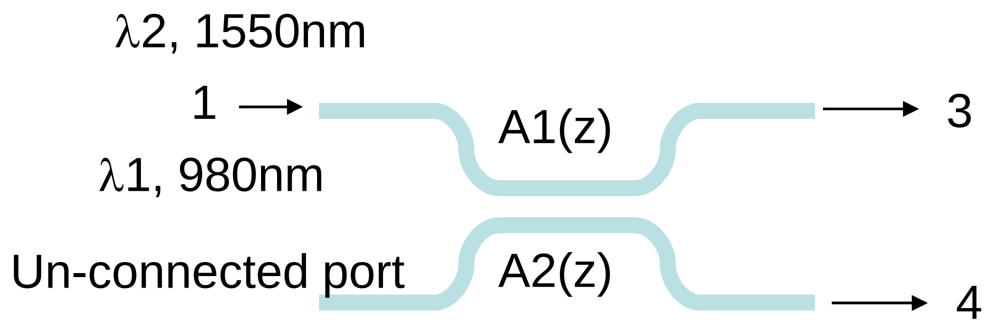
For λ2, 1550nm Initially, A1(0, λ2), A2(0, λ2)=0

$$A_1(z,1550) = A_1(0)\cos(\kappa_2 z)$$

For λ1, 980nm Initially, A1(0, λ1)=0, A2(0, λ1)

