Electro-optic effect:

$$\begin{cases} \nabla \bullet D = \rho, \\ \nabla \bullet B = 0, \end{cases} \quad \begin{cases} \nabla \times E = -\dfrac{\partial B}{\partial t}, \\ \nabla \times H = j + \dfrac{\partial D}{\partial t}, \end{cases} \qquad D = \varepsilon_0 E + P = \varepsilon E$$

Kerr effect

$$P = \varepsilon_0 (\chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 \ldots),$$

Pockels effect

Linear electro-optic effect:

$$\varepsilon_{ij} = \varepsilon_0 (1 + \chi_{ij}^{(1)} + \chi_{ijk}^{(2)} E_k) \;\; = \varepsilon_1 + r_{ijk} E_k,$$

When no E applied, $\varepsilon = \begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & \\ & & \varepsilon_{33} \end{pmatrix} = \varepsilon_0 \begin{pmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & \\ & & n_z^2 \end{pmatrix}$

Refractive indices on principle axes

Refractive index ellipsoid:

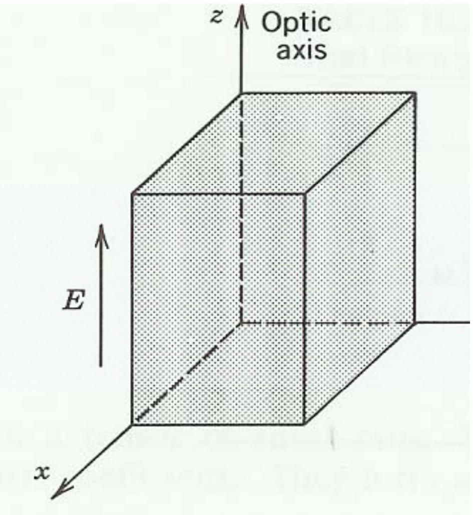
$$U_e = \frac{1}{2}\vec{E} \bullet \vec{D} = \frac{1}{2}\frac{\vec{D}}{\varepsilon} \bullet \vec{D}$$

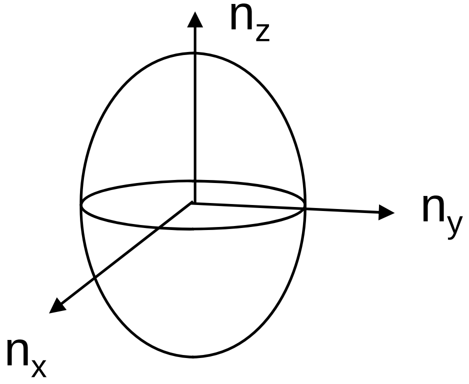
When no electric field applied,

$$U_e = \frac{1}{2}\left(\frac{D_x^2}{\varepsilon_{11}} + \frac{D_y^2}{\varepsilon_{22}} + \frac{D_z^2}{\varepsilon_{33}}\right) = \frac{D^2}{2\varepsilon_0 r^2}\left(\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2}\right),$$

$$\Rightarrow \left(\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2}\right) = 1,$$

Refractive index ellipsoid





Refractive index ellipsoid:

$U_e = \frac{1}{2}\vec{E}\bullet\vec{D} = \frac{1}{2}\frac{\vec{D}}{\varepsilon}\bullet\vec{D}$ Generally,

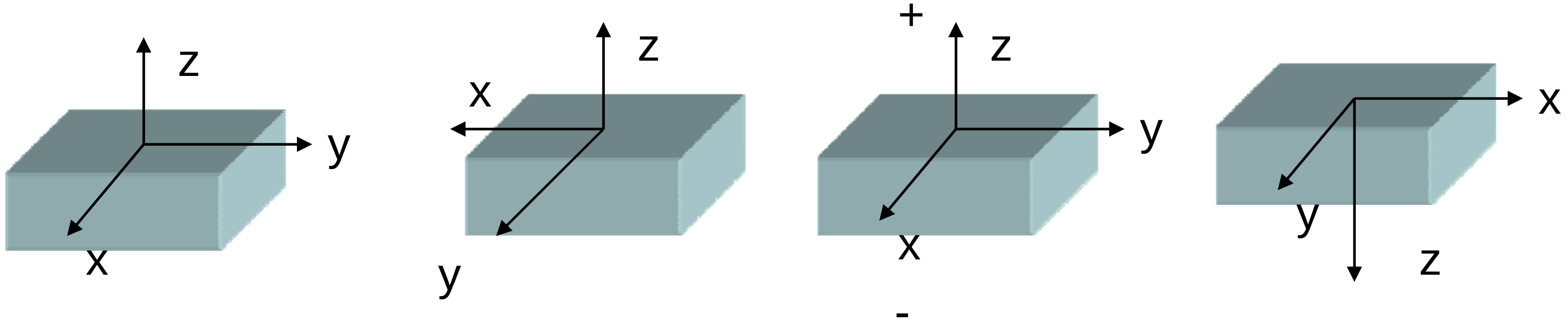
$1 = \sum_{i,j}\eta_{ij}x_i x_j,$ η is a 3x3 tensor, $\eta = \varepsilon^{-1}$ or $\varepsilon = \eta^{-1}$

When electric field applied, $\eta_{ij} = \eta_0 + r_{ijk}E_k,$

r_{ijk} : 3x3x3 tensor, 27 components

$r_{ijk} = r_{jik},$ Why?

No EO effect for crystal with central symmetry, why?



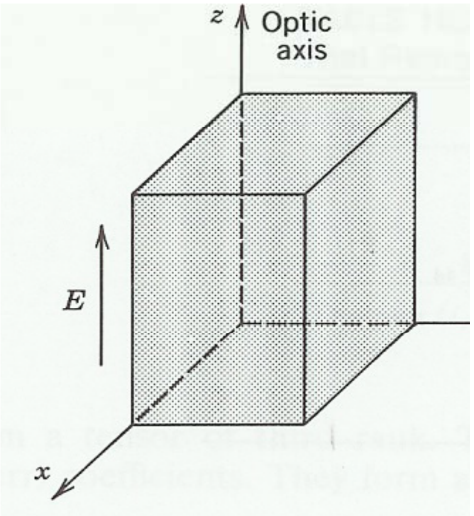
$r_{ijz}E_z = -r_{jiz}E_z,$

$$\varepsilon = \varepsilon_1 + \varepsilon_0 r_{ijk} \vec{E}_k \,, \qquad r_{ijk} = r_{jik} \,,$$

j	i = 1	2	3
1	1	6	5
2	6	2	4
3	5	4	3

$$\eta = \eta_0 + \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}$$

When E_z only,



$$\eta = \eta_0 + \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ E_3 \end{pmatrix}$$

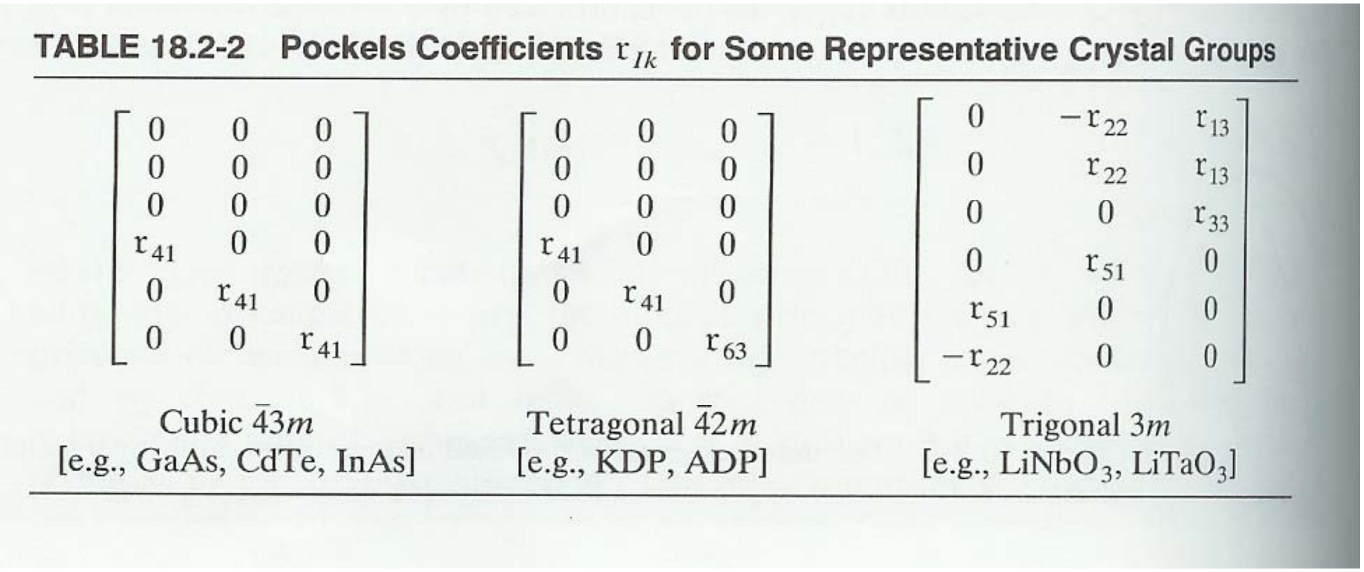
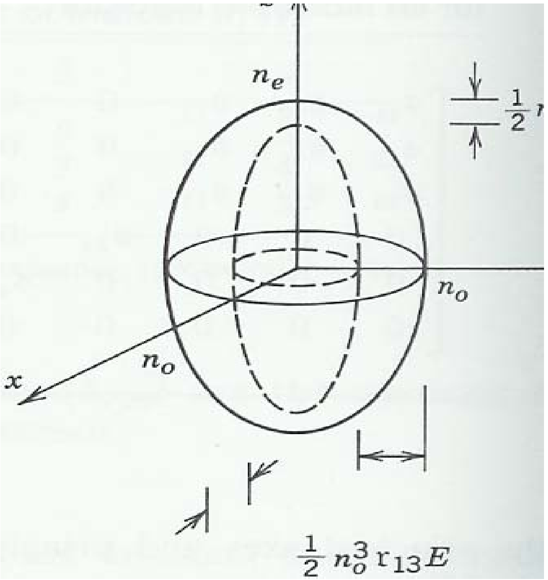


TABLE 18.2-2 Pockels Coefficients r_{Ik} for Some Representative Crystal Groups

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$	$\begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$
Cubic $\bar{4}3m$ [e.g., GaAs, CdTe, InAs]	Tetragonal $\bar{4}2m$ [e.g., KDP, ADP]	Trigonal $3m$ [e.g., $LiNbO_3$, $LiTaO_3$]

Take LiNbO$_3$ as an example :

$$\eta = \eta_0 + \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ E_3 \end{pmatrix} = \eta_0 + r_{33} E_3$$



$$\left(\frac{1}{n_0^2} + r_{13} E_z\right) x^2 + \left(\frac{1}{n_0^2} + r_{13} E_z\right) y^2 + \left(\frac{1}{n_0^2} + r_{33} E_z\right) z^2 = 1,$$

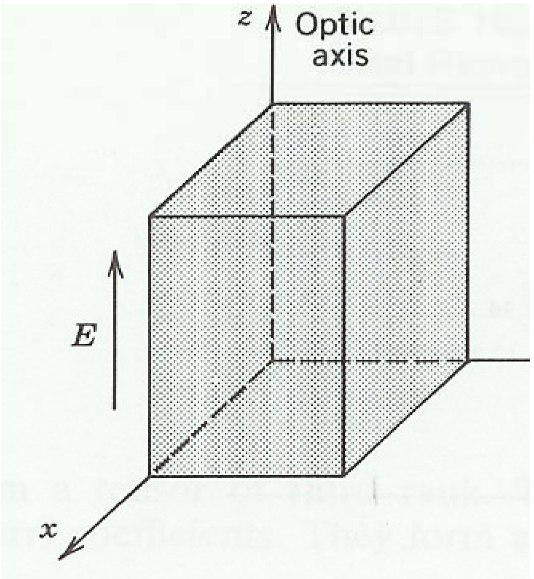
$$\frac{1}{n_e^2} = \left(\frac{1}{n_0^2} + r_{33} E_z\right), \qquad \frac{1}{n'^2_0} = \left(\frac{1}{n_0^2} + r_{13} E_z\right),$$

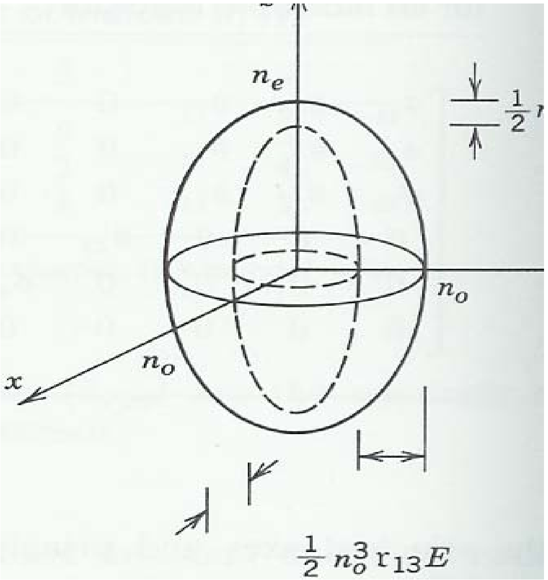
$$\boxed{\Delta n_0 = -\frac{1}{2} n_0^3 r_{13} E_z,}$$

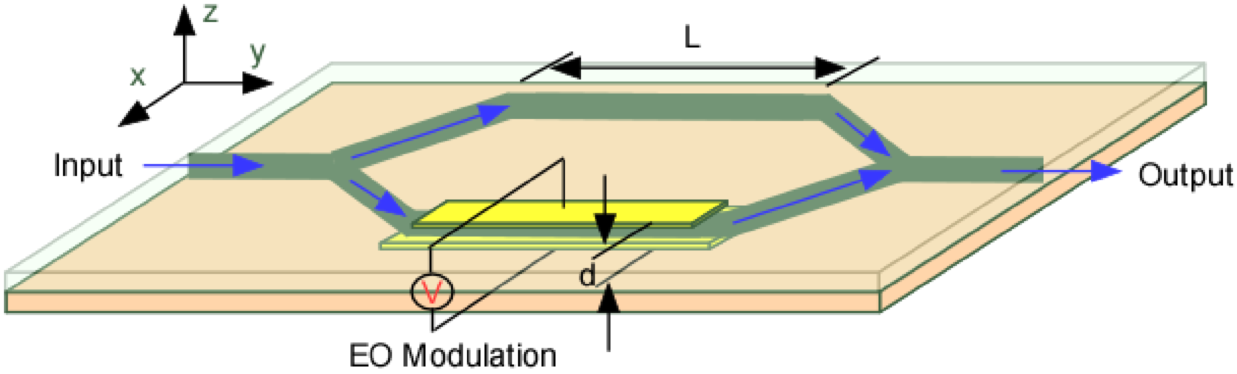
$$n_e = n_0 - \frac{1}{2} n_0^3 r_{33} E_z,$$

$$\boxed{\Delta n_e = -\frac{1}{2} n_0^3 r_{33} E_z,}$$

Take $LiNbO_3$ as an example: Z cut





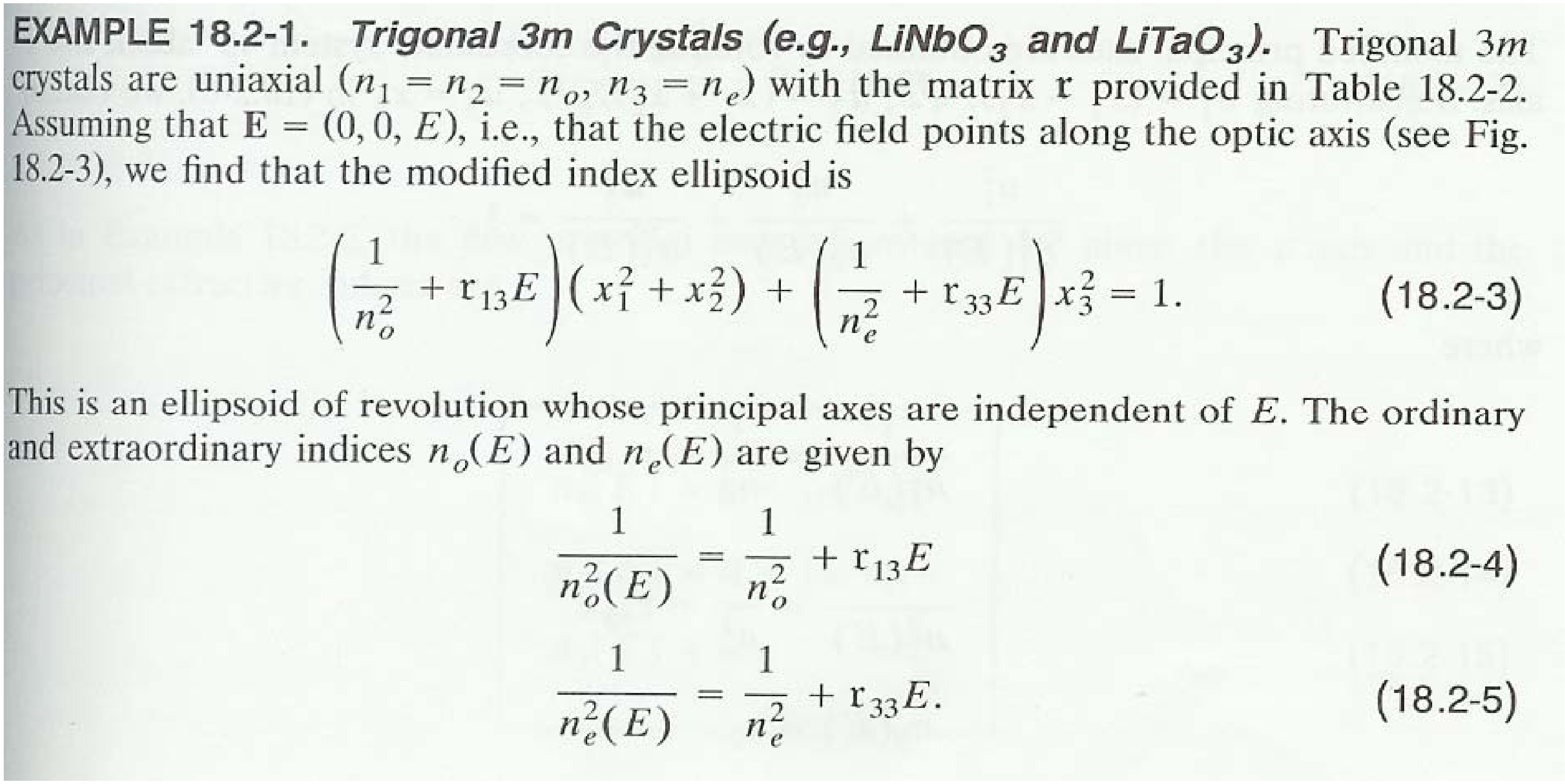


$$\Delta n = -\frac{1}{2} n_0^3 r_{33} E_z ,$$

TM polarization

$$\Delta n_0 = -\frac{1}{2} n_0^3 r_{13} E_z ,$$

TE polarization



EXAMPLE 18.2-1. ***Trigonal 3m Crystals (e.g., $LiNbO_3$ and $LiTaO_3$).*** Trigonal $3m$ crystals are uniaxial ($n_1 = n_2 = n_o$, $n_3 = n_e$) with the matrix $\mathfrak{r}$ provided in Table 18.2-2. Assuming that $\mathbf{E} = (0, 0, E)$, i.e., that the electric field points along the optic axis (see Fig. 18.2-3), we find that the modified index ellipsoid is

$$\left(\frac{1}{n_o^2} + \mathfrak{r}_{13}E\right)\left(x_1^2 + x_2^2\right) + \left(\frac{1}{n_e^2} + \mathfrak{r}_{33}E\right)x_3^2 = 1. \quad (18.2\text{-}3)$$

