Topics:

- Introduction to Optical Amplifier – types and applications
- Erbium-doped fiber amplifier (EDFA)
- External pumping and amplifier gain
- Power-conversion efficiency and gain profile

Some basic concepts

- Power in dBm

$$P(dBm) = 10\log[\frac{P(mW)}{1(mW)}],$$

- Examples:

$$P = 1(mW) = 10\log[\frac{P(mW)}{1(mW)}](dBm) = 0(dBm),$$

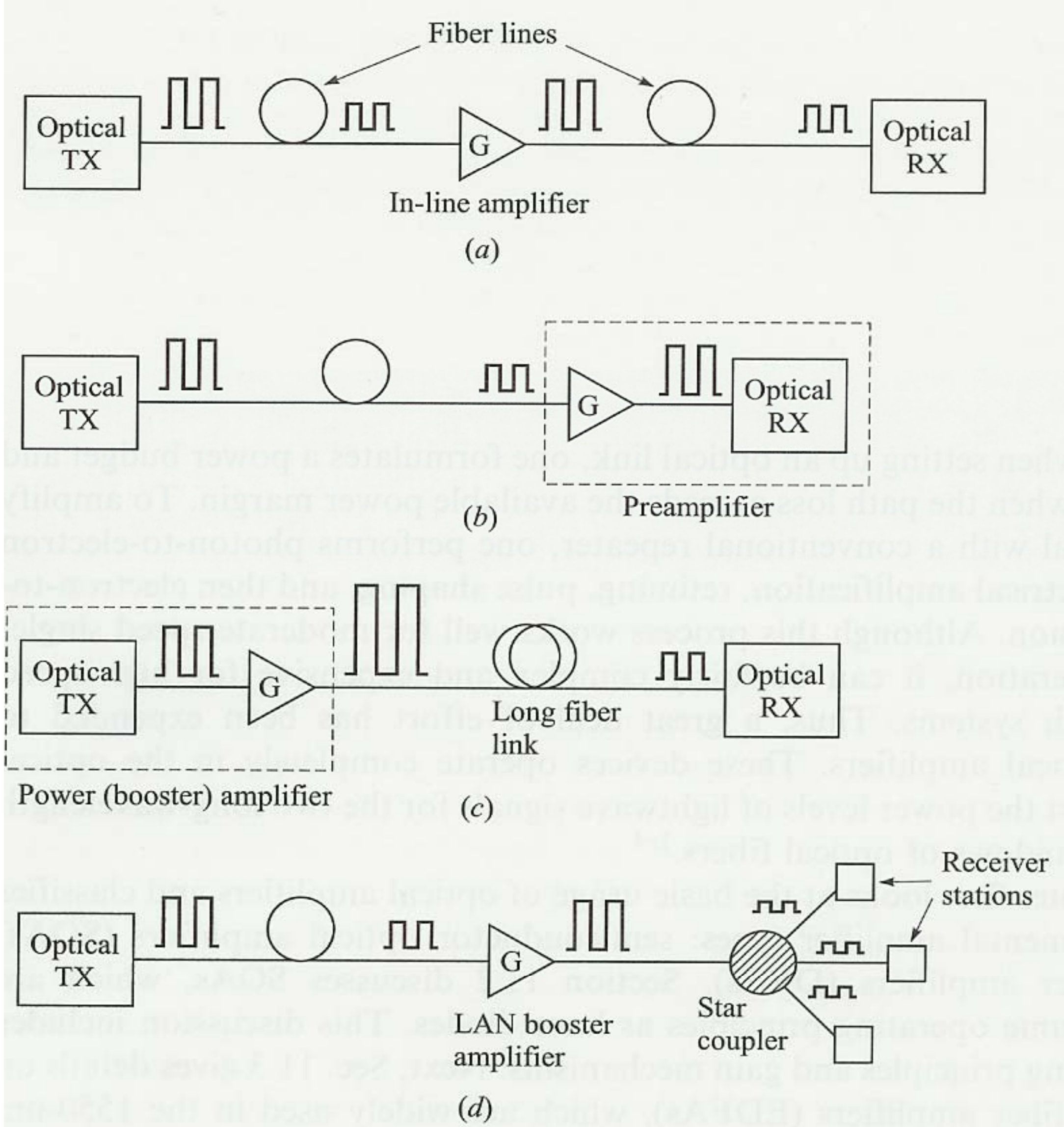
$$P = 2(mW) = 3(dBm), \quad P = 10(mW) = 10(dBm), \quad P = 1(\mu W) = -30(dBm),$$

- Noise figure (NF): SNR degradation

$$NF|_{linear} = \frac{(S/N)_{input}}{(S/N)_{output}},$$

$$NF|_{dB} = 10\log\frac{(S/N)_{input}}{(S/N)_{output}} = 10\log(S/N)_{input} - 10\log(S/N)_{output},$$