$$A_1(z,980) = iA_2(0,980)\sin(\kappa_1 z)$$





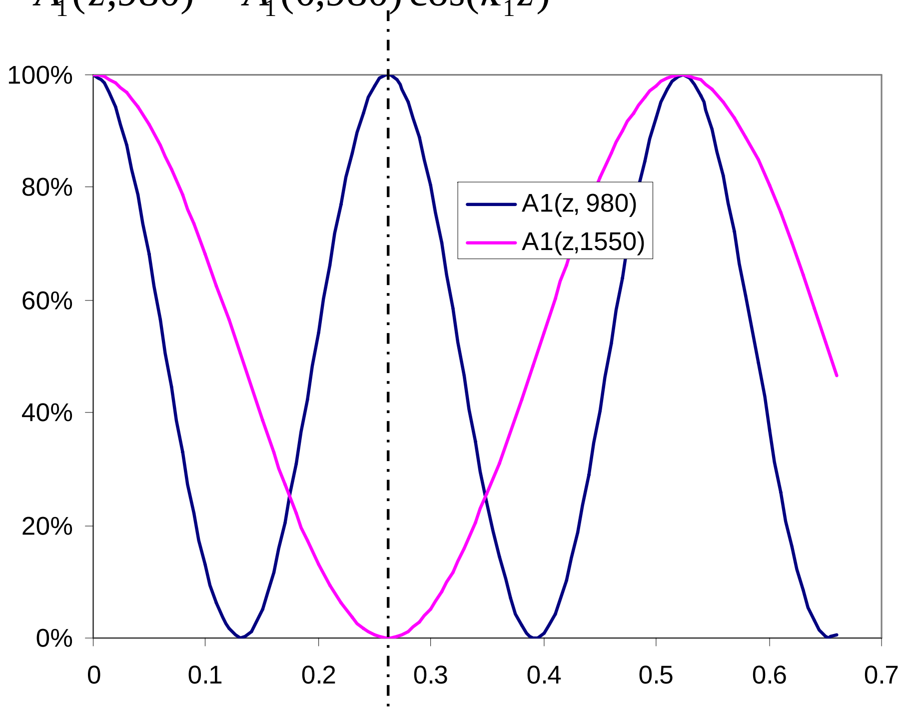
For λ2, 1550nm

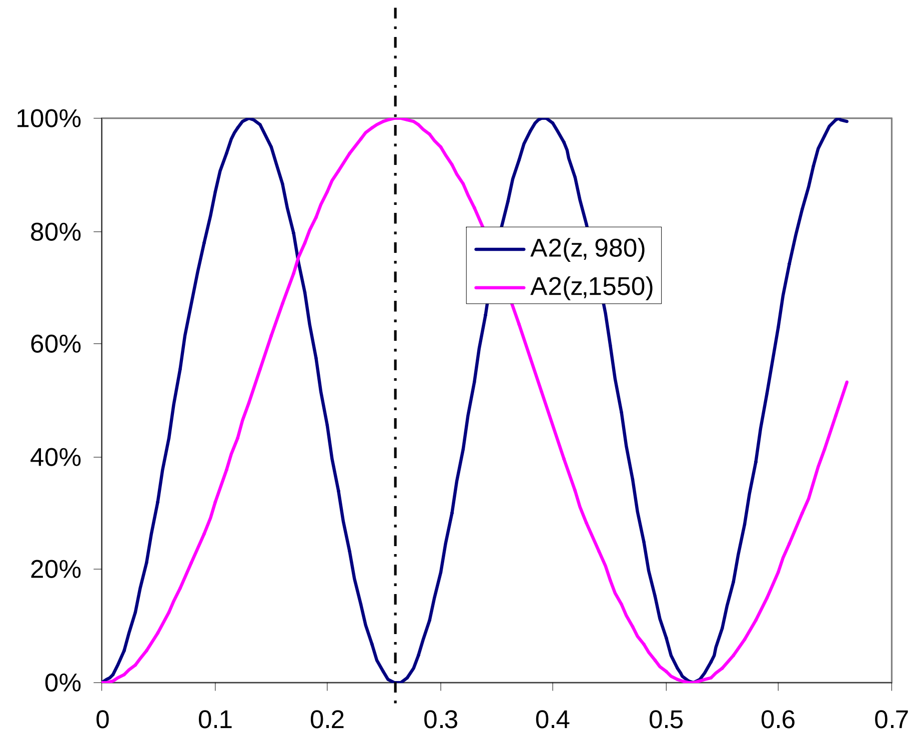
Initially, A1(0, λ2), A2(0, λ2)=0

$$A_1(z,1550) = A_1(0)\cos(\kappa_2 z)$$

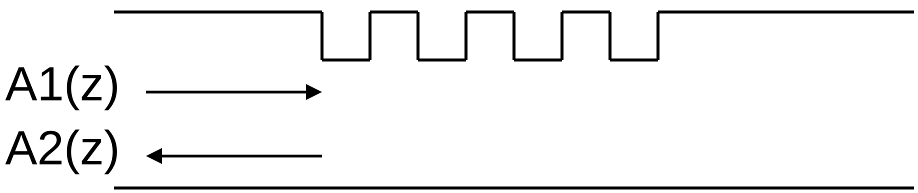
For λ1, 980nm Initially, A1(0, λ1), A2(0, λ1) = 0

$$A_1(z,980) = A_1(0,980)\cos(\kappa_1 z)$$





Waveguide gratings



$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)$$

$$\frac{dA_2(z)}{dz} = -i\kappa A_1(z)$$

$$\frac{d\,|A_1(z)|^2}{dz} = A_1^*(z)\frac{dA_1(z)}{dz} + A_1(z)\frac{dA_1^*(z)}{dz}$$

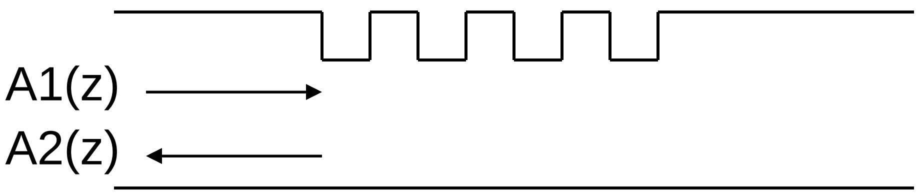
$$= A_1^*(z) i\kappa A_2(z) + A_1(z)(-i\kappa A_2^*(z))$$

$$\frac{d\,|A_2(z)|^2}{dz} = A_2^*(z)\frac{dA_2(z)}{dz} + A_2(z)\frac{dA_2^*(z)}{dz}$$