This is an ellipsoid of revolution whose principal axes are independent of E. The ordinary and extraordinary indices $n_o(E)$ and $n_e(E)$ are given by

$$\frac{1}{n_o^2(E)} = \frac{1}{n_o^2} + \mathfrak{r}_{13}E \quad (18.2\text{-}4)$$

$$\frac{1}{n_e^2(E)} = \frac{1}{n_e^2} + \mathfrak{r}_{33}E. \quad (18.2\text{-}5)$$

$$n_e = n_0 - \frac{1}{2}n_0^3 r_{33} E_z, \qquad \Delta n = -\frac{1}{2}n_0^3 r_{33} E_z,$$

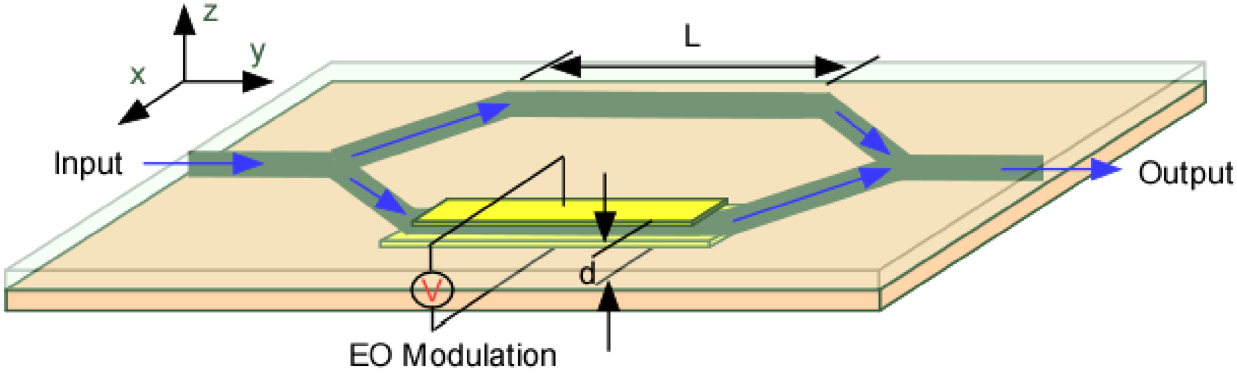
$$n_o = n_{o0} - \frac{1}{2}n_0^3 r_{13} E_z, \qquad \Delta n = -\frac{1}{2}n_0^3 r_{13} E_z,$$

Different index changes of TE and TM waves

Take LiNbO$_3$ Intensity modulator

Z cut

TM polarization



$$A_{out} = A_1 + A_2, \quad A_1 = \frac{1}{2} A_0 e^{i\beta n_0 L}, \qquad A_2 = \frac{1}{2} A_0 e^{i\beta (n_0 + \Delta n) L},$$

$$A_{out} = \frac{1}{2} A_0 e^{i\beta n_0 L} (1 + e^{i\beta \Delta n L}), \qquad I_{out} = \left|A_{out}\right|^2 = \frac{1}{4} I_0 \left|(1 + e^{i\beta \Delta n L})\right|^2 = I_0 \cos^2(\beta \Delta n L / 2),$$

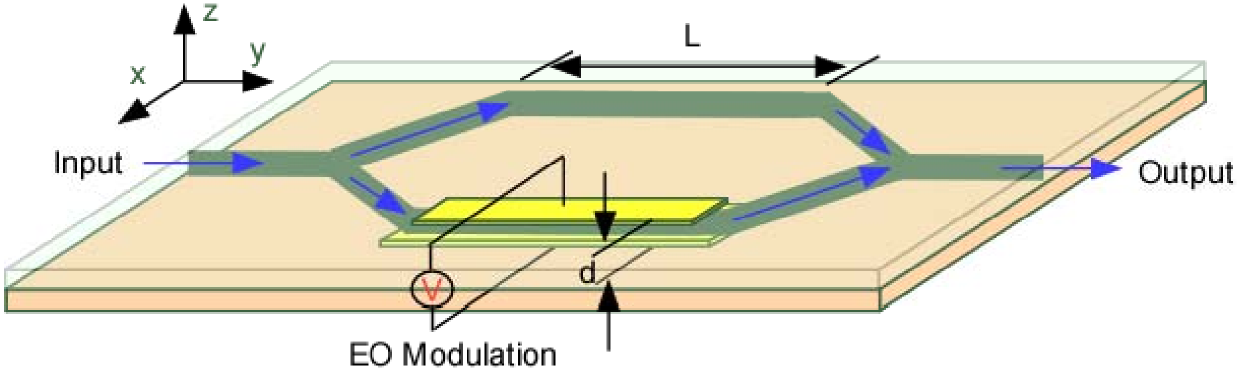
$$I_{out} = I_0 \cos^2(\frac{\pi}{\lambda} \Delta n L) = I_0 \cos^2(\frac{\pi}{\lambda}(-\frac{1}{2} n_0^3 \gamma_{33} \frac{V}{d}) L),$$

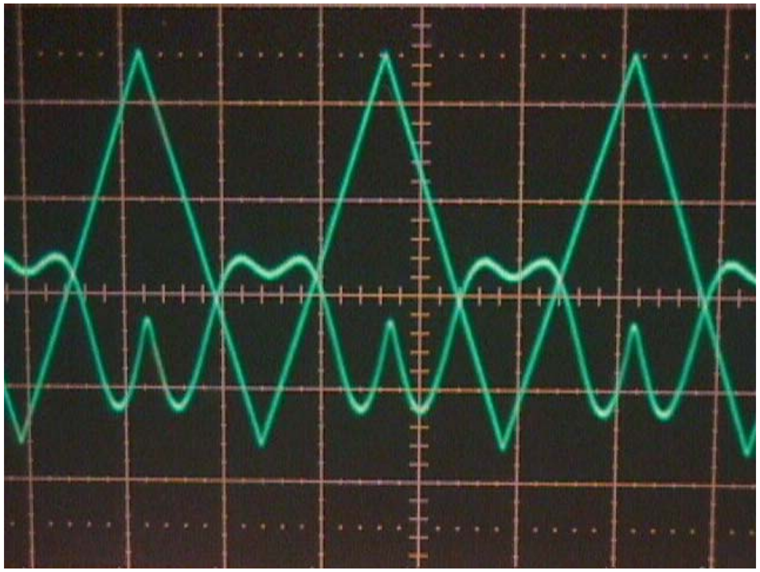
$$\frac{\pi}{\lambda}(-\frac{1}{2} n_0^3 \gamma_{33} \frac{V_\pi}{d}) L = -\pi / 2, \qquad V_\pi = \frac{\lambda d}{n_0^3 \gamma_{33} L}$$

Take $LiNbO_3$ Intensity modulator

Z cut

TM polarization





$$I_{out} = I_0 \cos^2(\frac{\pi}{\lambda}\Delta nL) = I_0 \cos^2(\frac{\pi}{\lambda}(-\frac{1}{2}n_0^3\gamma_{33}\frac{V}{d})L),$$

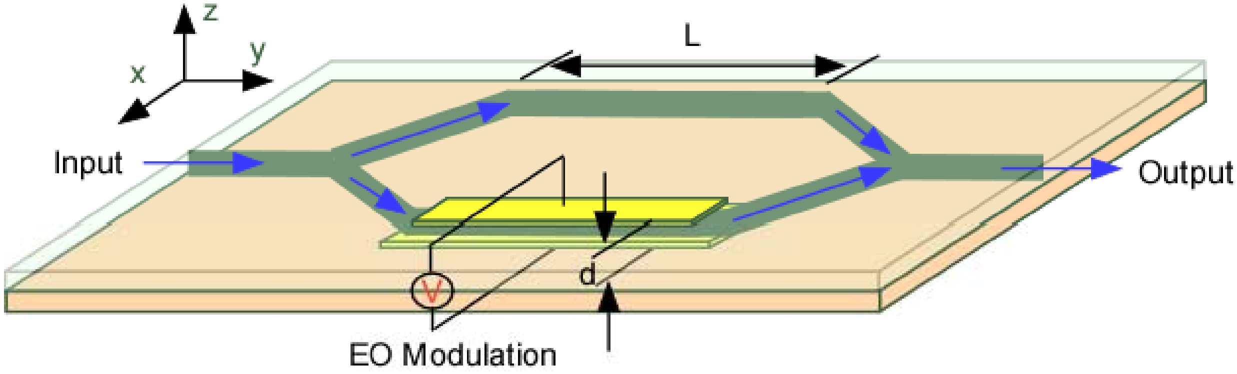
Linear region

Frequency double region

Operating characteristics

- Modulation depth $\eta = \frac{I_{\max} - I_{\min}}{I_{\max}}$, $\eta = 100\%$, $I_{\min} = 0$

- Bandwidth: the highest frequency the modulator can operate, R and C



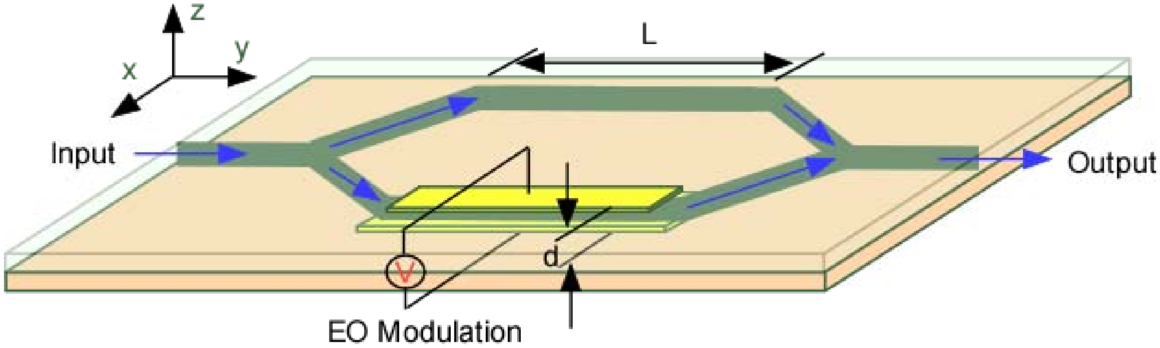
- Insertion loss (dB):

$$IL(dB) = -10\log(\frac{I_{out}}{I_{in}}),$$

- Power consumption: P/Δf, Δf: bandwidth

$$P_e = \Delta f W = \Delta f \frac{1}{2}\int \varepsilon E^2(\omega) dV,$$

for a channel waveguide, assuming the E filed is uniform,



$$C = \varepsilon \frac{A}{d} = \varepsilon \frac{W_d L}{d}$$

C ~ 0.4pF

$$W = \frac{1}{2}\int \varepsilon E^2(\omega) dV = \frac{L d W_d}{2} \varepsilon E^2(\omega),$$

Take $LiNbO_3$ as an example: $E = V_\pi / d = \frac{\lambda}{n_0^3 \gamma_{33} L}$

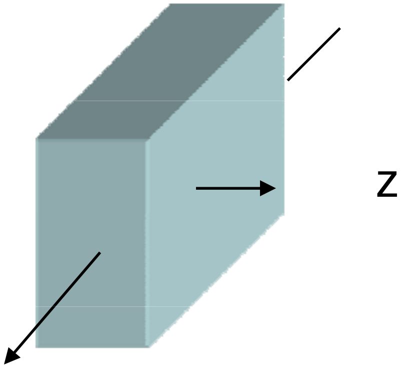
$$W = \frac{1}{2}\int \varepsilon E^2(\omega) dV = \frac{d W_d}{2L} \varepsilon (\frac{\lambda}{n_0^3 \gamma_{33}})^2,$$

P/Δf ~ 2μW/MHz

- Power consumption: P/Δf, Δf: bandwidth

$$P_e = \Delta f W = \Delta f \, \frac{1}{2}\int \varepsilon E^2(\omega) dV,$$

for a bulk EO modulators, assuming the E filed is uniform,



$$\Delta n = -\frac{1}{2} n_0^3 r_{33} E_z,$$

$$C = \varepsilon \frac{A}{d} = \varepsilon \frac{W_d L}{d}$$

C ~ 3pF

$$W = \frac{1}{2}\int \varepsilon E^2(\omega) dV = \frac{L d W_d}{2} \varepsilon E^2(\omega),$$

Take LiNbO$_3$ as an example: $E = V_\pi / d = \dfrac{\lambda}{n_0^3 \gamma_{33} L}$

$$W = \frac{1}{2}\int \varepsilon E^2(\omega) dV = \frac{d W_d}{2L} \varepsilon \left(\frac{\lambda}{n_0^3 \gamma_{33}}\right)^2,$$

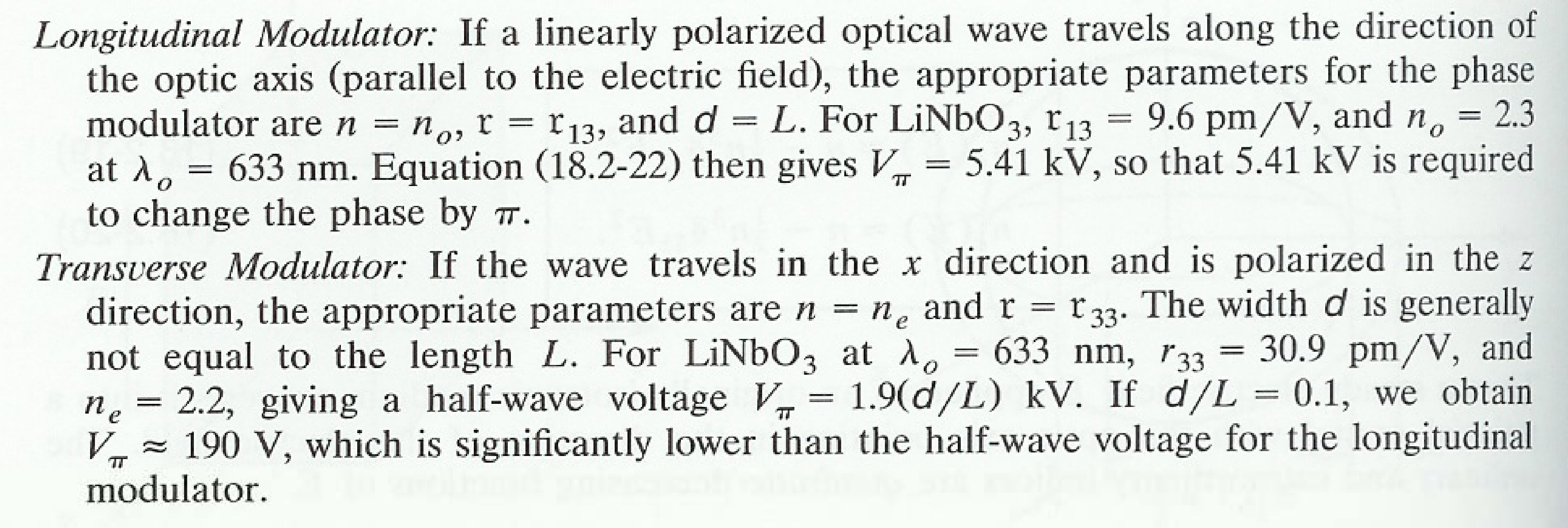
P/Δf ~ 2W/MHz

Phase modulator:

$\Delta n = -\frac{1}{2} n_0^3 r_{33} E_z,$ Phase shift due to the applied voltage:

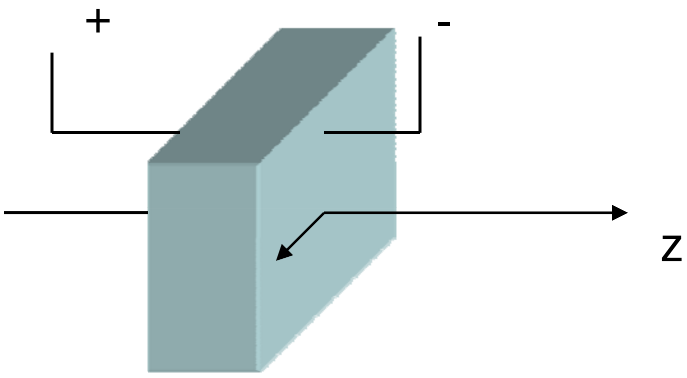
$$\Delta\varphi = -\frac{1}{2} n_0^3 r_{33} E_z \frac{2\pi}{\lambda_0} L, \qquad V_\pi = \frac{d}{L} E_z \frac{\lambda_0}{n_0^3 r_{33}},$$

Examples:

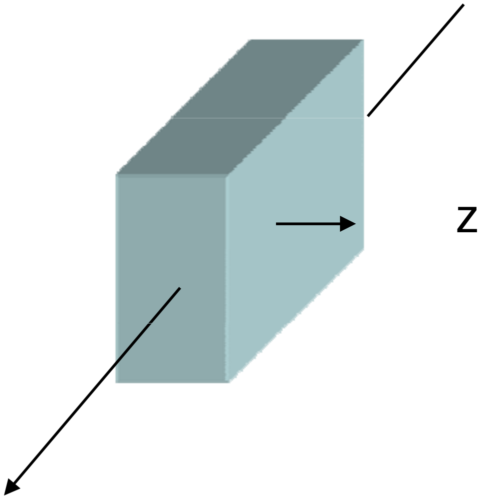


Longitudinal Modulator: If a linearly polarized optical wave travels along the direction of the optic axis (parallel to the electric field), the appropriate parameters for the phase modulator are $n = n_o$, $r = r_{13}$, and $d = L$. For LiNbO$_3$, $r_{13} = 9.6$ pm/V, and $n_o = 2.3$ at $\lambda_o = 633$ nm. Equation (18.2-22) then gives $V_\pi = 5.41$ kV, so that 5.41 kV is required to change the phase by π.