Introduction to optical amplifier

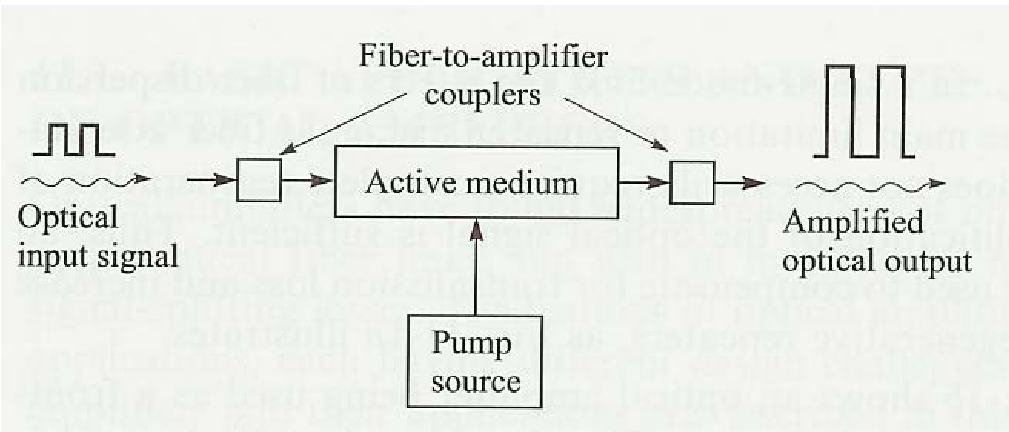


Applications:

- in-line amplifier
  - input –25 to -20dBm dBm
  - output ~ 0 to 5dBm
  - Gain ~ 20dB
  - Noise figure <4dB
- Preamplifier
  - input –35 to -30dBm dBm
  - output ~ -15 to -10dBm
  - Gain ~ 20dB, Noise figure <5dB
- Power (booster) amplifier
  - input –5 to -0dBm dBm
  - output ~ 10 to 20dBm
  - Gain ~ 15dB, Noise figure <4dB
- LAN-booster
  - input -25 to -20dBm dBm
  - output ~ 5 to 15dBm
  - Gain ~ 30dBm, Noise figure <4dB