$$= A_2^*(z)(-i\kappa A_1(z) + A_2(z)(i\kappa A_1^*(z))$$

$$\frac{d}{dz}\left(\left|A_1(z)\right|^2 - \left|A_2(z)\right|^2\right) = 0$$

Waveguide gratings



$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)$$

$$\frac{dA_2(z)}{dz} = -i\kappa A_1(z)$$

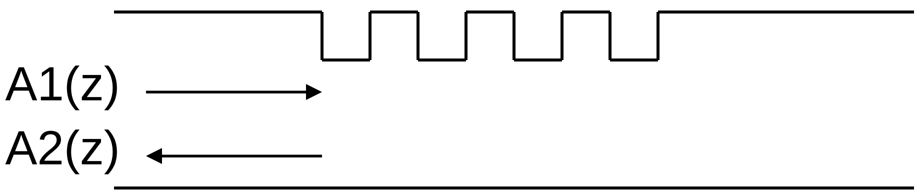
$$\frac{d^2 A_1(z)}{dz^2} = i\kappa \frac{d}{dz} A_2(z) = \kappa^2 A_1(z)$$

$$A_1(z) = A\cosh(\kappa z) + B\sinh(-\kappa z)$$

$$A_2(z) = C\cosh(\kappa z) + D\sinh(-\kappa z)$$

A, B, C, D are determined by the initial conditions

Waveguide gratings



$A1(z)|_{z=0} = A1(0)$

$A2(z)\ )|_{z=L} = 0$

$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)$$

$$\frac{dA_2(z)}{dz} = -i\kappa A_1(z)$$

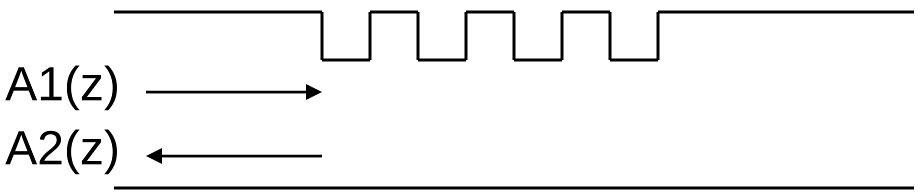
$$A_1(z) = A1(0)\cosh(\kappa z) + B\sinh(\kappa z)$$

$$A1(0)\sinh(\kappa L) + B\cos(\kappa zL) = 0$$

$$A_1(z) = A1(0)\left[\cosh(\kappa z) - \frac{\sinh(\kappa L)}{\cosh(\kappa L)}\sinh(\kappa z)\right] = A1(0)\frac{\cosh\kappa(z-L)}{\cosh(\kappa L)}$$