Transverse Modulator: If the wave travels in the x direction and is polarized in the z direction, the appropriate parameters are $n = n_e$ and $r = r_{33}$. The width d is generally not equal to the length L. For LiNbO$_3$ at $\lambda_o = 633$ nm, $r_{33} = 30.9$ pm/V, and $n_e = 2.2$, giving a half-wave voltage $V_\pi = 1.9(d/L)$ kV. If $d/L = 0.1$, we obtain $V_\pi \approx 190$ V, which is significantly lower than the half-wave voltage for the longitudinal modulator.

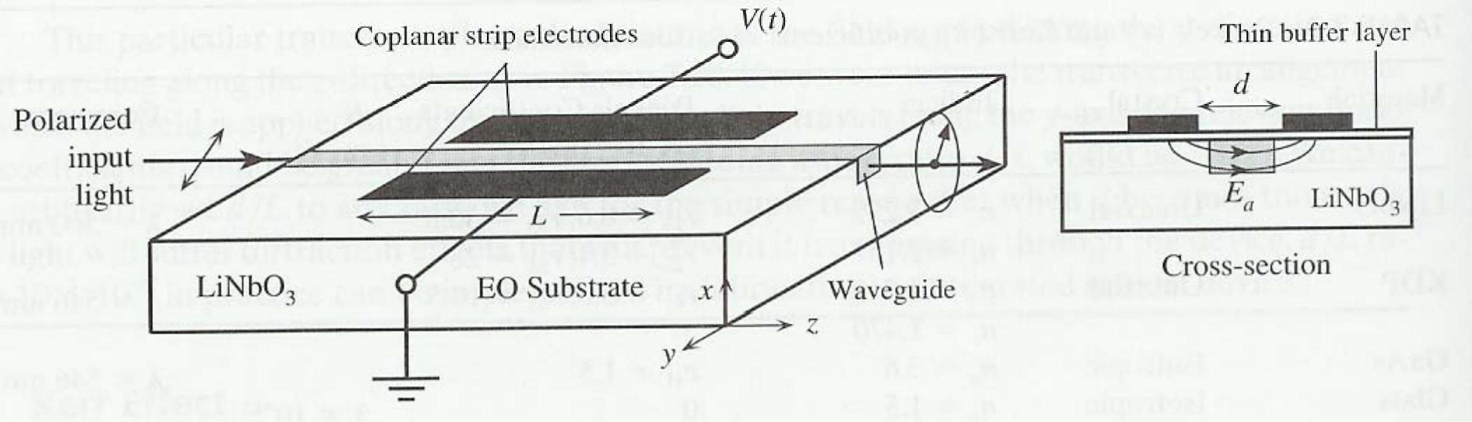


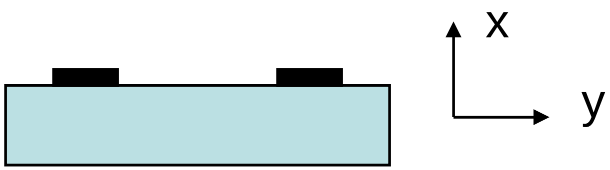
$$\Delta n = -\frac{1}{2} n_0^3 r_{13} E_z ,$$



$$\Delta n = -\frac{1}{2} n_0^3 r_{33} E_z ,$$

Polarization modulator:





$$\eta = \eta_0 + \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ E_2 \\ 0 \end{pmatrix}$$

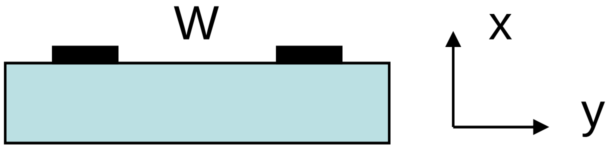
$$\Delta n_x = -\frac{1}{2} n_0^3 r_{22} E_y,$$

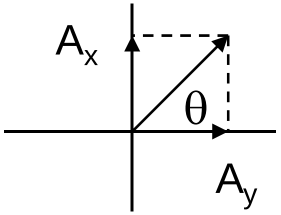
$$\Delta n_y = \frac{1}{2} n_0^3 r_{22} E_y,$$

$$V_\pi = \frac{d}{2L} E \frac{\lambda_0}{n_0^3 r_{33}},$$

Phase shift due to the applied voltage:

Polarization modulator:





$$\Delta n_x = -\frac{1}{2} n_0^3 r_{22} E_y,$$

$$\Delta n_y = \frac{1}{2} n_0^3 r_{22} E_y,$$

$$A_{\pi,in} = A_0 \sin\theta,$$

$$A_{\pi,out} = A_0 \sin\theta \exp(i\beta n_x L) = A_0 \sin\theta \exp(i\beta L(n_0 - \frac{1}{2} n_0^3 r_{22} E_y),$$

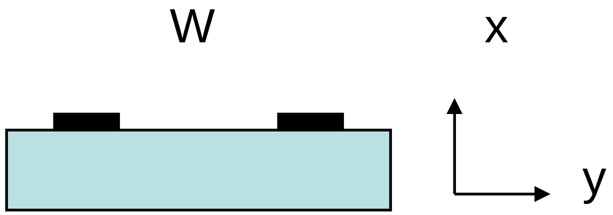
$$A_{y,in} = A_0 \cos\theta,$$

$$A_{y,out} = A_0 \cos\theta \exp(i\beta n_y L) = A_0 \cos\theta \exp(i\beta L(n_0 + \frac{1}{2} n_0^3 r_{22} E_y),$$

Phase difference due to the applied voltage:

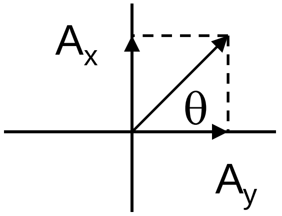
$$\phi = \beta L n_0^3 r_{22} E_y = \beta L n_0^3 r_{22} V_y / W,$$

Polarization modulator:

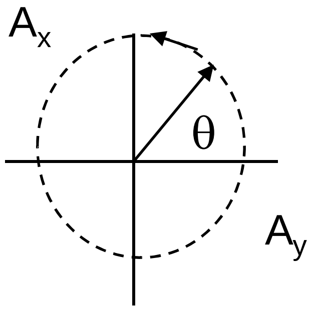


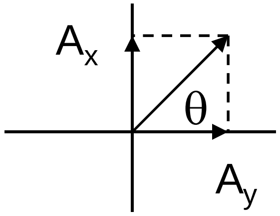
Phase difference due to the applied voltage:

$$\phi = \beta L n_0^3 r_{22} E_y = \beta L n_0^3 r_{22} V_y / W,$$

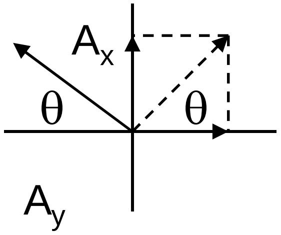


When φ = π/2,

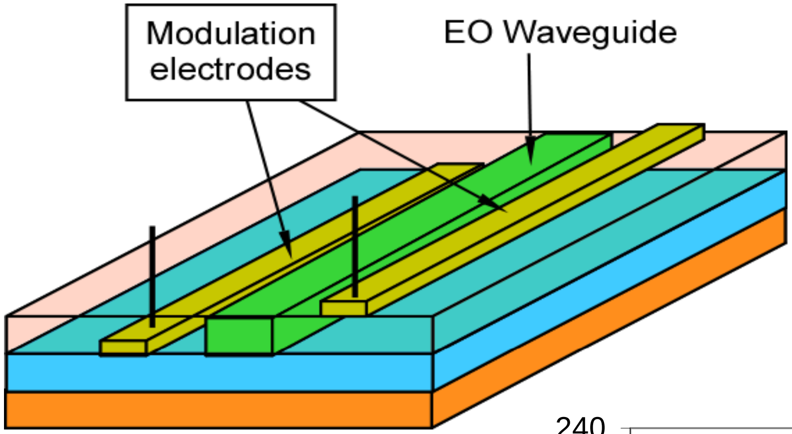


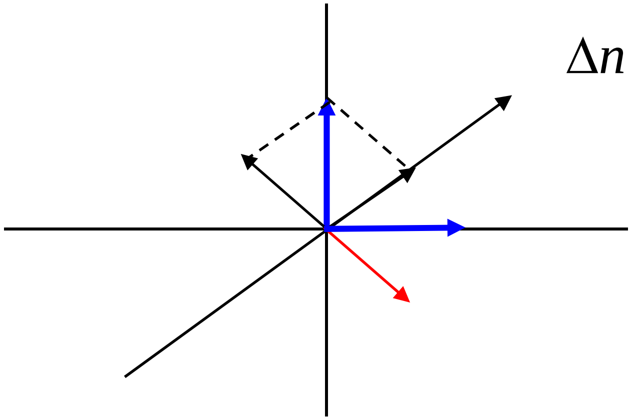


When φ = π,

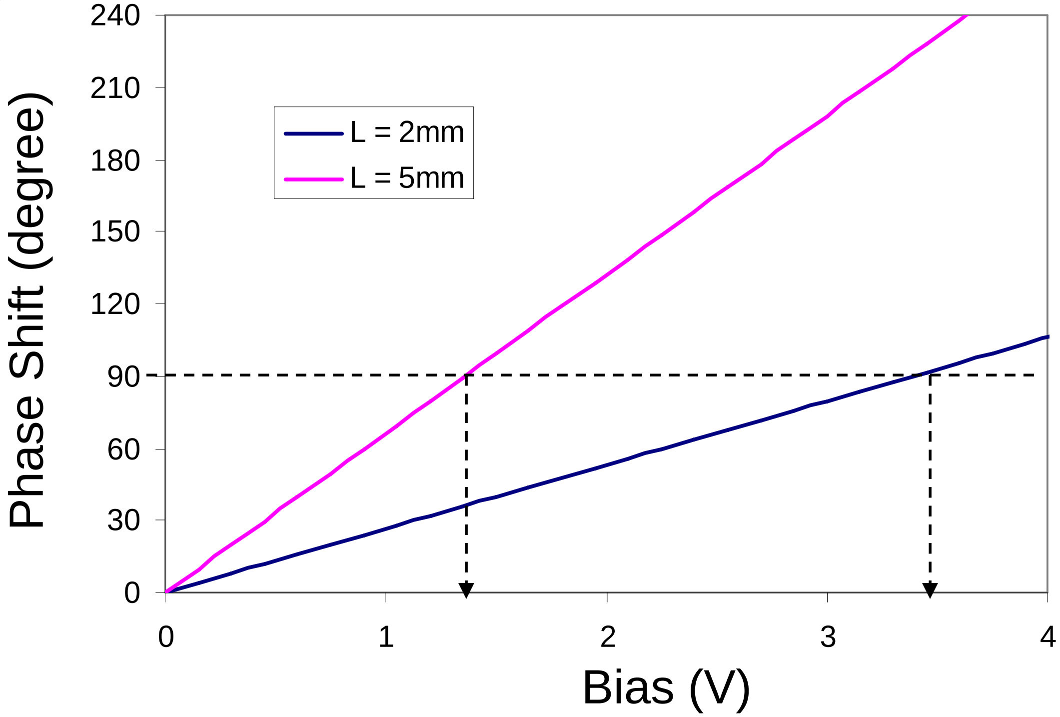


TE-TM converter:



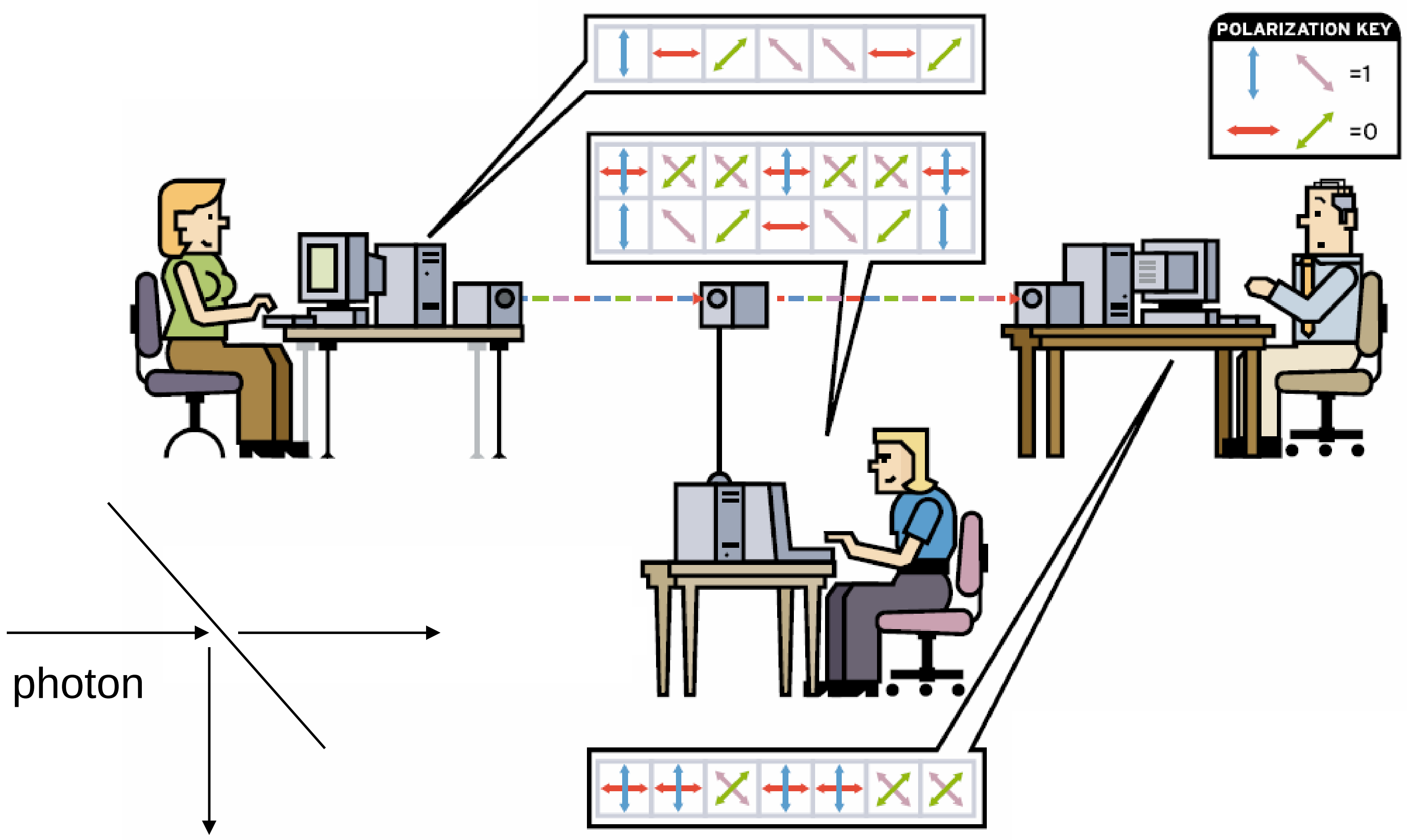


$$\Delta n = -\frac{1}{2} n_0^3 V / d,$$



Quantum Key Distribution (QKD) Bennett and Brassard 1984 protocol

Proc. IEEE Int. Conf. Computers, Systems and Signal Processing, 1984, pp. 175–179.

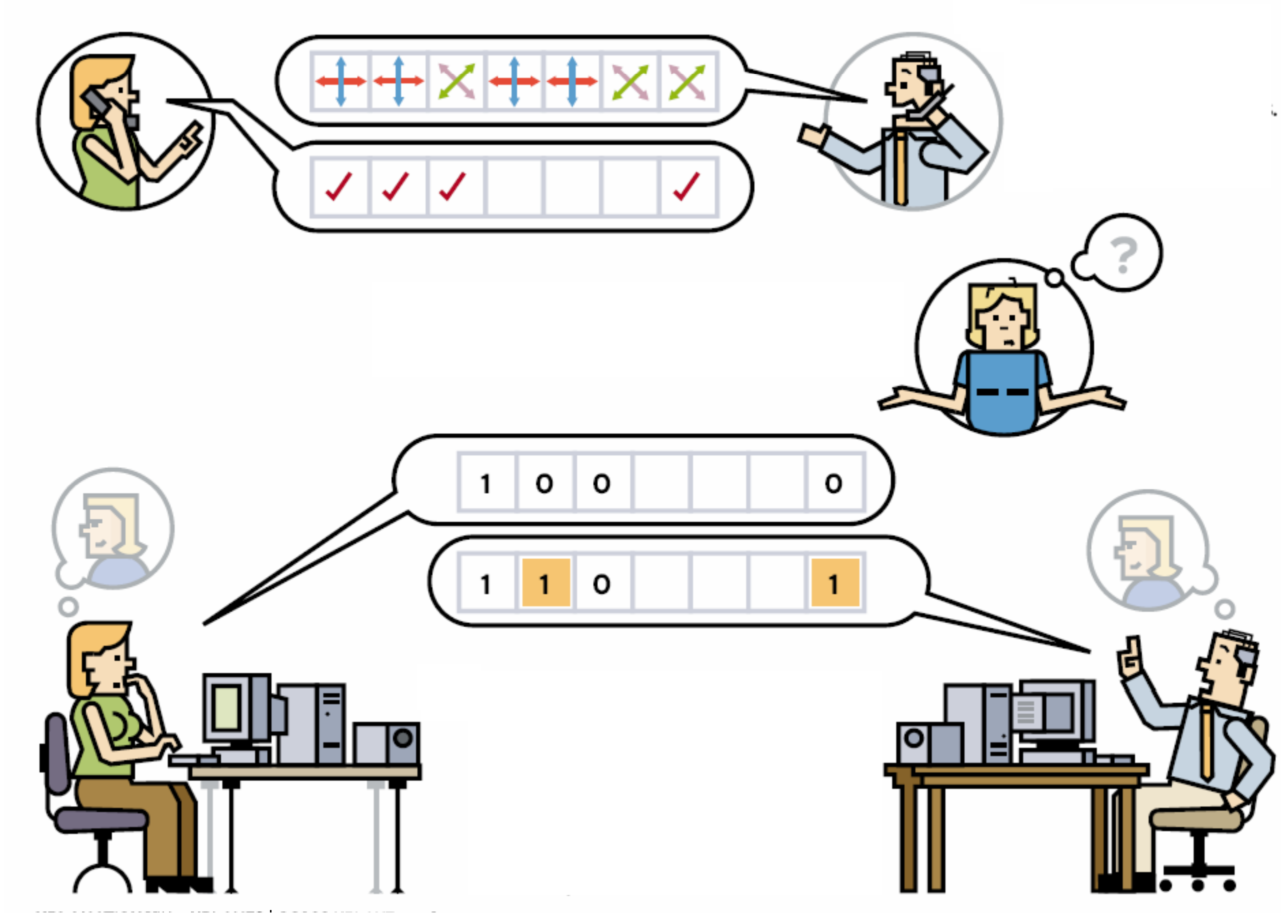


Single photon

JUSTIN MULLINS**,** IEEE SPECTRUM, p. 40, May 2002

Quantum Key Distribution (QKD) Bennett and Brassard 1984 protocol

Proc. IEEE Int. Conf. Computers, Systems and Signal Processing, 1984, pp. 175–179.