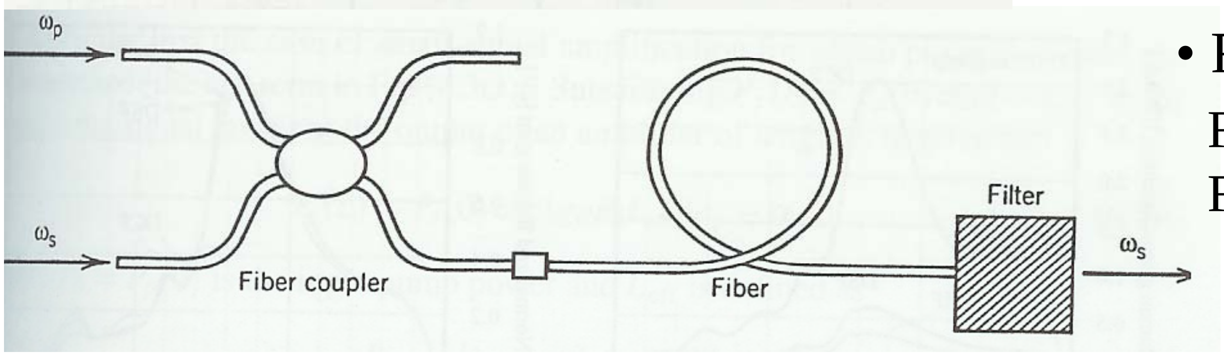
Types of Optical Amplifiers

- Erbium doped fiber amplifier
- Raman amplifier
- Semiconductor amplifier



- Erbium doped fiber amplifier

Active gain medium needed



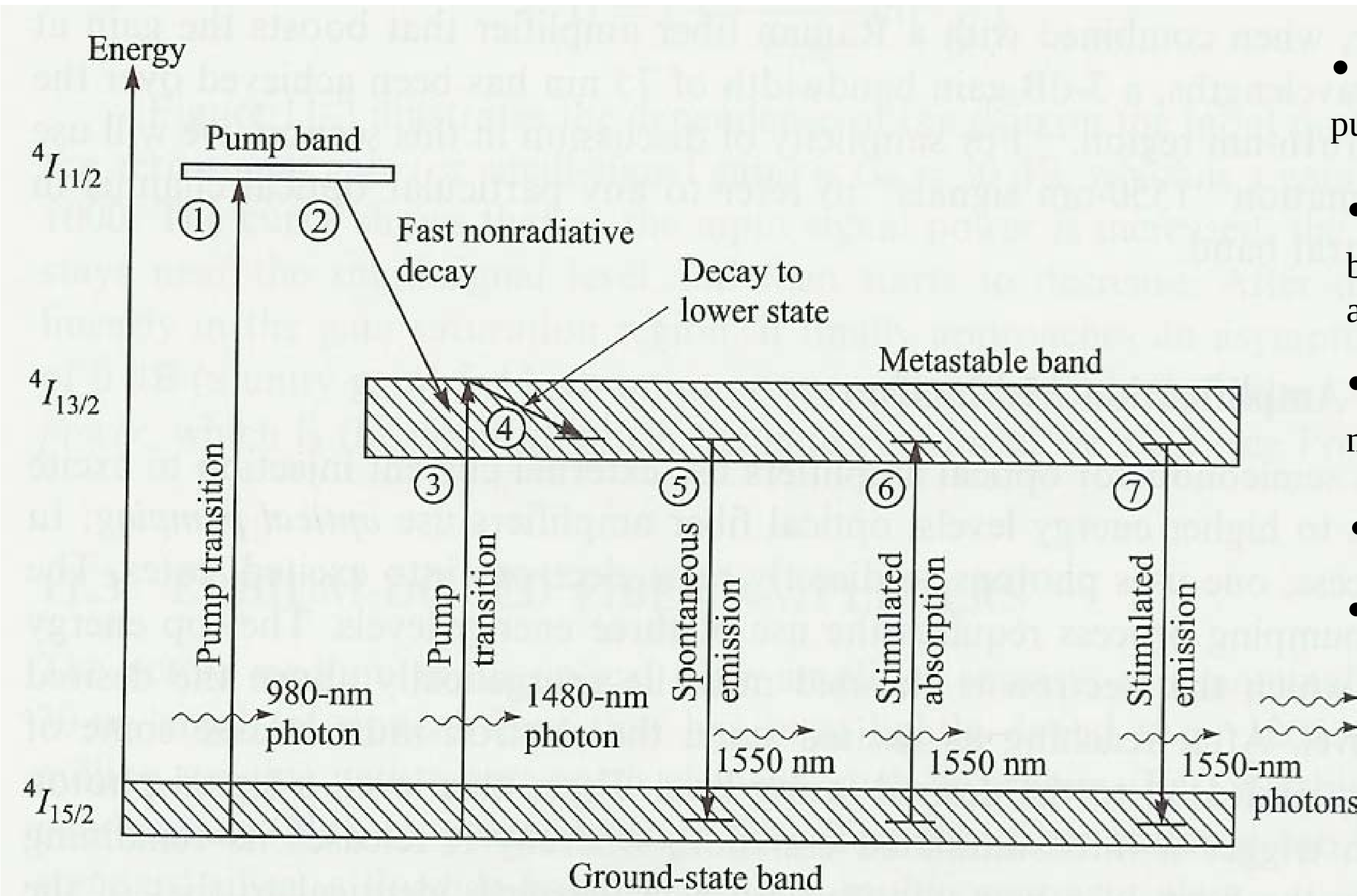
- Raman amplifier

Fiber nonlinearity

Filter needed

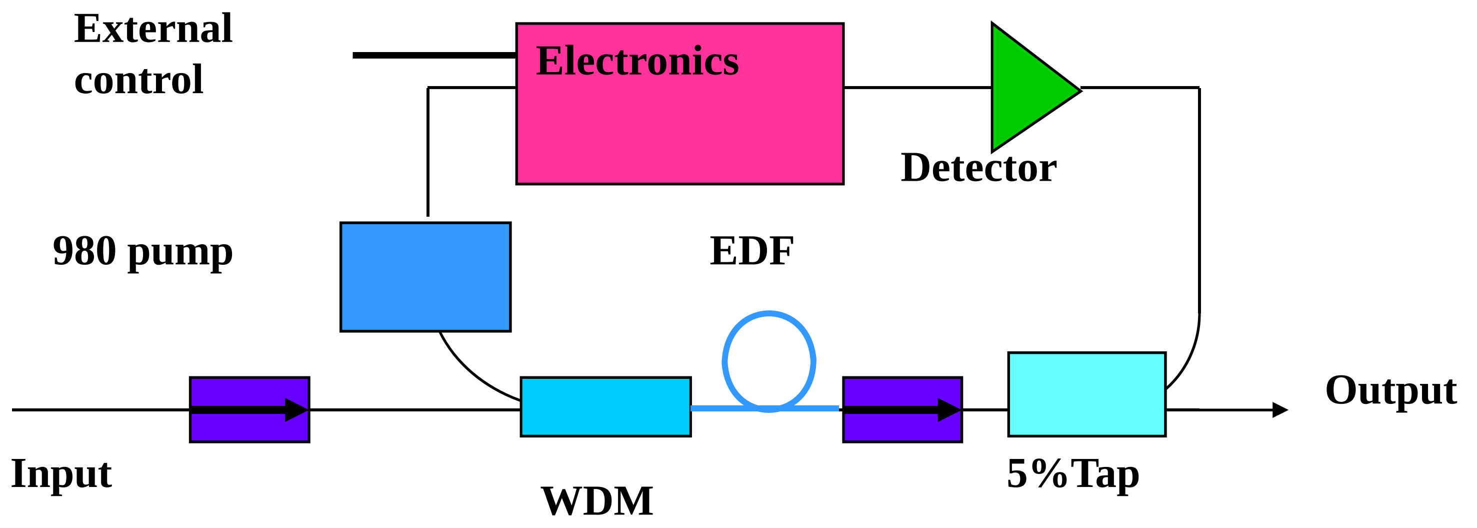
Erbium doped fiber amplifier