$$A_2(z) = A_1(0)\frac{\sinh\kappa(z-L)}{\cosh(\kappa L)}$$

Waveguide gratings



$A1(z)|_{z=0} = A1(0)$

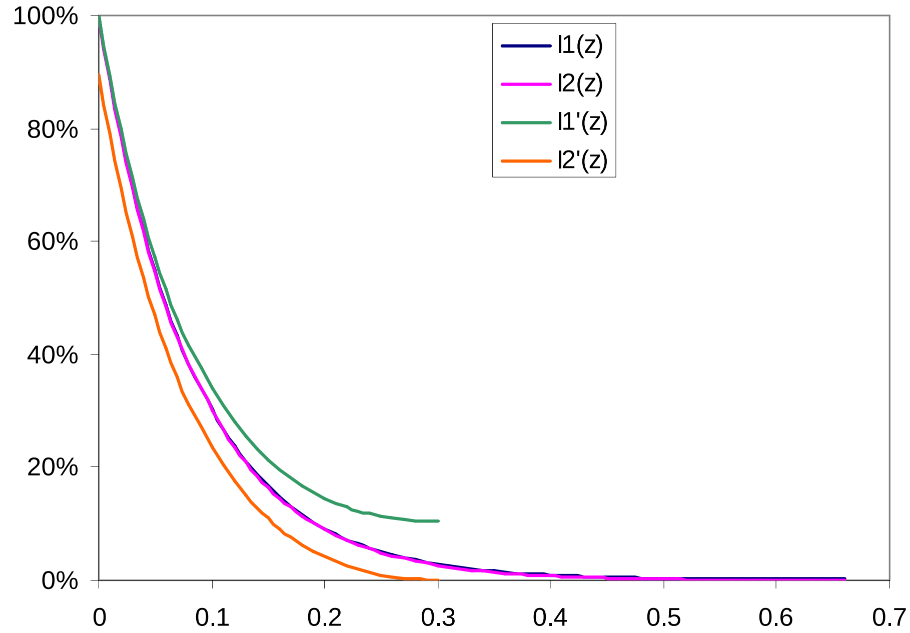
$A2(z)\,)|_{z=L} = 0$

$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)$$

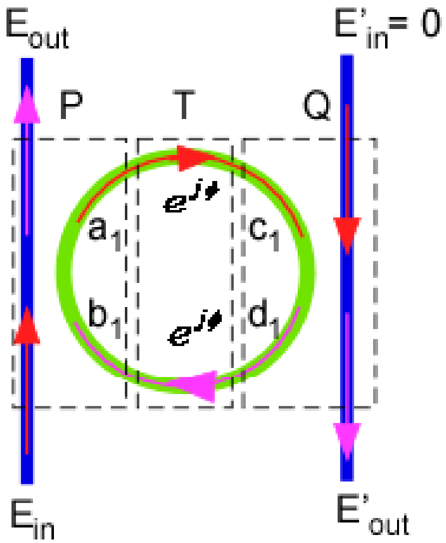
$$\frac{dA_2(z)}{dz} = -i\kappa A_1(z)$$

$$A_1(z) = A1(0)\frac{\cosh\kappa(z-L)}{\cosh(\kappa L)}$$

$$A_2(z) = A_1(0)\frac{\sinh\kappa(z-L)}{\cosh(\kappa L)}$$



Ring resonators



$$E_{out} = j\kappa b_1 + t^* E_{in},$$

$$a_1 = -j\kappa^* E_{in} - t b_1,$$

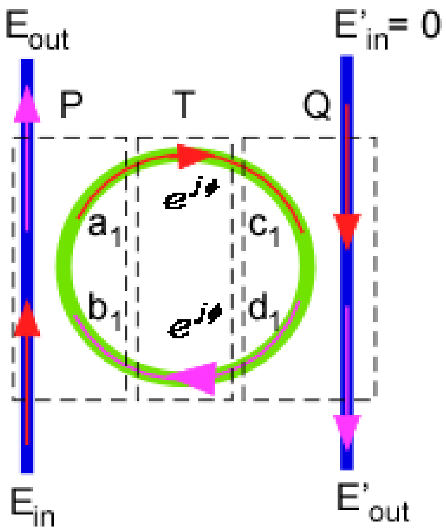
$$|E_{out}|^2 = (j\kappa b_1 + t^* E_{in})(-j\kappa^* b_1^* + t E_{in}^*)$$
$$= |\kappa|^2 |b_1|^2 + |E_{in}|^2 |t|^2 - j\kappa^* b_1^* t^* E_{in} + j\kappa b_1 t E_{in}^*$$

$$|a_1|^2 = (-j\kappa^* E_{in} - t b_1)(j\kappa E_{in}^* - t^* b_1^*)$$
$$= |\kappa|^2 |E_{in}|^2 + |b_1|^2 |t|^2 + j\kappa^* b_1^* t^* E_{in} - j\kappa b_1 t E_{in}^*$$

$$|E_{out}|^2 + |a_1|^2 = |\kappa|^2 |b_1|^2 + |E_{in}|^2 |t|^2 + |\kappa|^2 |E_{in}|^2 + |b_1|^2 |t|^2$$
$$= (|\kappa|^2 + |t|^2)(|E_{in}|^2 + |b_1|^2) = (|E_{in}|^2 + |b_1|^2)$$

$$|\kappa|^2 + |t|^2 = 1$$

Ring resonators



$$E_{out} = j\kappa b_1 + t^* E_{in},$$

$$a_1 = -j\kappa^* E_{in} - t b_1,$$

$$|\kappa|^2 + |t|^2 = 1$$

$$E_{in} = \frac{1}{-j\kappa^*}(a_1 + t b_1),$$

$$E_{out} = j\kappa b_1 + t^* \frac{1}{-j\kappa^*}(a_1 + t b_1) = \frac{1}{-j\kappa^*} t^* a_1 + |t|^2 b_1 + |\kappa|^2 b_1)$$