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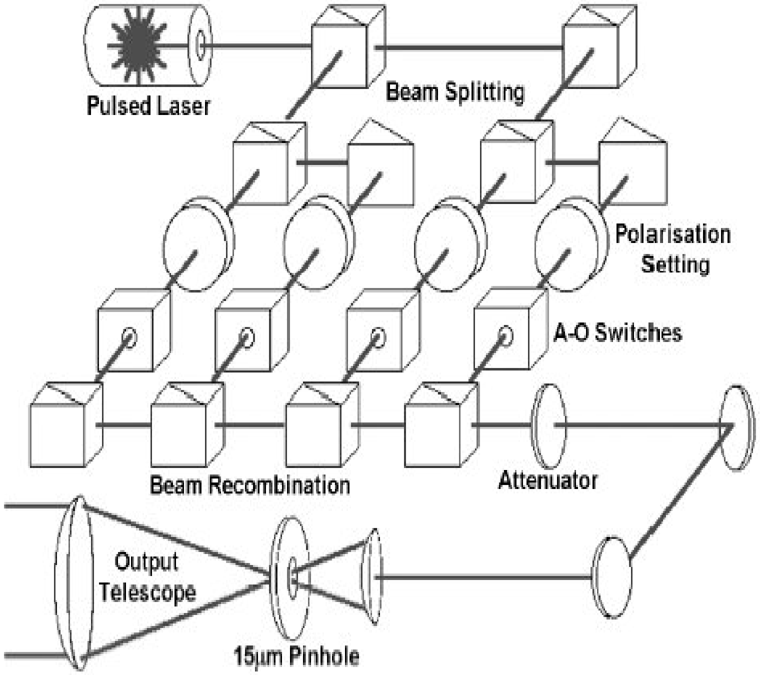
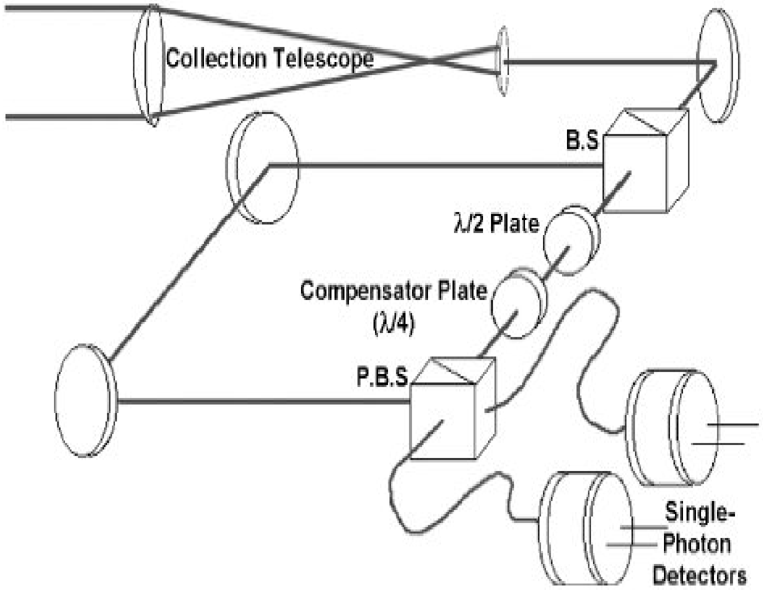
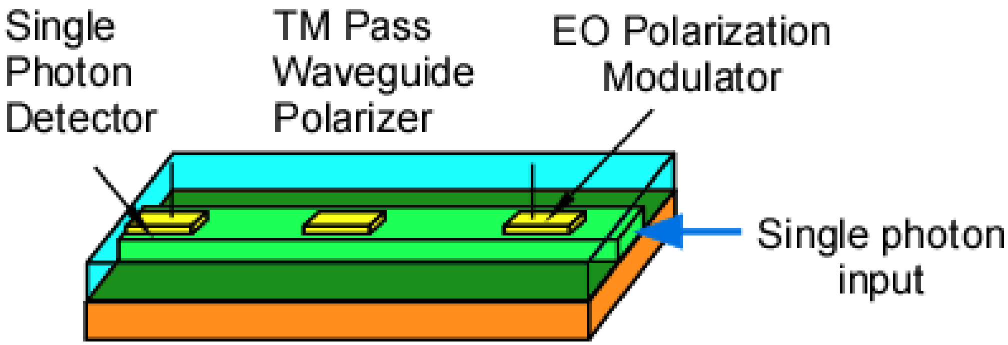
Quantum Key Distribution (QKD)

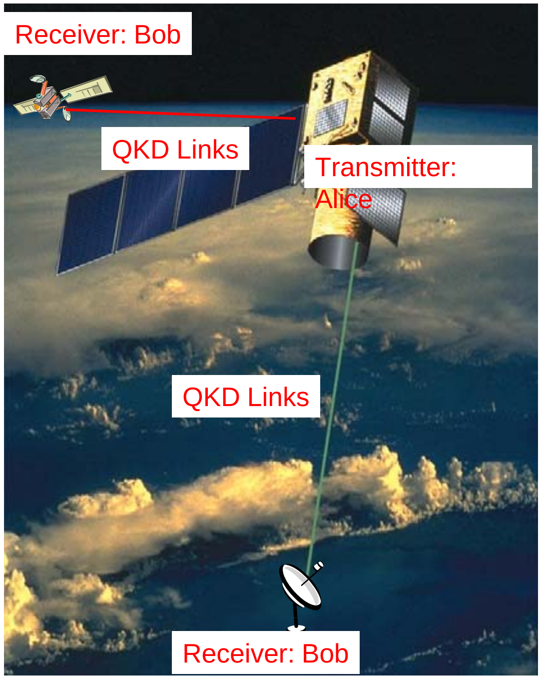




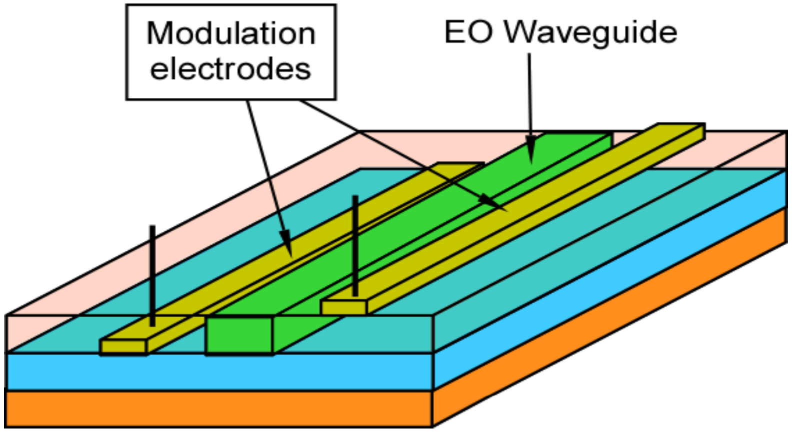
W. T. Buttler, etal. Physical Review A, Vol 57, 1998, pp. 2379-2382

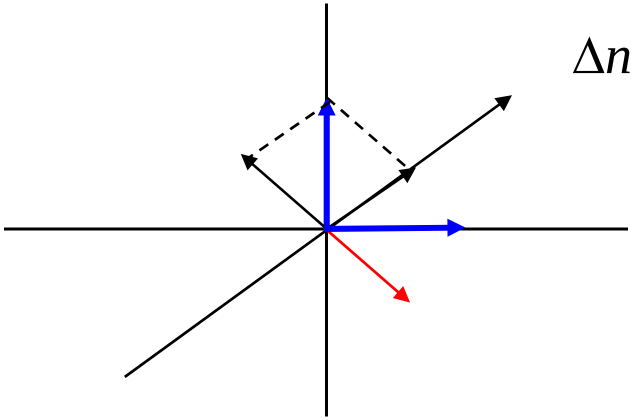
Integrated QKD Receiver



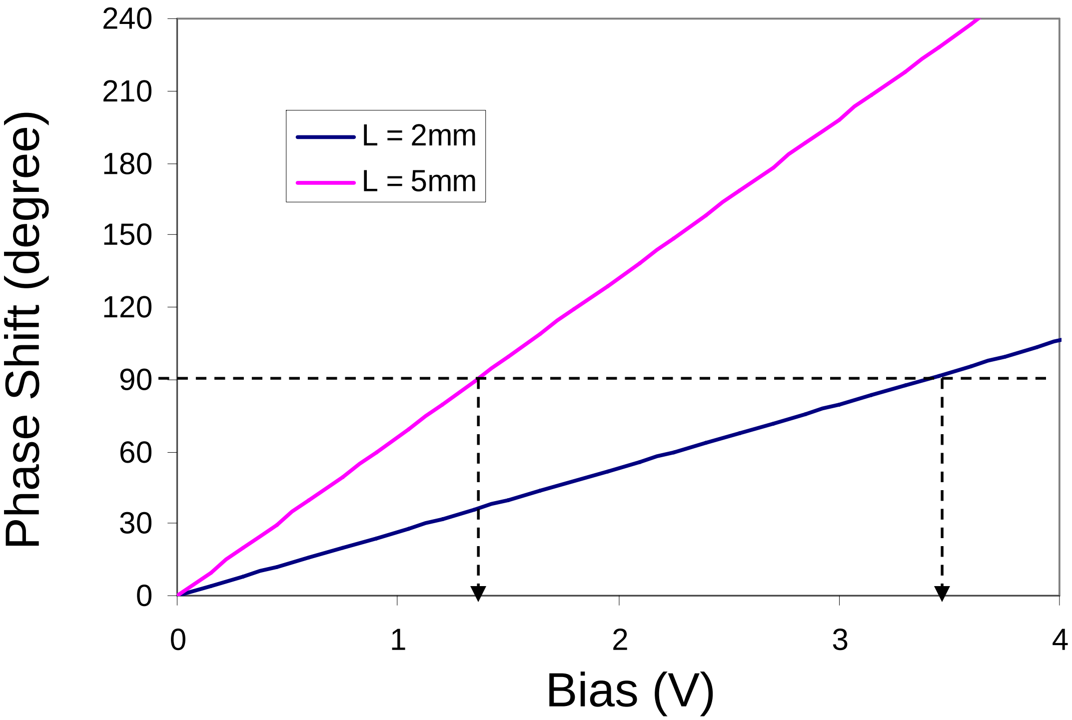


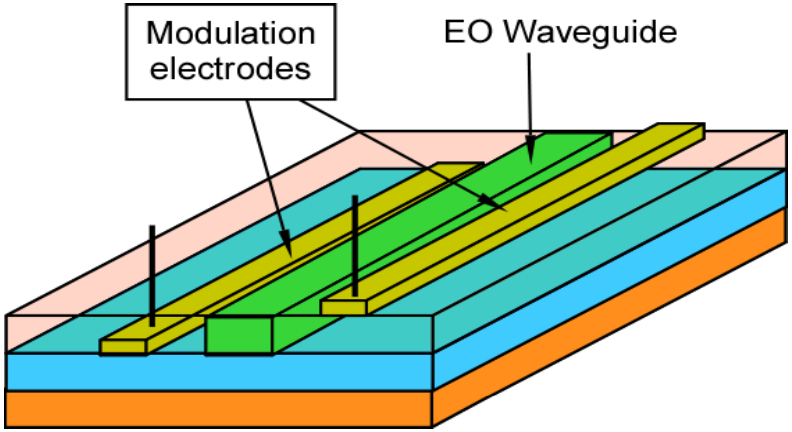
Input photon polarization states	Detection polarization basis	Final polarization states	Si single photon detector signal
TE	Linear	TE	0
TE	Circular	TE & TM	x
TM	Linear	TM	1
TM	Circular	TE & TM	x
Circular – left-hand	Linear	TE & TM	x
Circular – left-hand	Circular	TE	0
Circular – right-hand	Linear	TE & TM	x
Circular – right-hand	Circular	TM	1

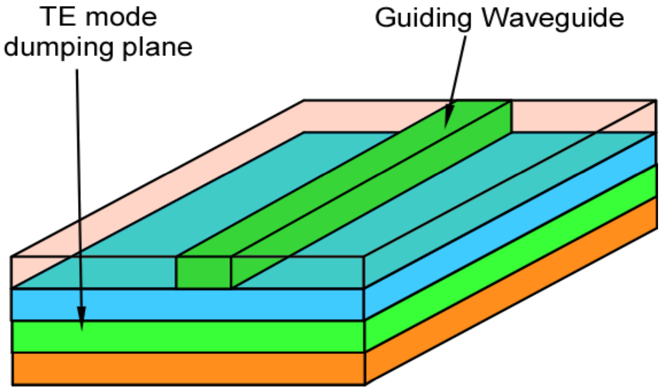




$$\Delta n = -\frac{1}{2} n^3 V / d,$$



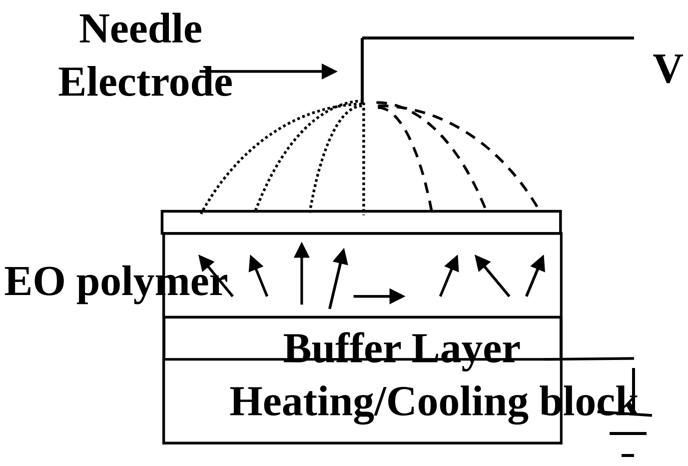


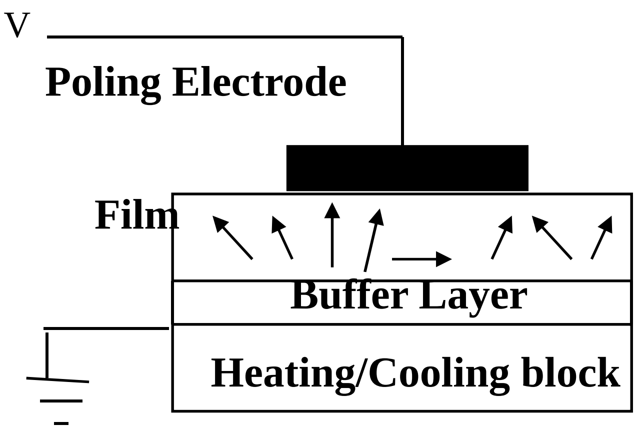


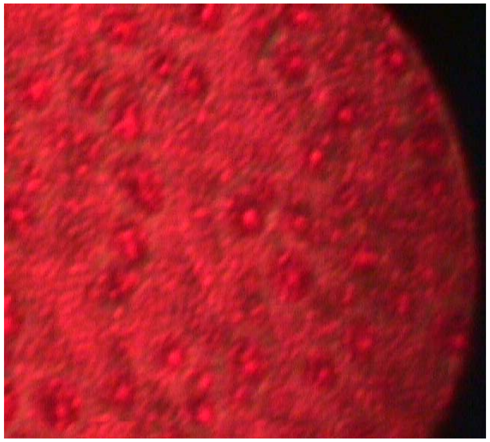
Input photon polarization states	Applied voltage	Detection polarization basis	Final polarization states	detected signal
TE	0	Linear	TE	0
TE	V	Circular	Circular	x
TM	0	Linear	TM	1
TM	V	Circular	Circular	x
Circular – left-hand	0	Linear	Circular	x
Circular – left-hand	V	Circular	TE	0
Circular – right-hand	0	Linear	Circular	x
Circular – right-hand	V	Circular	TM	1

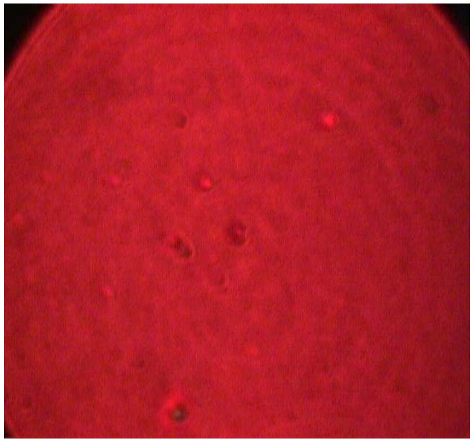
EO Polymer poling

Poling of polymer materials, contact poling and corona poling:







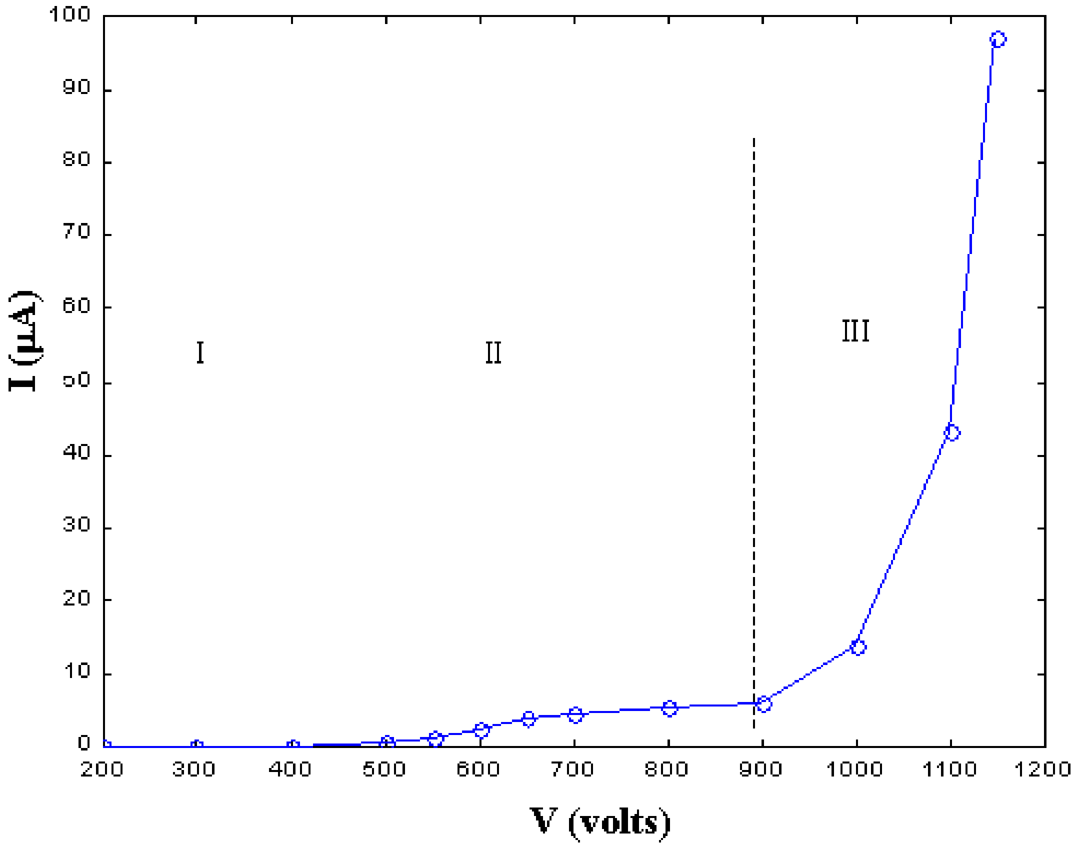


Advantages:

- Lower voltage ~ 800V
- Good film quality
- Uniform poling voltage
- Easy control of poling voltage
- Select poling area

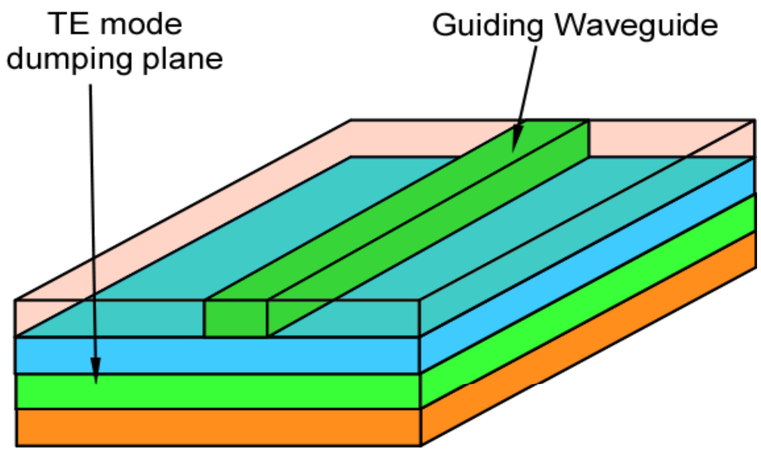
I-V curve during poling:

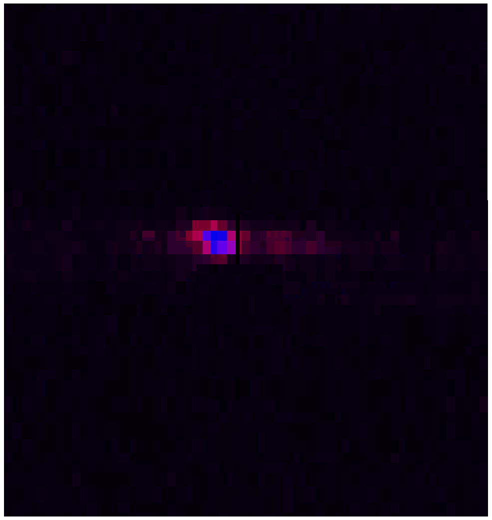
- Poling voltage : 900V
- EO coefficient ~ 22pm/V

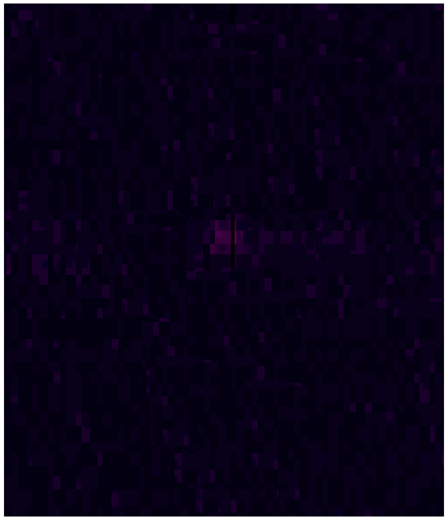


TM-pass waveguide Polarizer

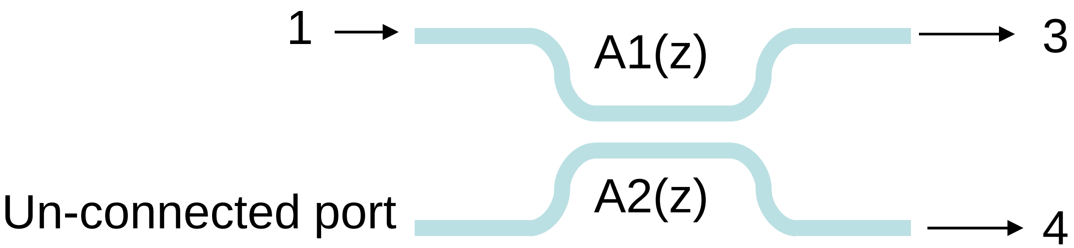
	Before poling	After poling
n (TE)	1.594	1.591
n (TM)	1.594	1.598




TM input


TE input

Directional coupler:



For A1(0) = A1, A2(0) =0

$$\boxed{\begin{aligned}\frac{dA_1(z)}{dz} &= i\kappa A_2(z)e^{i\Delta\beta z}\\ \frac{dA_2(z)}{dz} &= i\kappa A_1(z)e^{-i\Delta\beta z}\end{aligned}}$$

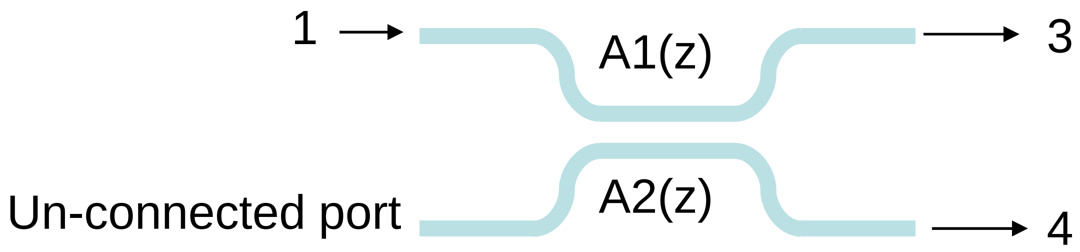
$$\Delta\beta = \beta_2 - \beta_1$$

$$\frac{d}{dz}\left(e^{-i\Delta\beta z}\frac{dA_1(z)}{dz}\right) = i\kappa\frac{d}{dz}A_2(z) = -\kappa^2 A_1(z)e^{-i\Delta\beta z}$$

$$\frac{d^2A_1(z)}{dz^2} - i\Delta\beta\frac{dA_1(z)}{dz} + \kappa^2 A_1(z) = 0$$

$$\frac{d^2A_2(z)}{dz^2} + i\Delta\beta\frac{dA_2(z)}{dz} + \kappa^2 A_2(z) = 0$$

Directional coupler:



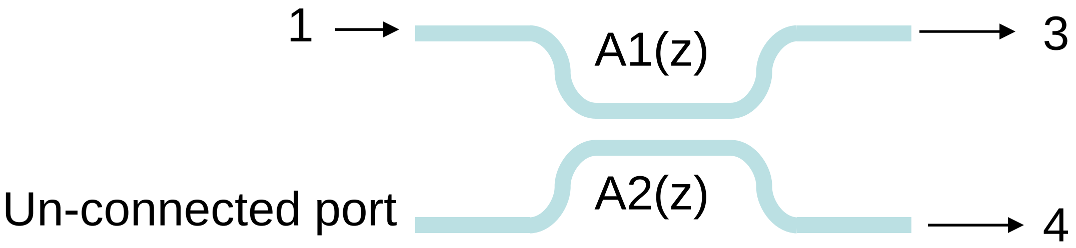
For A1(0) = A1, A2(0) =0

$$\frac{d^2A_1(z)}{dz^2} - i\Delta\beta\frac{dA_1(z)}{dz} + \kappa^2 A_1(z) = 0$$

$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(A\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)\right)$$

$$\frac{d^2A_2(z)}{dz^2} + i\Delta\beta\frac{dA_2(z)}{dz} + \kappa^2 A_2(z) = 0$$

$$A_2(z) = e^{-\frac{i\Delta\beta}{2}z}\left(C\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)\right)$$



1

3

A1(z)

Un-connected port

A2(z)

4

$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(A\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)\right)$$

$$A_2(z) = e^{-\frac{i\Delta\beta}{2}z}\left(C\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)\right)$$

For A1(0) = A1, A2(0) =0 $A_2(z) = e^{\frac{i\Delta\beta}{2}z} iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)$

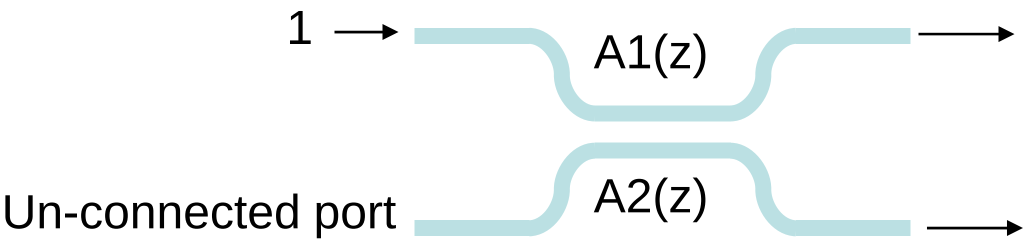
$$\frac{d}{d}A_2(z) = e^{\frac{i\Delta\beta}{2}z} iD\sqrt{(\Delta\beta/2)^2+\kappa^2}\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) - D\frac{\Delta\beta}{2}e^{\frac{i\Delta\beta}{2}z}\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)$$

$$= i\kappa A_1(z)$$

$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}A_1(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iBe^{\frac{i\Delta\beta}{2}z}\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)$$

$$D = \frac{\kappa A_1(0)}{\sqrt{(\Delta\beta)^2+4\kappa^2}} \qquad B = \frac{\frac{\Delta\beta}{2}A_1(0)}{\sqrt{(\Delta\beta)^2+4\kappa^2}}$$

Directional coupler:



For A1(0) = A1, A2(0) =0

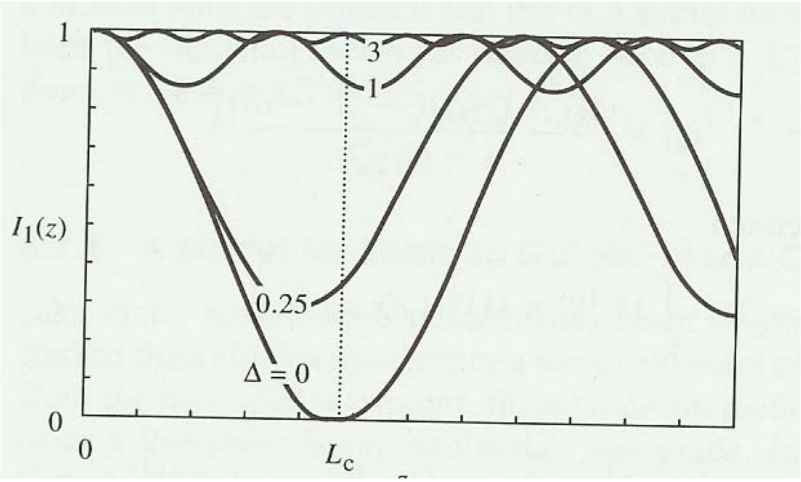
$$A_1(z) = A_1(0)\cos(\kappa z)$$

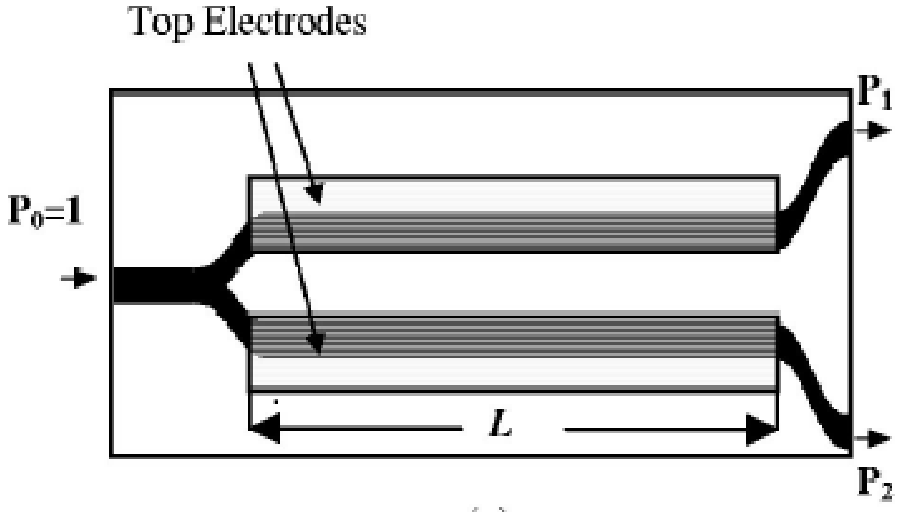
$$A_2(z) = iA_1(0)\sin(\kappa z)$$

$$A_2(z) = e^{\frac{i\Delta\beta}{2}z} i \frac{\kappa A_1(0)}{\sqrt{(\Delta\beta/2)^2 + \kappa^2}} \sin(\sqrt{(\Delta\beta/2)^2 + \kappa^2}\, z)$$

$$P_2 / P_0 = \frac{|\kappa|^2}{|\kappa|^2 + \left(\frac{\Delta\beta}{2}\right)^2} \sin^2\left(\sqrt{|\kappa|^2 + \left(\frac{\Delta\beta}{2}\right)^2}\right) L$$

$$\Delta\beta = \frac{2\pi}{\lambda}\Delta n = \frac{2\pi}{\lambda}(-\frac{1}{2}n_0^3 r_{33} E_z),$$





$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(A\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$A_2(z) = e^{-\frac{i\Delta\beta}{2}z}\left(C\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

For A1(0)= A2(0) = ½ A(0),

$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(A_1(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$A_2(z) = e^{-\frac{i\Delta\beta}{2}z}\left(A_2(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(A_1(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$\frac{d}{dz}A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(-A_1(0)\sqrt{(\Delta\beta/2)^2+\kappa^2}\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iB\sqrt{(\Delta\beta/2)^2+\kappa^2}\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$+\frac{i\Delta\beta}{2}e^{\frac{i\Delta\beta}{2}z}\left(A_1(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)e^{i\Delta\beta z} = i\kappa e^{\frac{i\Delta\beta}{2}z}\left(A_2(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$i\kappa A_2(0) = iB\sqrt{(\Delta\beta/2)^2+\kappa^2} + \frac{i\Delta\beta}{2}A_1(0)$$

$$B = \frac{\kappa - \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}}A_2(0)$$

$$\frac{d}{dz}A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(-A_1(0)\sqrt{(\Delta\beta/2)^2+\kappa^2}\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iB\sqrt{(\Delta\beta/2)^2+\kappa^2}\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

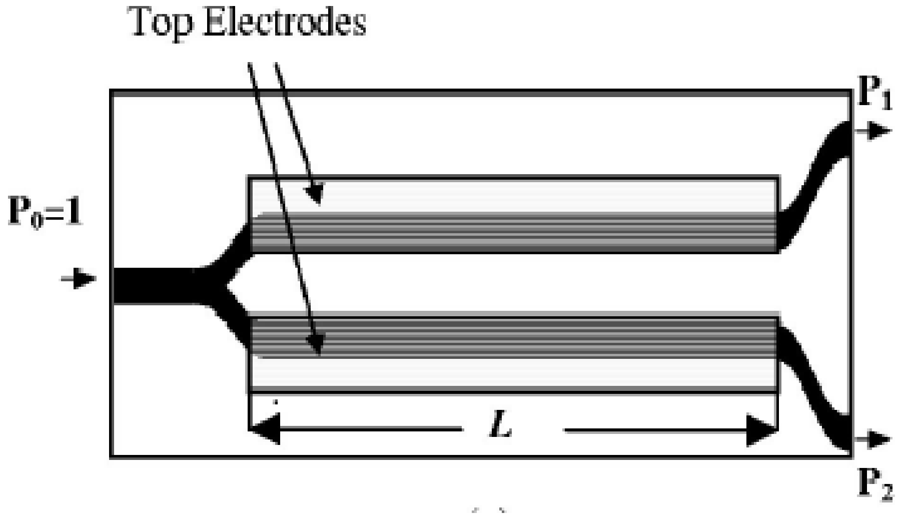
$$+\frac{i\Delta\beta}{2}e^{\frac{i\Delta\beta}{2}z}\left(A_1(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$\frac{dA_1(z)}{dz} = i\kappa A_2(z)e^{i\Delta\beta z} = i\kappa e^{\frac{i\Delta\beta}{2}z}\left(A_2(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(A_1(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

$$A_2(z) = e^{-\frac{i\Delta\beta}{2}z}\left(A_2(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}z)\right)$$