- Optical pumping, 980nm or 1480nm laser

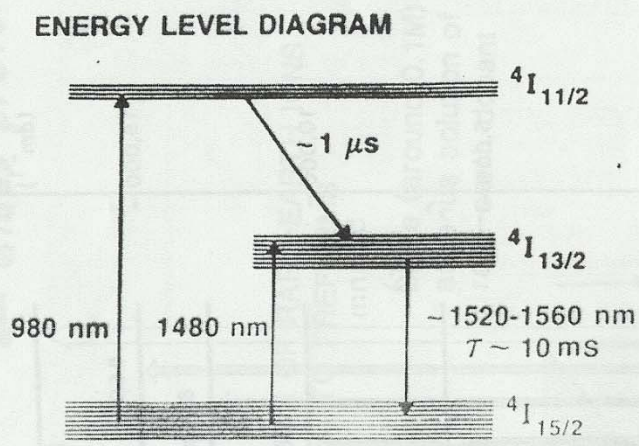


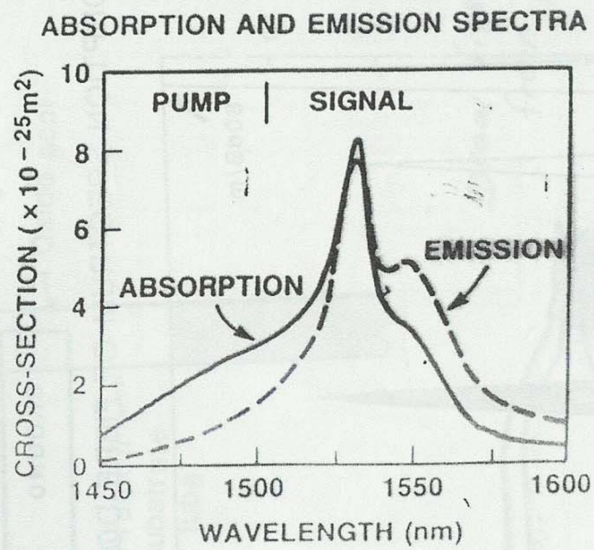
- Pump band is narrow, so the pumping laser has to be narrow
- Population inversion between the metastable band and the ground band
- pump band lifetime ~1us metastable lifetme ~10ms.
- Multi-channel amplification
- cross-talk issues