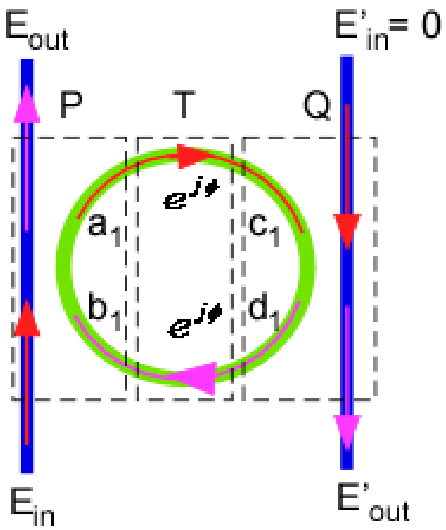
$$E_{out} = \frac{1}{-j\kappa^*}(t^* a_1 + b_1)$$

$$E_{in} = \frac{1}{-j\kappa^*}(a_1 + t b_1),$$

$$\begin{pmatrix} E_{out} \\ E_{in} \end{pmatrix} = \frac{1}{-j\kappa^*}\begin{pmatrix} t^* & 1 \\ 1 & t \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

P

Ring resonators



$$b_1 = d_1 e^{j\phi}$$

$$c_1 = a_1 e^{j\phi}$$

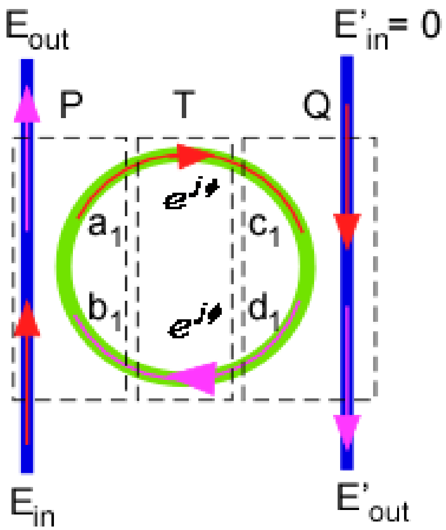
$$\phi = \beta(\pi R)$$

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} e^{-j\phi} & 0 \\ 0 & e^{j\phi} \end{pmatrix} \begin{pmatrix} c_1 \\ d_1 \end{pmatrix}$$

T

$$\begin{pmatrix} E_{out} \\ E_{in} \end{pmatrix} = \frac{1}{-j\kappa^*} \begin{pmatrix} t^* & 1 \\ 1 & t \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \frac{1}{-j\kappa^*} \begin{pmatrix} t^* & 1 \\ 1 & t \end{pmatrix} \begin{pmatrix} e^{-j\phi} & 0 \\ 0 & e^{j\phi} \end{pmatrix} \begin{pmatrix} c_1 \\ d_1 \end{pmatrix} = PT \begin{pmatrix} c_1 \\ d_1 \end{pmatrix}$$

Ring resonators



$$E'_{out} = -j\kappa^* c_1 + t^* E'_{in}\ ,$$

$$d_1 = j\kappa E'_{in} - t c_1,$$

$$|E'_{out}|^2 = |\kappa|^2 |c_1|^2 + |E'_{in}|^2 |t|^2 + j\kappa c_1^* t^* E_{in} - j\kappa^* c_1 t E'^*_{in}$$

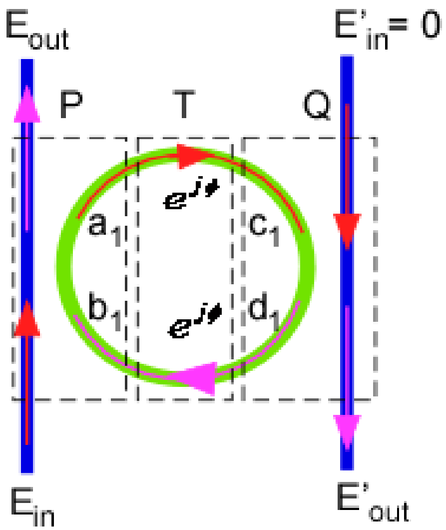
$$|d_1|^2 = |\kappa|^2 |E_{in}|^2 + |c_1|^2 |t|^2 + j\kappa^* c_1 t E'^*_{in} - j\kappa c_1^* t^* E'_{in}$$

$$c_1 = \frac{1}{-j\kappa^*}(t^* E'_{in} + E'_{out}),$$

$$d_1 = \frac{1}{-j\kappa^*}(|\kappa|^2 E'_{in} + |t| E'_{in} + t E'_{out}),$$

$$\begin{pmatrix} c_1 \\ d_1 \end{pmatrix} = \frac{1}{-j\kappa^*} \begin{pmatrix} t^* & 1 \\ 1 & t \end{pmatrix} \begin{pmatrix} E'_{in} \\ E'_{out} \end{pmatrix}$$