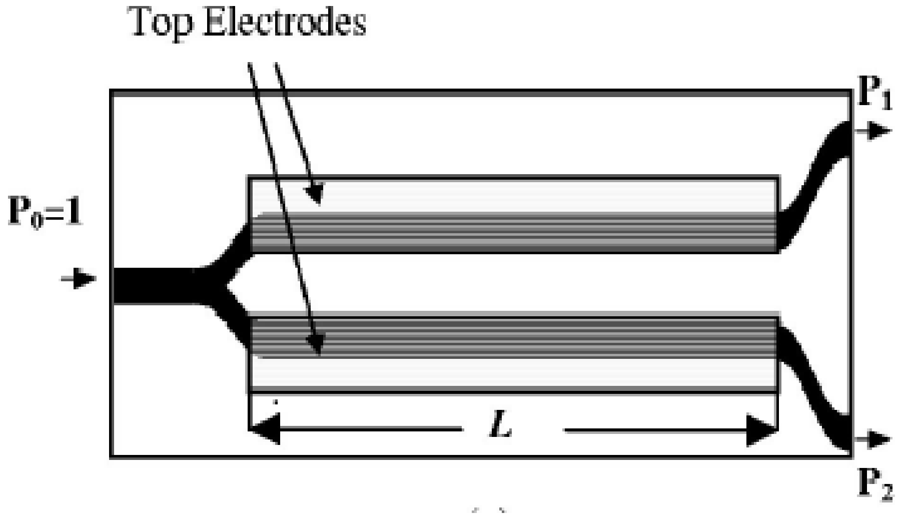
$$B = \frac{\kappa - \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}}A_1(0) \qquad D = \frac{\kappa + \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}}A_1(0)$$



$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(A_1(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)\right)$$

$$A_2(z) = e^{-\frac{i\Delta\beta}{2}z}\left(A_2(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)\right)$$

$$B = \frac{\kappa - \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}}A_1(0) \qquad D = \frac{\kappa + \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}}A_1(0)$$



When Δβ = 0,

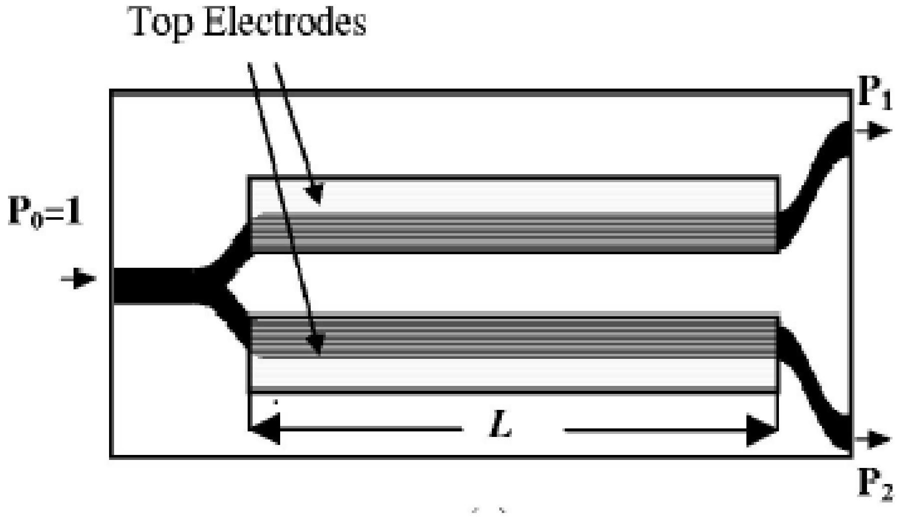
$$A_1(z) = A_1(0)\left(\cos(\kappa z) + i\sin(\kappa z)\right)$$

$$A_2(z) = A_1(0)\left(\cos(\kappa z) + i\sin(\kappa z)\right)$$

$$A_1(z) = e^{\frac{i\Delta\beta}{2}z}\left(A_1(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iB\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)\right)$$

$$A_2(z) = e^{-\frac{i\Delta\beta}{2}z}\left(A_2(0)\cos(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z) + iD\sin(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z)\right)$$

$$B = \frac{\kappa - \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}}A_1(0) \qquad D = \frac{\kappa + \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}}A_1(0)$$

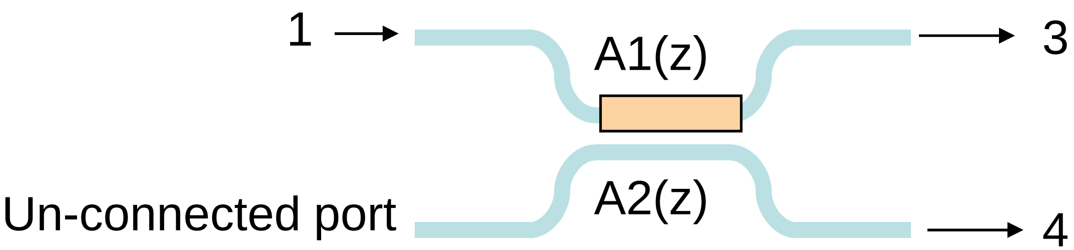


When $\Delta\beta \neq 0$,

$$P_1(z) = P_1(0)\left(\cos^2\left(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z\right) + \left(\frac{\kappa - \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}} \right)^2 \sin^2\left(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z\right) \right)$$

$$P_2(z) = P_2(0)\left(\cos^2\left(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z\right) + \left(\frac{\kappa + \frac{\Delta\beta}{2}}{\sqrt{(\Delta\beta/2)^2+\kappa^2}} \right)^2 \sin^2\left(\sqrt{(\Delta\beta/2)^2+\kappa^2}\,z\right) \right)$$

Traveling-wave electrodes:

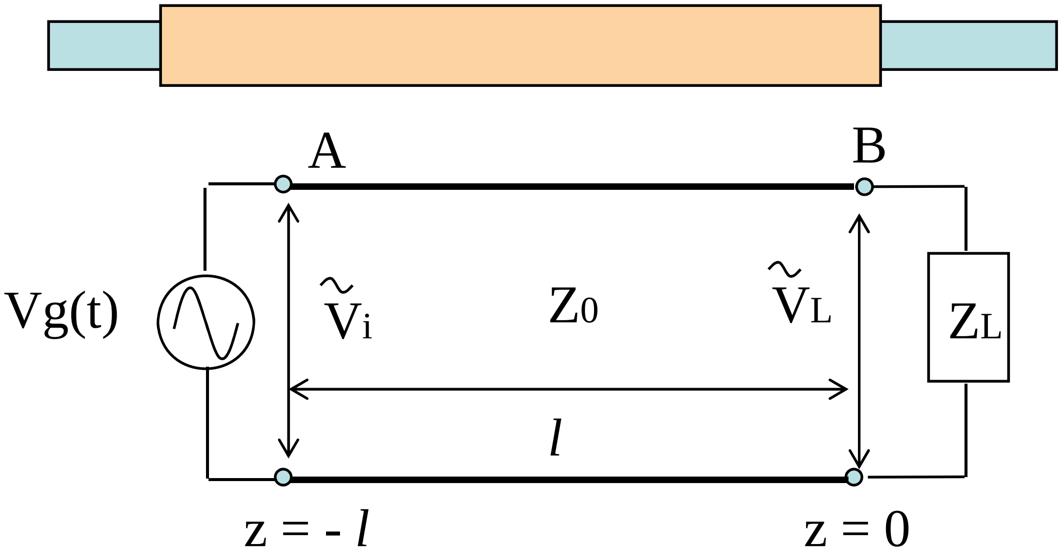


For A1(0) = A1, A2(0) =0

$$A_1(z) = A_1(0)\cos(\kappa z)$$

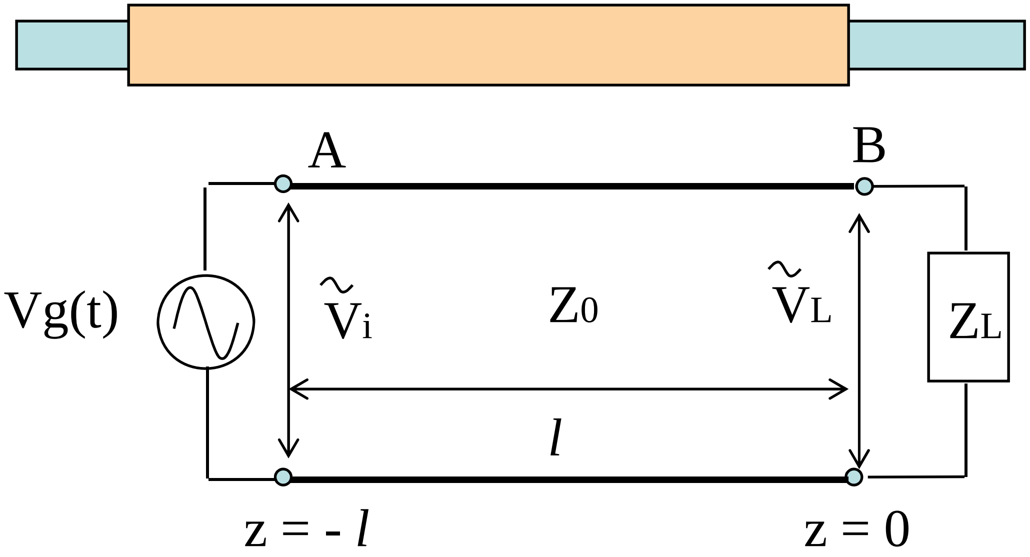
$$A_2(z) = iA_1(0)\sin(\kappa z)$$

For high-speed electrode using transmission lines:

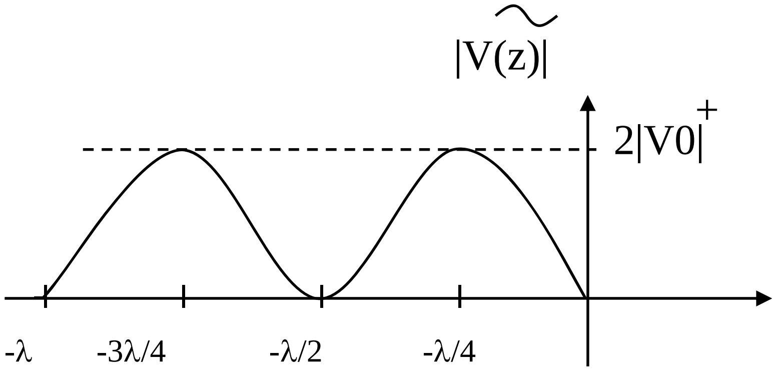


$$\Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Standing waves for impedance mismatching:



$$\Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

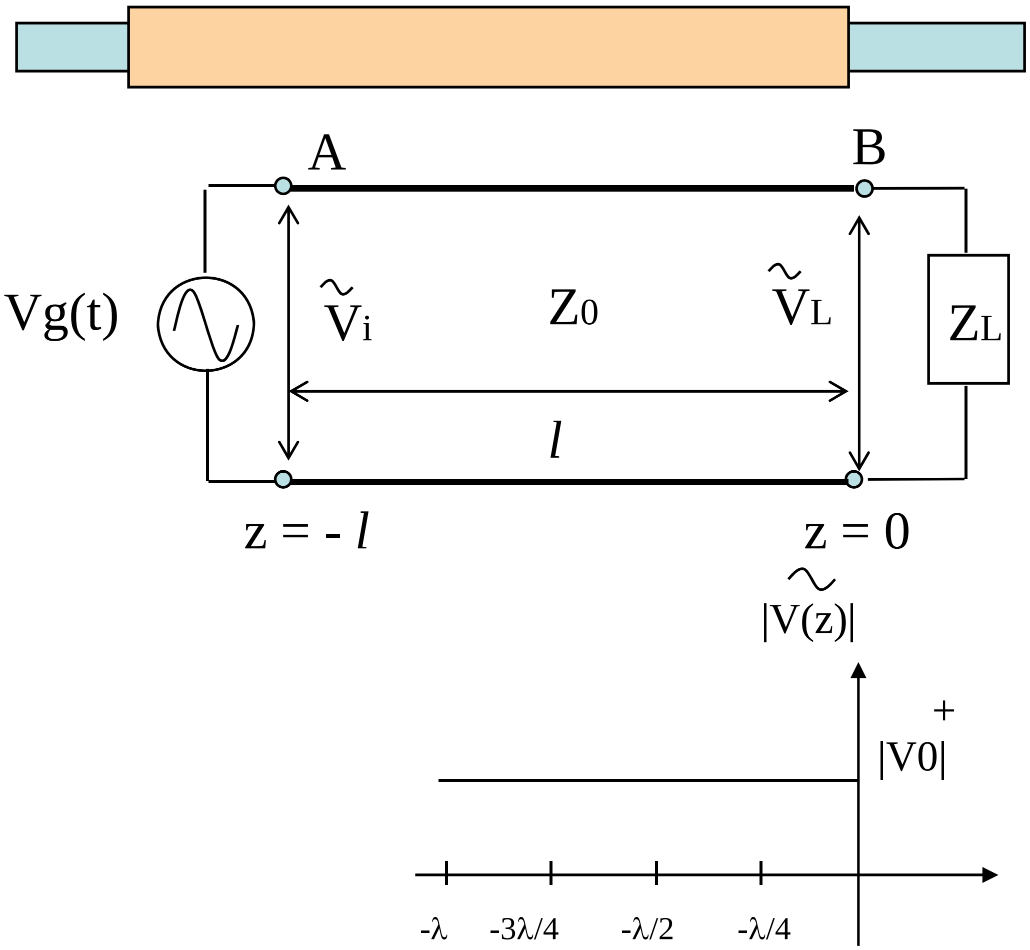


Standing waves

$$\Delta n = -\frac{1}{2} n_0^3 r_{33} E_z ,$$

Cancelled

Traveling waves for impedance matching:



$$\Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

traveling waves

$$\Delta n = -\frac{1}{2} n_0^3 r_{33} E_z ,$$

Kerr effect :

$$\Delta n = \lambda K E_a^2$$

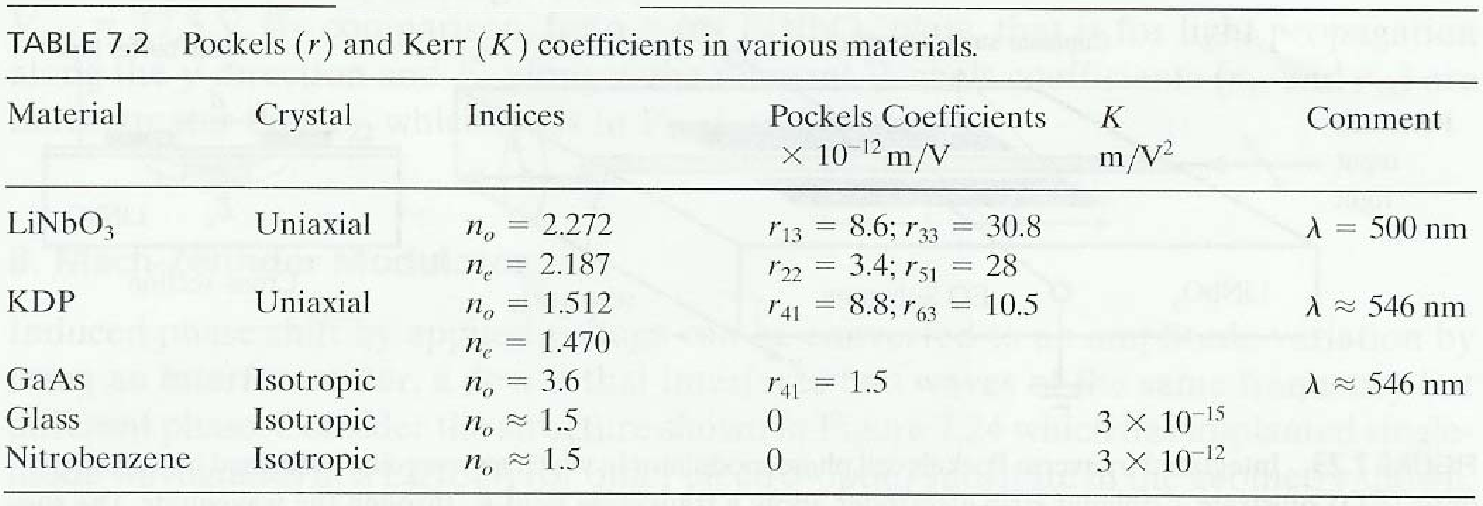
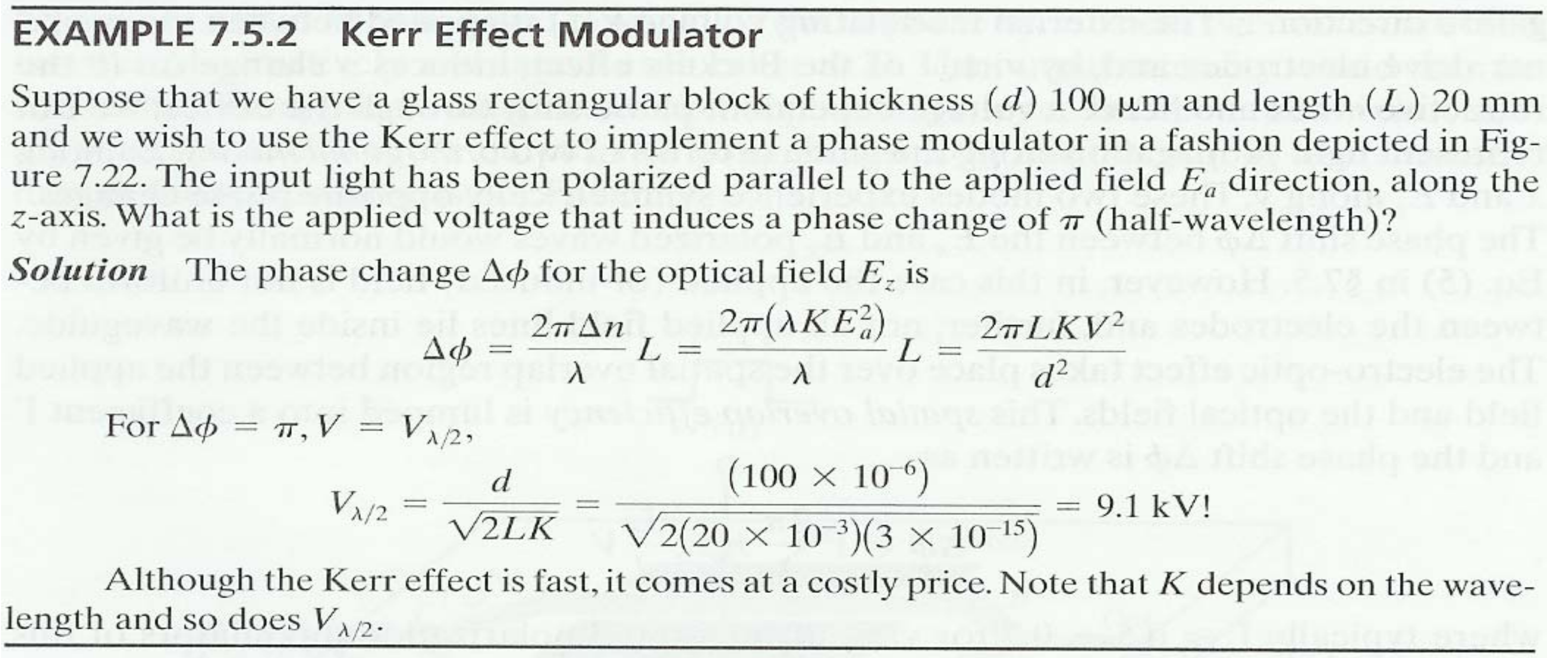


TABLE 7.2 Pockels (r) and Kerr (K) coefficients in various materials.