Amplifier architecture



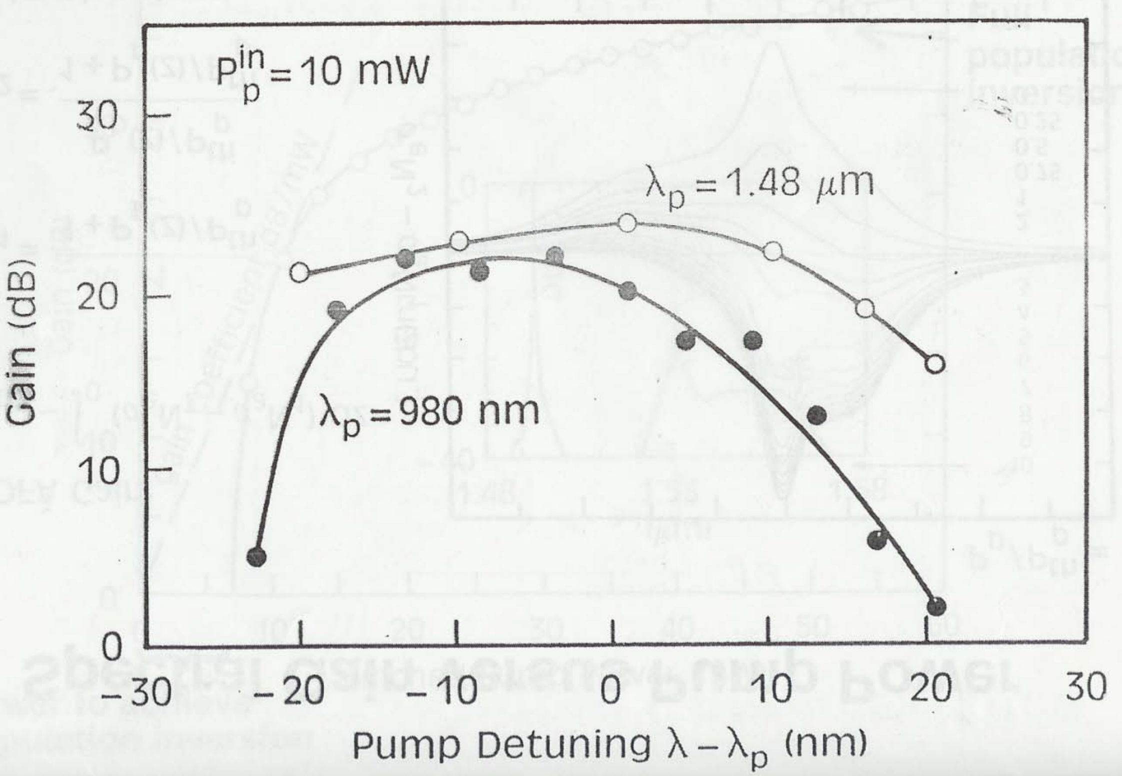
Er-doped Glass Pump Bands



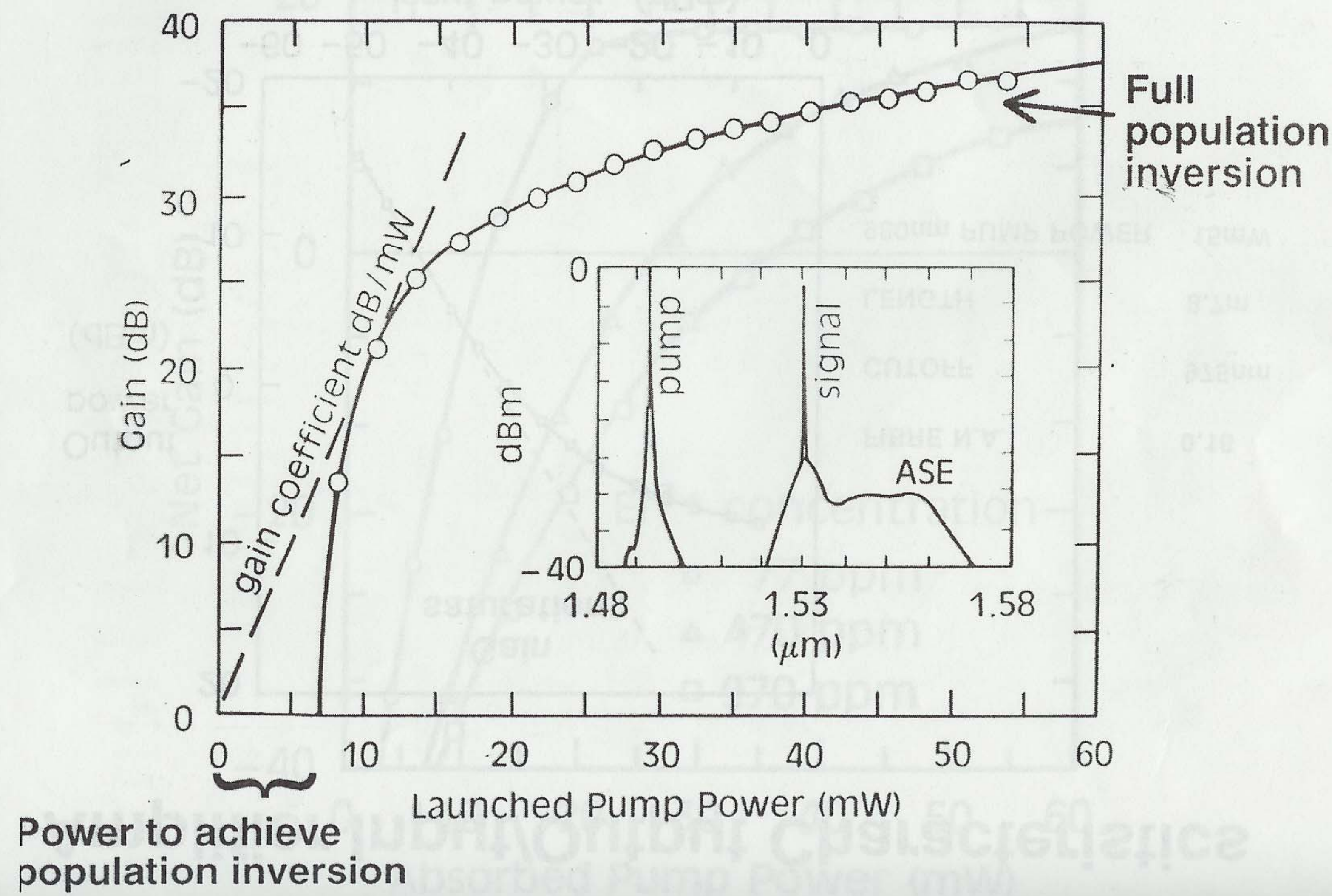


	980 (nm)	1490 (nm)
Pump/signal λMUX	Easy	Difficult
Pump efficiency	~~4~~ 11 dB/mW	2.~~1~~6 dB/mW
Spectral gain	Peaky	Smoother
Noise figure	~ 3 dB	~ 5 dB