Ring resonators



$$\begin{pmatrix} E_{out} \\ E_{in} \end{pmatrix} = \frac{1}{-j\kappa^*}\begin{pmatrix} t^* & 1 \\ 1 & t \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \frac{1}{-j\kappa^*}\begin{pmatrix} t^* & 1 \\ 1 & t \end{pmatrix}\begin{pmatrix} e^{-j\phi} & 0 \\ 0 & e^{j\phi} \end{pmatrix}\begin{pmatrix} c_1 \\ d_1 \end{pmatrix}$$

$$= \frac{1}{-j\kappa^*}\begin{pmatrix} t^* & 1 \\ 1 & t \end{pmatrix}\begin{pmatrix} e^{-j\phi} & 0 \\ 0 & e^{j\phi} \end{pmatrix}\frac{1}{-j\kappa^*}\begin{pmatrix} t^* & 1 \\ 1 & t \end{pmatrix}\begin{pmatrix} E'_{in} \\ E'_{out} \end{pmatrix}$$

$$\begin{pmatrix} E_{out} \\ E_{in} \end{pmatrix} = PTP\begin{pmatrix} E'_{in} \\ E'_{out} \end{pmatrix} = M\begin{pmatrix} E'_{in} \\ E'_{out} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} E'_{in} \\ E'_{out} \end{pmatrix}$$

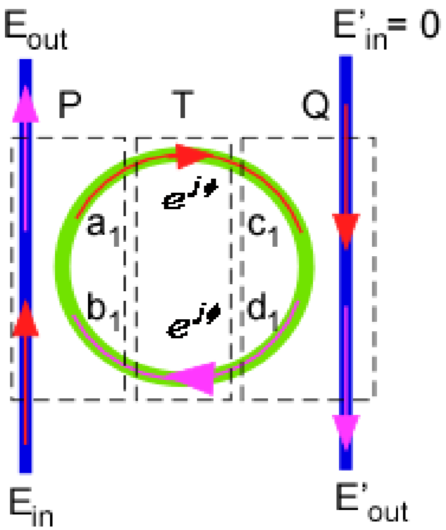
$$E_{out} = M_{11}E'_{in} + M_{12}E'_{out} = M_{12}E'_{out}$$

$$\frac{E'_{out}}{E_{out}} = \frac{1}{M_{12}} \qquad \frac{|E'_{out}|^2}{|E_{out}|^2} = \left|\frac{1}{M_{12}}\right|^2$$

$$E_{in} = M_{21}E'_{in} + M_{22}E'_{out} = M_{22}E'_{out}$$

$$\frac{E_{out}}{E_{in}} = \frac{M_{12}}{M_{22}} \qquad \left|\frac{E_{out}}{E_{in}}\right|^2 = \left|\frac{M_{12}}{M_{22}}\right|^2$$

Ring resonators




```
n1 = 1;
lamda = 1.538:0.0000001:1.539; %in um
beta = 2*pi./lamda;
R = 10/(2*pi*n1);
kapa = 0.005;
F = 1/kapa^2;
pha = beta*pi*R*n1;
t = sqrt(1-kapa^2);
P(1,1) = t;
P(1,2) = 1;
P(2,1) = 1;
P(2,2) = t;
P = P/kapa;
for m = 1:length(lamda)
   T(1,1) = exp(-j*pha(m));
   T(1,2) = 0;
   T(2,1) = 0;
   T(2,2) = exp(j*pha(m));
   M = P*T*P;
   Tr(m) = (abs(M(1,2)/M(2,2)))^2;
   Trr(m) =  Tr(m)*(abs(1/M(1,2)))^2;
end
plot(lamda,Tr);
hold on;
plot(lamda,Trr,'r');
zoom on;
hold off;
```