Material	Crystal	Indices	Pockels Coefficients $\times 10^{-12}$ m/V	K m/V^2	Comment
LiNbO$_3$	Uniaxial	$n_o = 2.272$ $n_e = 2.187$	$r_{13} = 8.6; r_{33} = 30.8$ $r_{22} = 3.4; r_{51} = 28$		$\lambda = 500$ nm
KDP	Uniaxial	$n_o = 1.512$ $n_e = 1.470$	$r_{41} = 8.8; r_{63} = 10.5$		$\lambda \approx 546$ nm
GaAs	Isotropic	$n_o = 3.6$	$r_{41} = 1.5$		$\lambda \approx 546$ nm
Glass	Isotropic	$n_o \approx 1.5$	0	3×10^{-15}	
Nitrobenzene	Isotropic	$n_o \approx 1.5$	0	3×10^{-12}	



EXAMPLE 7.5.2 Kerr Effect Modulator

Suppose that we have a glass rectangular block of thickness (d) 100 μm and length (L) 20 mm and we wish to use the Kerr effect to implement a phase modulator in a fashion depicted in Figure 7.22. The input light has been polarized parallel to the applied field E_a direction, along the z-axis. What is the applied voltage that induces a phase change of π (half-wavelength)?

Solution The phase change $\Delta\phi$ for the optical field E_z is

$$\Delta\phi = \frac{2\pi\Delta n}{\lambda} L = \frac{2\pi(\lambda K E_a^2)}{\lambda} L = \frac{2\pi L K V^2}{d^2}$$

For $\Delta\phi = \pi, V = V_{\lambda/2}$,

$$V_{\lambda/2} = \frac{d}{\sqrt{2LK}} = \frac{(100 \times 10^{-6})}{\sqrt{2(20 \times 10^{-3})(3 \times 10^{-15})}} = 9.1 \text{ kV!}$$

Although the Kerr effect is fast, it comes at a costly price. Note that K depends on the wavelength and so does $V_{\lambda/2}$.

Applications of Kerr effect :

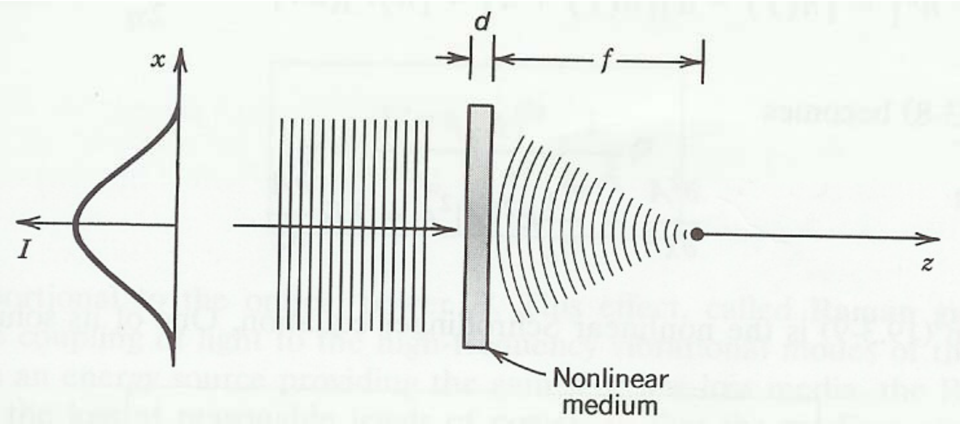
$$\Delta n = \lambda K E^{2} = \propto I = n_2 I ,$$

- Self-phase modulation

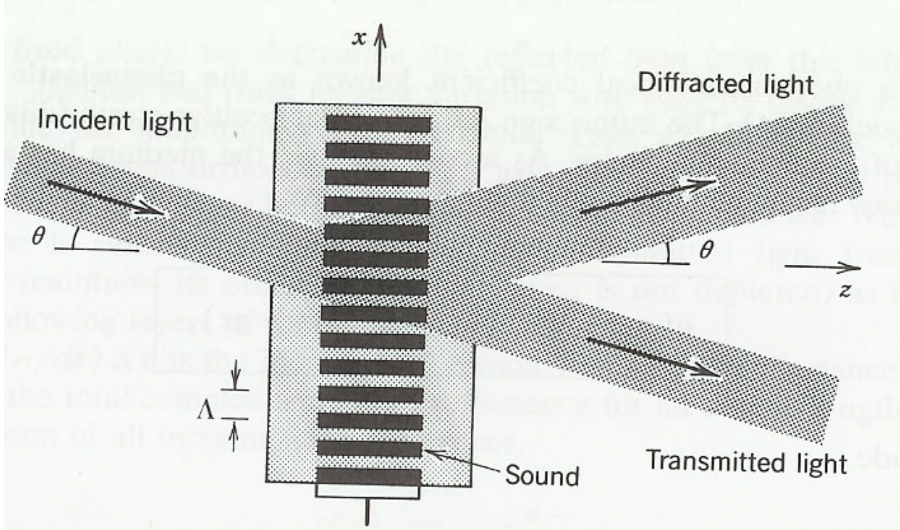
$$\Delta\varphi = 2\pi n_2 \frac{L}{\lambda_o A} P, \qquad P_\pi = \lambda_o A/2Ln_2.$$

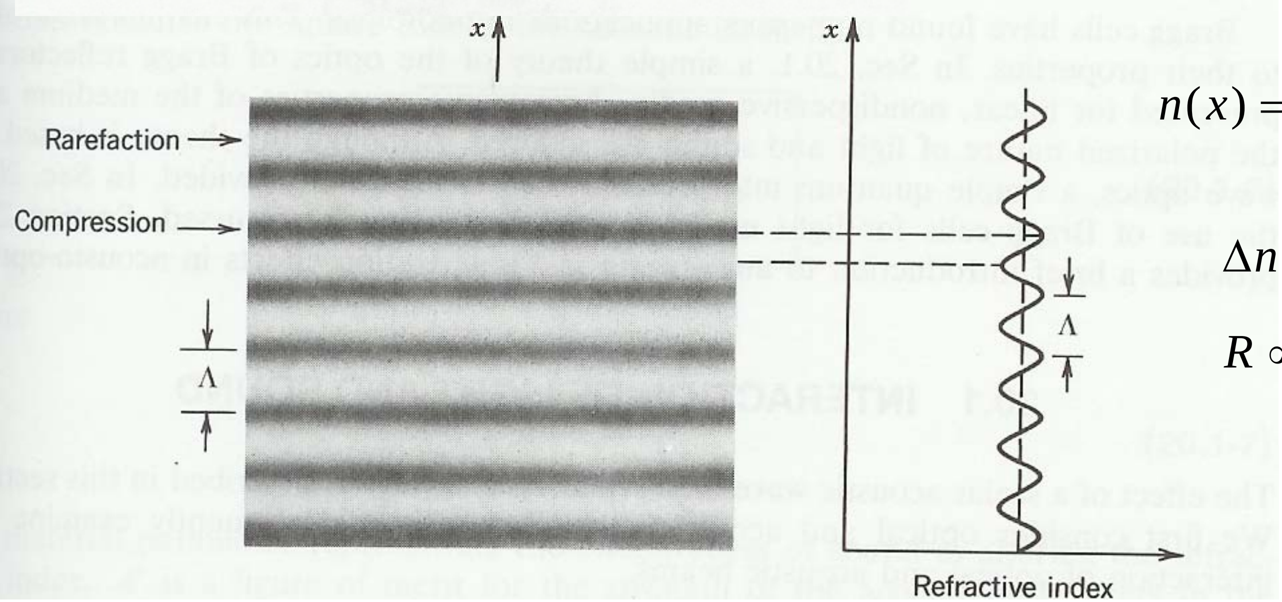
$$A = 10^{-2}\ \mathrm{mm}^2, \qquad n_2 = 10^{-10}\ \mathrm{cm}^2/\mathrm{W} \qquad L = 1\ \mathrm{m}, \qquad P_\pi = 0.5\ \mathrm{W}.$$

- Self-Focusing

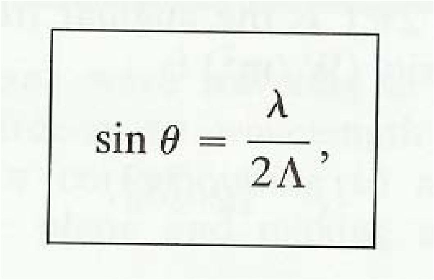


Acoustic-optic Modulators

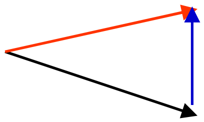




- Bragg condition:



$$\boxed{\sin\theta = \frac{\lambda}{2\Lambda},}$$



$$2\frac{2\pi}{\lambda}\sin\theta = \frac{2\pi}{\Lambda},$$

$$n(x) = n_0 - \Delta n_0 \cos(\Omega t - \frac{2\pi}{\Lambda}x),$$

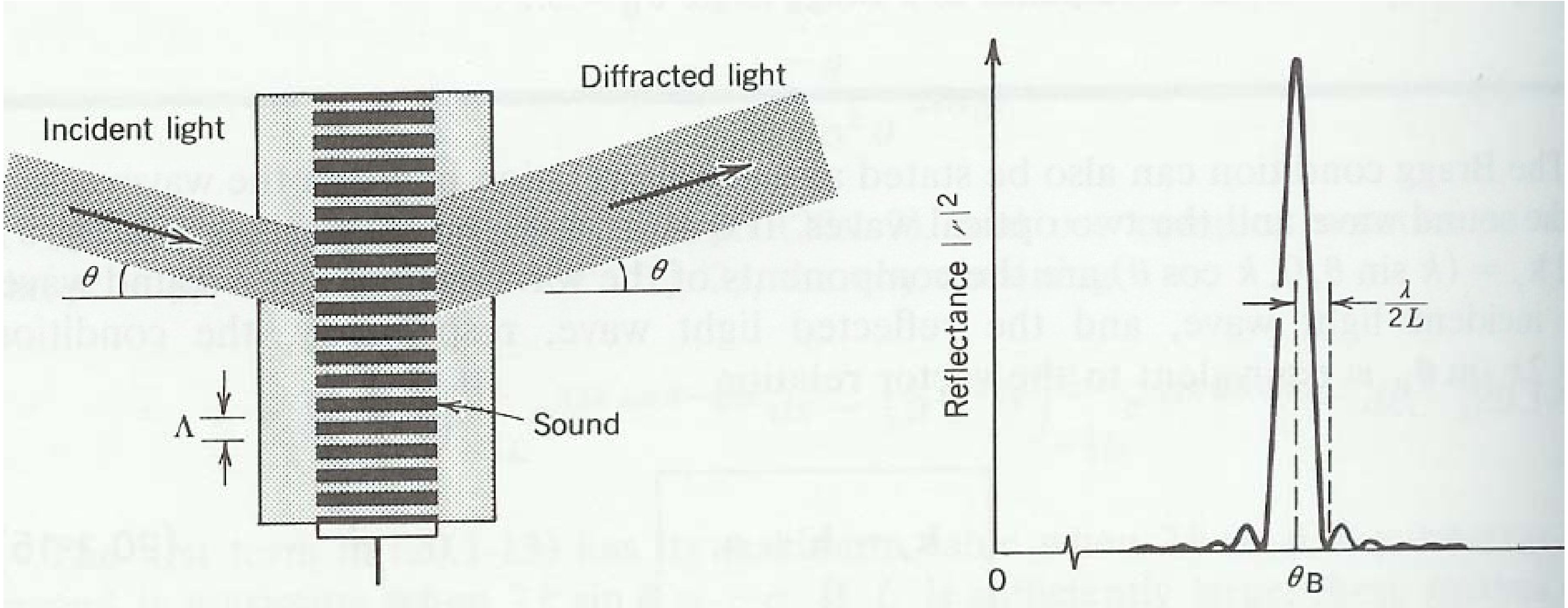
$$\Delta n \propto I_s^{\frac{1}{2}},$$

$$R \propto \Delta n^2 = \propto I_s,$$

Doppler shift

$$\boxed{\omega_r = \omega + \Omega.}$$

Angle mismatch

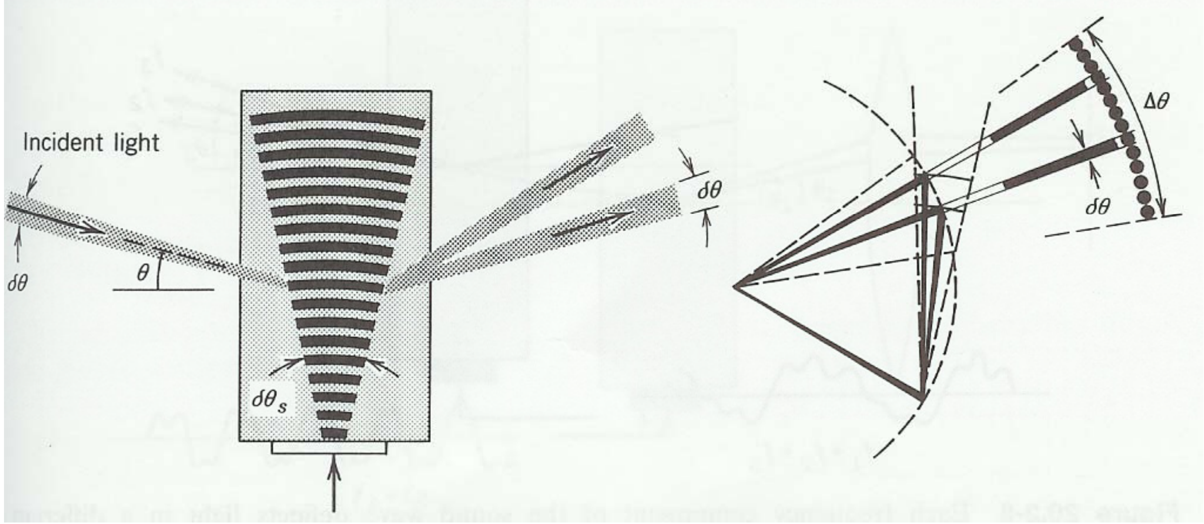


Acoustic –optics devices

- Modulator

Bandwidth: $\sin\theta \approx \theta \approx \frac{\lambda}{2\Lambda} = \frac{\lambda}{2v_s} f, \qquad \delta\theta = \frac{\lambda}{v_s} B = \frac{\lambda}{D},$

- Beam Scanner

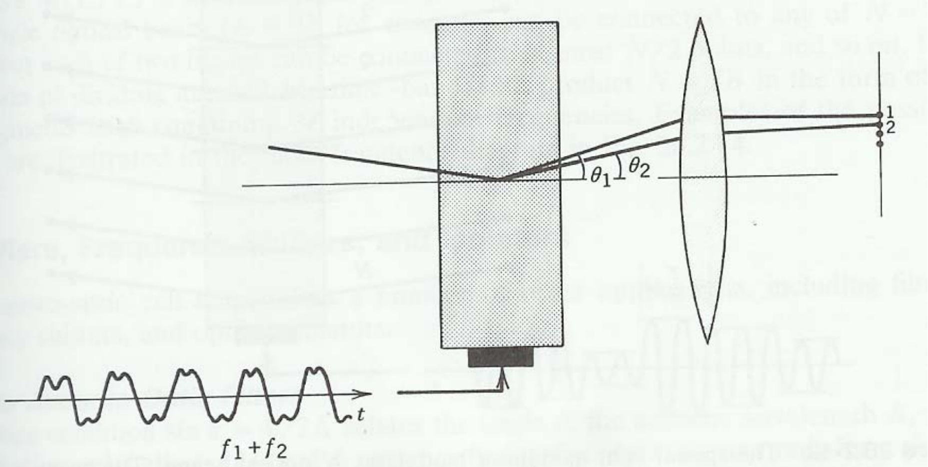


Number of resolvable spots:

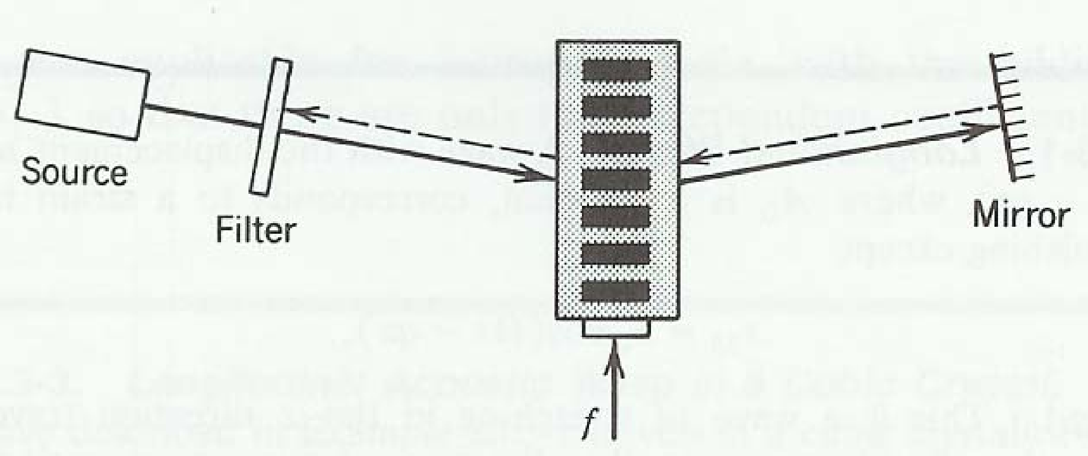
$$N = \frac{\Delta\theta}{\delta\theta}$$