Spectral Gain Dependence on Pump Wavelength



Optical Gain versus Pump Power



Spectral Gain versus Pump Power

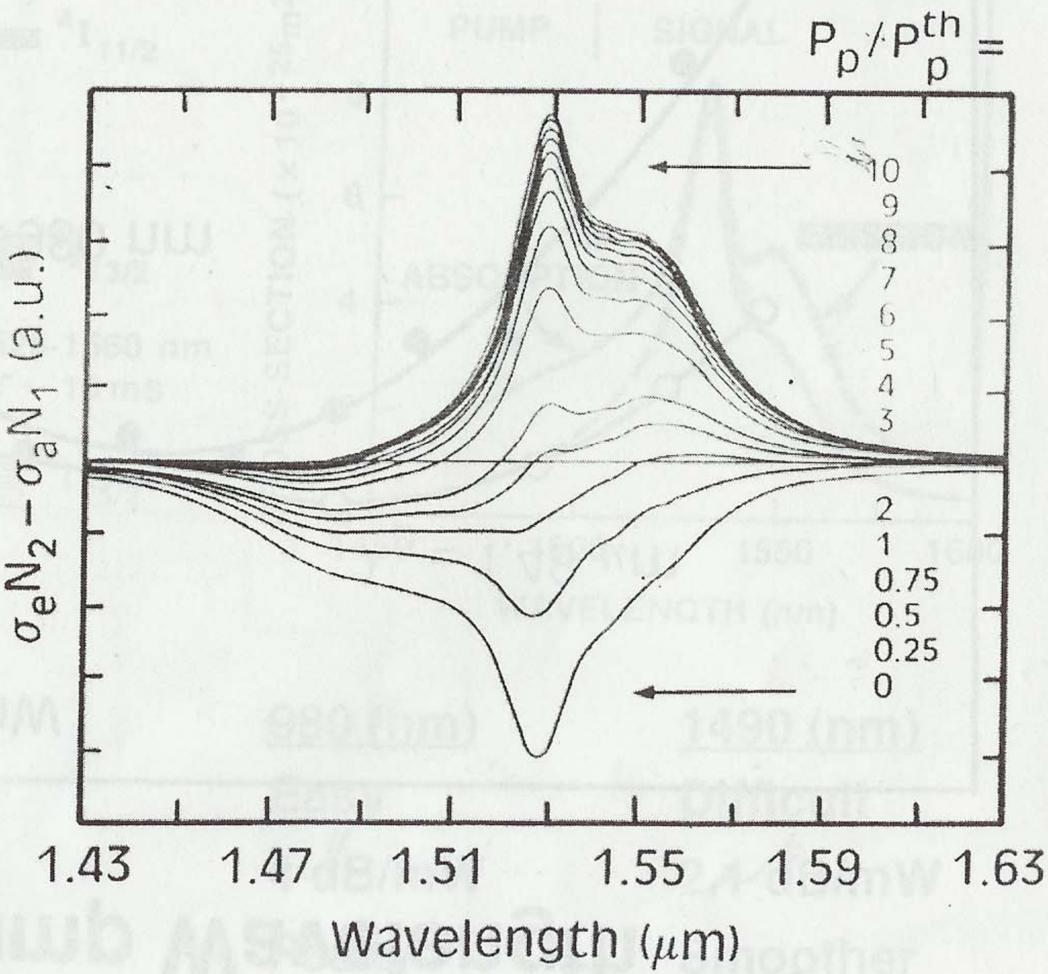
EDFA Gain:

$$G_{dB} \sim \int_0^L (\sigma_e N_2 - \sigma_a N_1)\, dz$$

$$N_1 = \frac{1}{1 + P_p(z)/P_p^{th}}$$

$$N_2 = \frac{P_p(z)/P_p^{th}}{1 + P_p(z)/P_p^{th}}$$

(3-level system)



Power conversion efficiency

$$P_{s,out} \le P_{s,in} + \frac{\lambda_p}{\lambda_s} P_{p,in},$$

$$PCE = \frac{P_{s,out} - P_{s,in}}{P_{p,in}} \approx \frac{P_{s,out}}{P_{s,in}} \le \frac{\lambda_p}{\lambda_s},$$

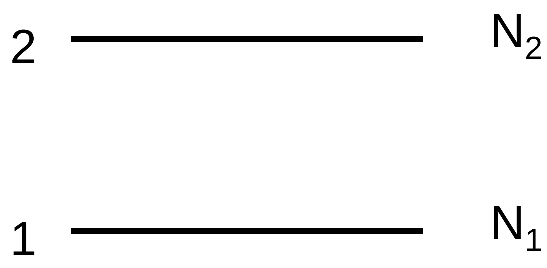
$$P_{s,in} \le \frac{(\lambda_p / \lambda_s) P_{p,in}}{G-1},$$

Consider an EDFA being pumped at 980nm with a 30-mW pump. If the gain at 1550nm is 20dB, then what's the maximum input power?

$$P_{s,in} \le \frac{(\lambda_p / \lambda_s) P_{p,in}}{G-1} = 190\mu W = -7.2dBm,$$

Einstein's A and B coefficients

Assuming a two-level system:



Spontaneous emission rate:

$$R_{sp} = A_{21} N_2,$$

Stimulated emission rate:

$$R_{st} = B_{21} N_2 P(h\nu),$$

Absorption rate:

$$R_{ab} = B_{12} N_1 P(h\nu),$$

Steady state:

$$R_{sb} = R_{st} + R_{sp}$$

$$A_{21} N_2 + B_{21} N_2 P(h\nu) = B_{12} N_1 P(h\nu)$$

$$A_{21} N_2 = B_{12} N_1 P(h\nu) - B_{21} N_2 P(h\nu)$$

Einstein's A and B coefficients

Assuming a two-level system:

2 N_2

1 N_1

$$A_{21} N_1 e^{-\frac{E_2 - E_1}{kT}} = N_2 P(h\nu) d\nu \left(B_{12} e^{\frac{E_2 - E_1}{kT}} - B_{21} \right)$$

$$P(h\nu) d\nu = \frac{8\pi h \nu^3}{c^3} \frac{d\nu}{e^{\hbar\omega/kT} - 1}$$

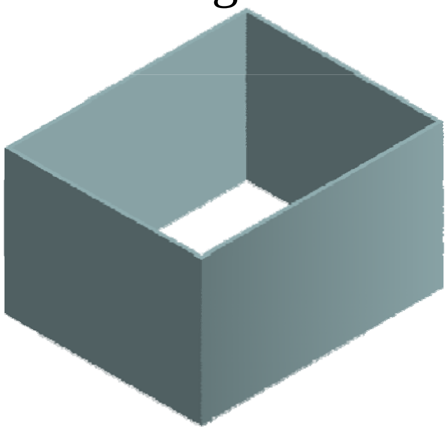
$$A_{21} N_1 e^{-\frac{E_2 - E_1}{kT}} = N_1 e^{-\frac{E_2 - E_1}{kT}} \frac{8\pi \nu^3}{c^3} \frac{d\nu}{e^{\hbar\omega/kT} - 1} \left(B_{12} e^{\frac{E_2 - E_1}{kT}} - B_{21} \right)$$

$$B_{21} = B_{12}$$

$$A_{21} = B_{12} \frac{8\pi h \nu^3}{c^3} d\nu$$

Photon energy density

Assuming a blackbody cavity:

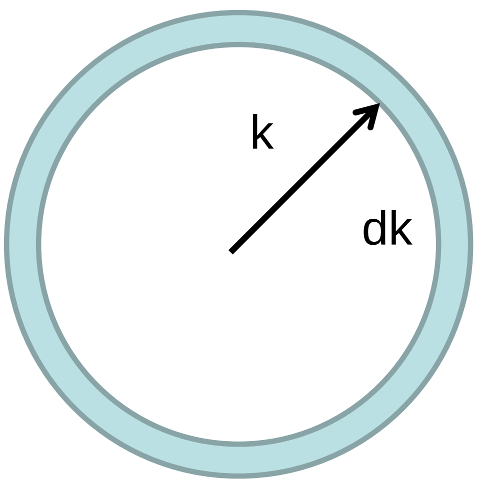


L_x L_y L_z

$$\sin k_m L_x = 0 \qquad k_m = \pm \frac{m\pi}{L_x}$$

Volume of each mode at k-space

$$\frac{2\pi}{L_x}\frac{2\pi}{L_y}\frac{2\pi}{L_z}$$



Volume between k to k+dk

$$2 \times 4\pi k^2 dk$$

# of modes between k to k+dk

$$\frac{8\pi k^2 dk}{\left(\frac{2\pi}{L_x}\frac{2\pi}{L_y}\frac{2\pi}{L_z}\right)} = \frac{8\pi k^2 dk}{(2\pi)^3} V$$

Photon energy density

of modes between k to k+dk per volume

$$\frac{8\pi k^2 dk}{(2\pi)^3}$$

E and k relation $$k = \frac{2\pi}{\lambda} = \frac{2\pi v}{c}$$

# of modes between v to v+dv per volume

$$\frac{8\pi}{(2\pi)^3}\left(\frac{2\pi v}{c}\right)^2 \frac{2\pi}{c} dv = \frac{8\pi v^2}{c^3} dv$$

Photon energy density for hv:

$$\left(\frac{8\pi v^2}{c^3} dv\right) hv \times N_{photon/\mathrm{mode}} = \frac{8\pi h v^3}{c^3} \frac{dv}{e^{hv/kT} - 1}$$

$$\boxed{P(hv)dv = \frac{8\pi h v^3}{c^3} \frac{dv}{e^{hv/kT} - 1}}$$

Photon energy density

$$P(hv)dv = \frac{8\pi hv^3}{c^3}\frac{dv}{e^{hv/kT}-1}$$

For blackbody

$$B_{21} = B_{12}$$

$$A_{21} = B_{12}\frac{8\pi hv^3}{c^3}dv$$

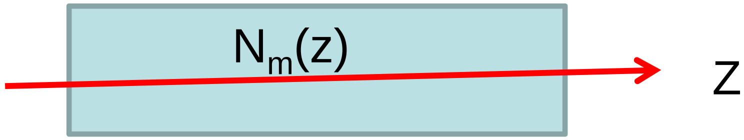
Einstein's A and B coefficients for Blackbody

$$B_{21} = B_{12}$$

$$A_{21} = B_{12}N_{\mathrm{mod}\,e}hv$$

Einstein's A and B coefficients for general cases

For a two-level amplifier



$$\frac{dN_m}{dz} = \left(N_2 - N_1\right)P_m(hv)dvB_{21} + A_{21}N_2\,|_{N_m},$$