Acoustic –optics devices

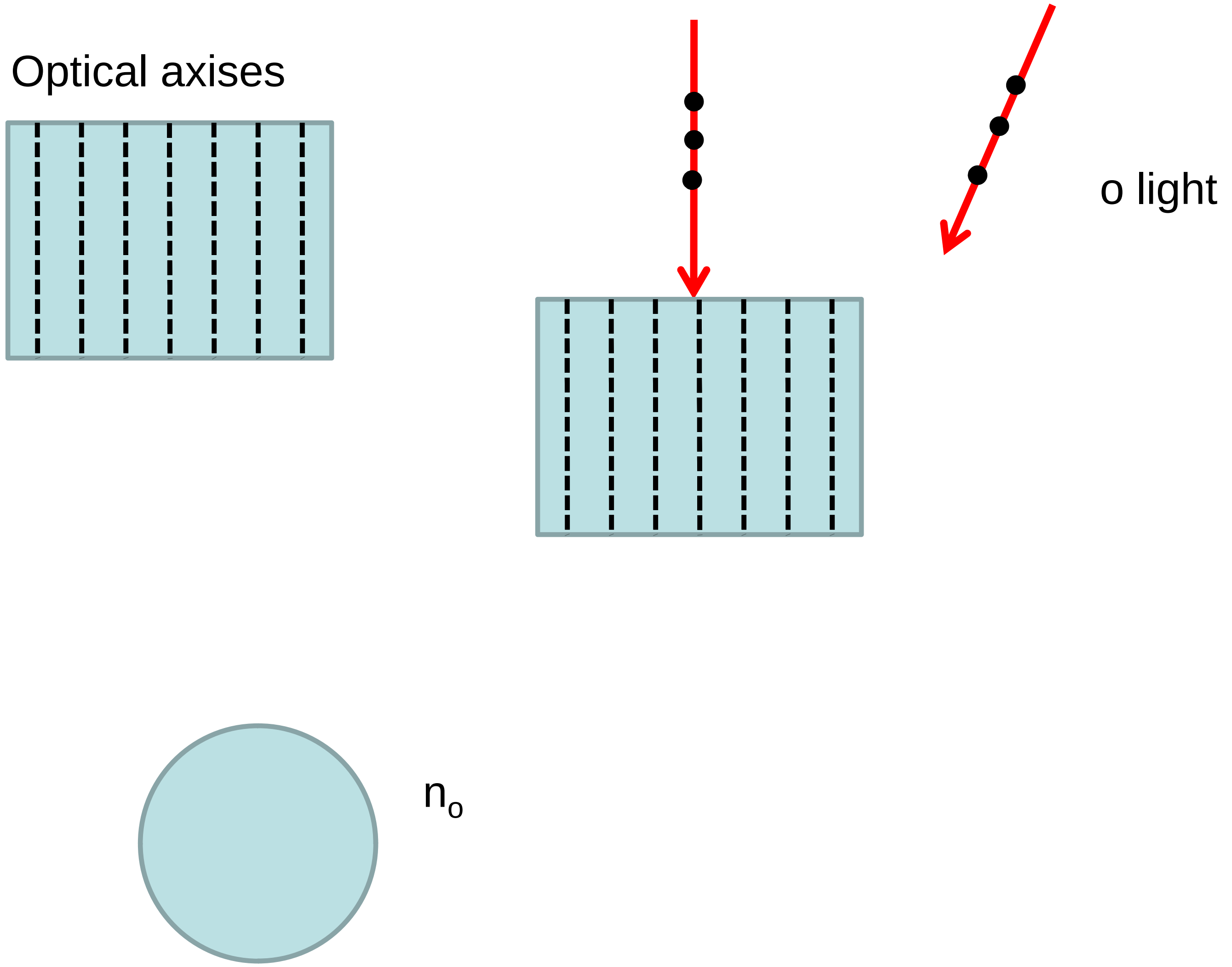
- Free space inter-connector



- Isolator



Light propagation in anisotropic crystals

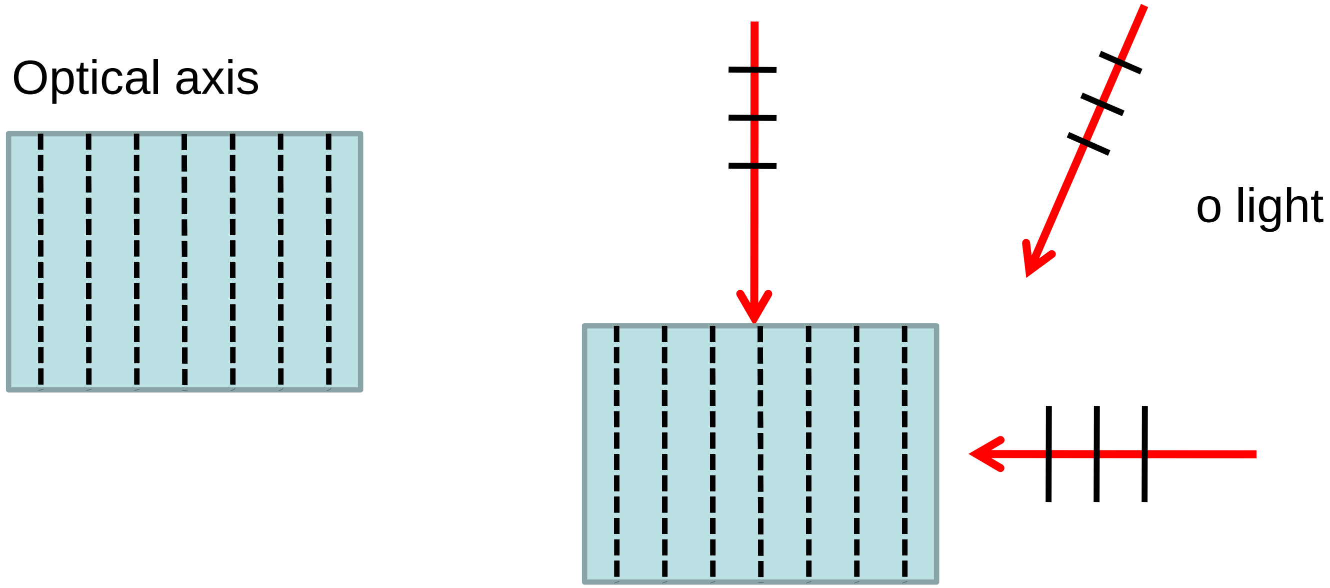


Optical axises

o light

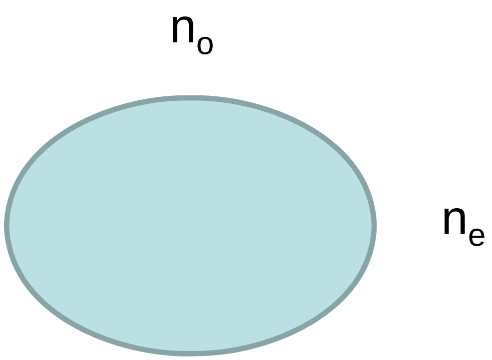
n_o

Light propagation in anisotropic crystals



Optical axis

o light

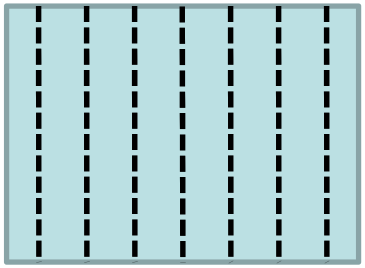


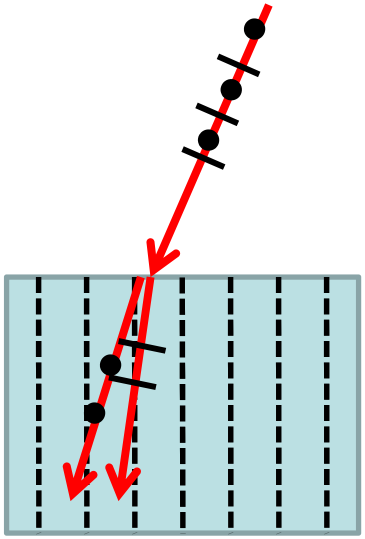
n_o

n_e

Light propagation in anisotropic crystals

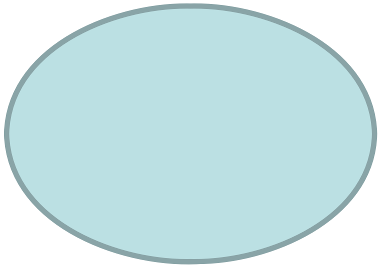
Optical axis





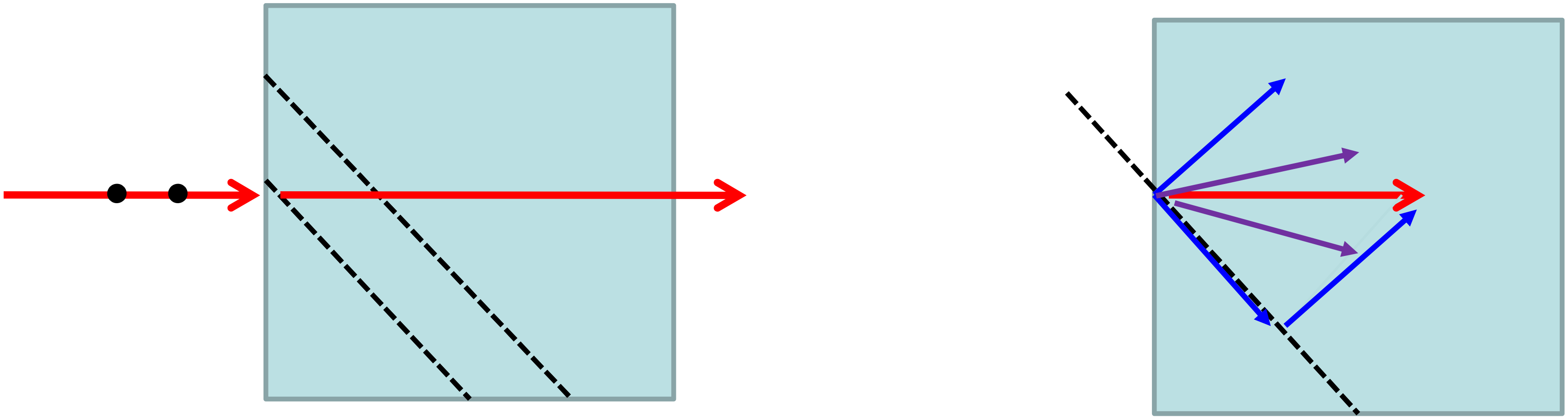
Polarization beam splitter

n_o

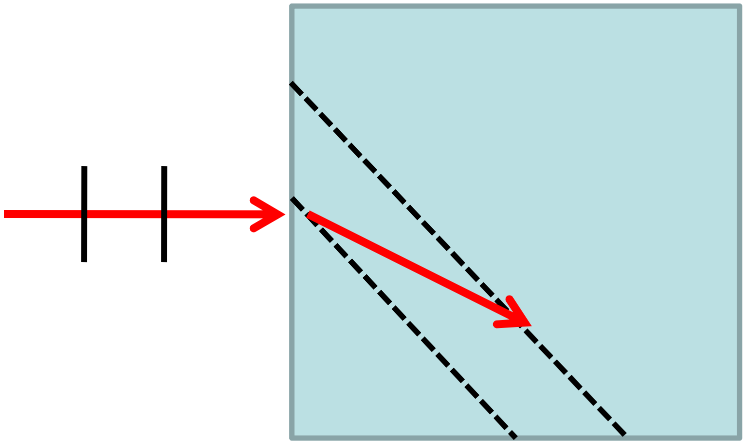


n_e

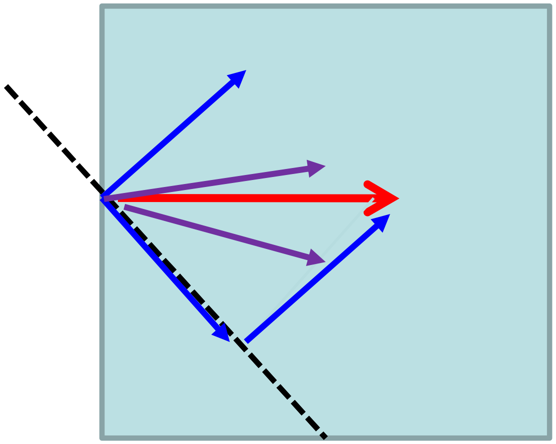
Light propagation in anisotropic crystals



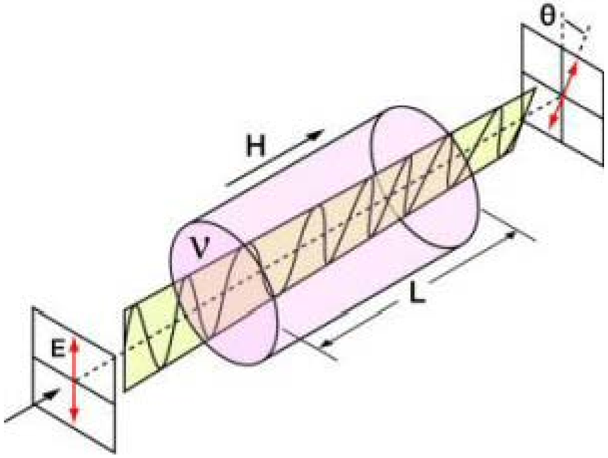
Light propagation in anisotropic crystals



$n_e > n_o$



Faraday rotator



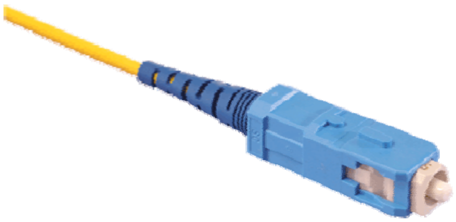
Fiber–optics devices

- Coupler

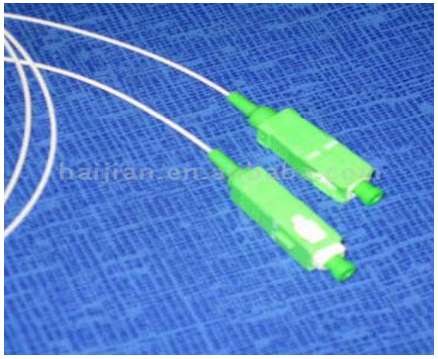
FC/PC

FC/APC

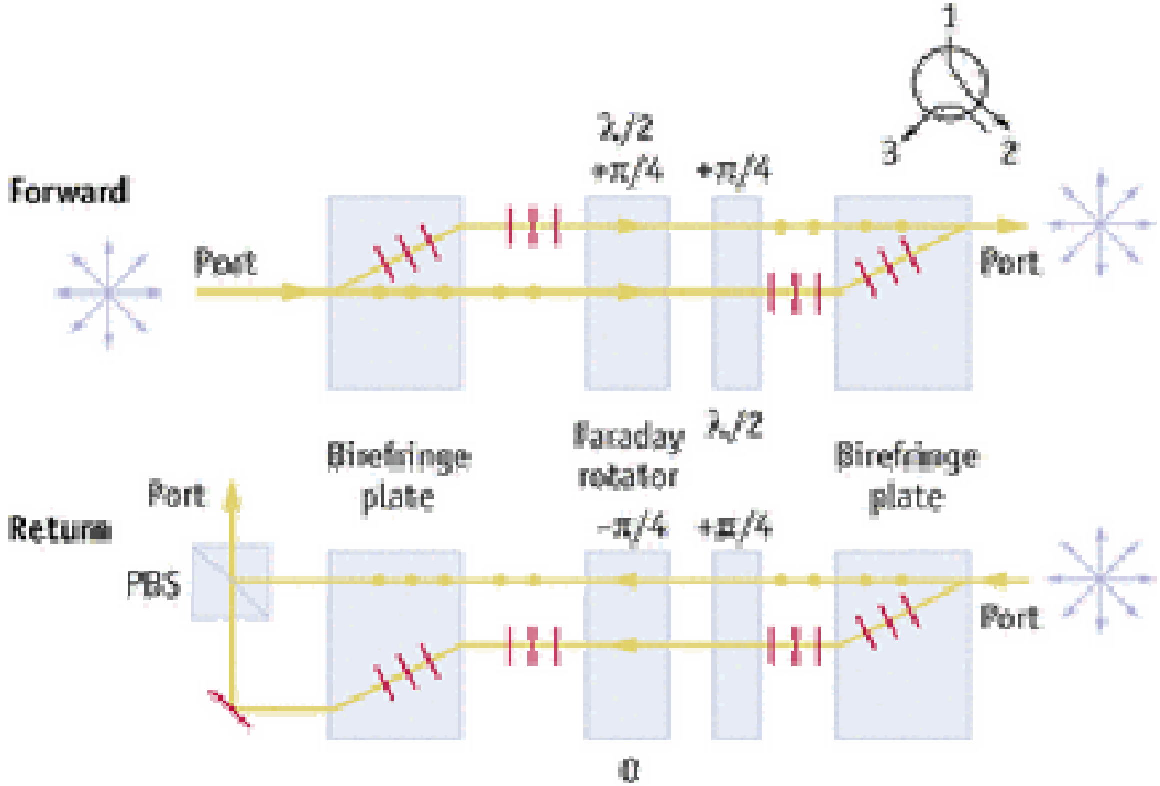
SC/PC



SC/APC



Circulator


1
3
2
Forward
Port
λ/2
+π/4
+π/4
Port
Birefringe plate
Faraday rotator
λ/2
Birefringe plate
Return
Port
PBS
-π/4
+π/4
Port
α