$$A_{21} = N_{\mathrm{mode}}B_{21}hv, \qquad P(hv)dv = N_{\mathrm{mode}}hvN_m$$

Assuming only one mode

$$\frac{dN_m}{dz} = (N_2 - N_1)B_{21}hvN_m + B_{21}N_2hv,$$

$$\frac{dN_m}{(N_2 - N_1)N_m + N_2} = B_{21}hvdz,$$

Noise Power

$$\frac{dN_m}{N_m + \dfrac{N_2}{(N_2 - N_1)}} = B_{21}hv(N_2 - N_1)dz,$$

$$\frac{dN_m}{N_m + n_{sp}} = B_{21}hv(N_2 - N_1)dz, \qquad n_{sp} = \frac{N_2}{N_2 - N_1},$$ is the spontaneous factor.

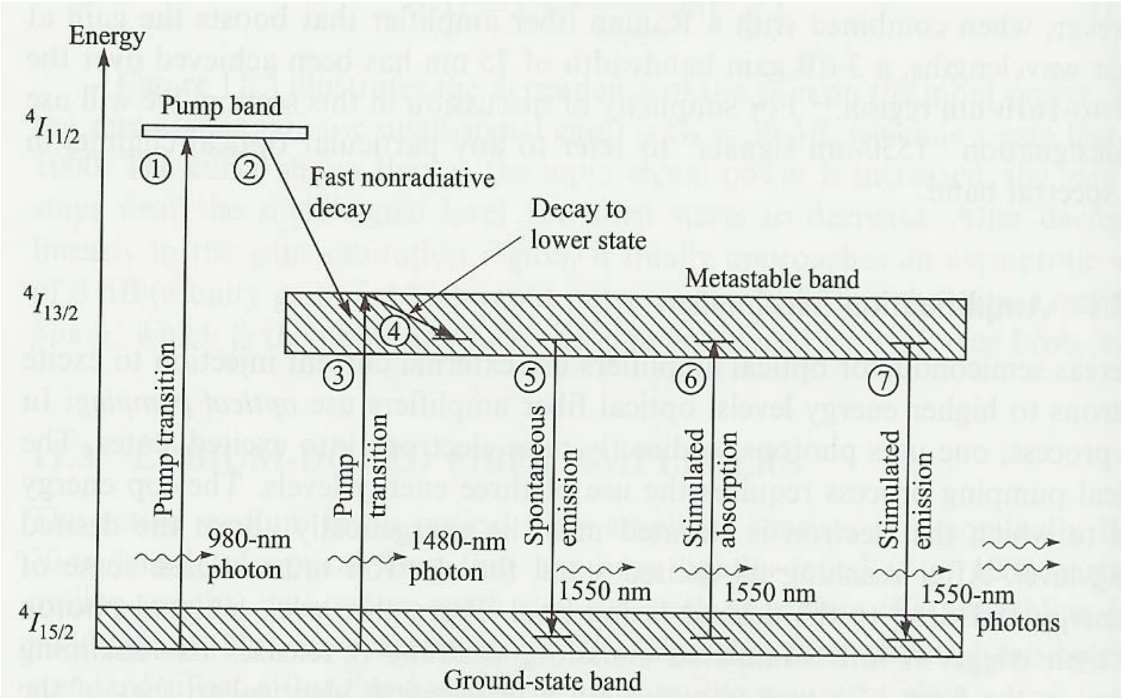
$$\ln(N_m + n_{sp})\big|_0^L = B_{21}hv(N_2 - N_1)L,$$

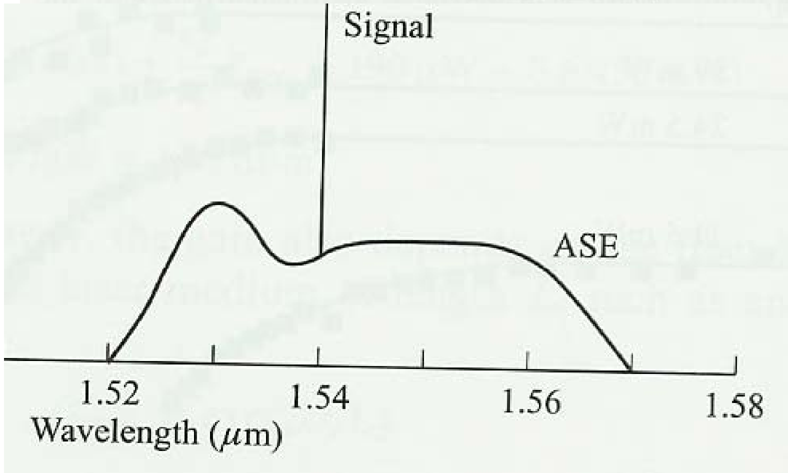
$$\frac{N_m(L) + n_{sp}}{N_m(0) + n_{sp}} = e^{gL} = G,$$

$$N_m(L) = N_m(0)G + n_{sp}(G-1),$$

$$P_{ASE} = n_{sp}(G-1)hv,$$ Power spectral density

Amplified spontaneous noise





Noise Power spectral density:

$$S(\nu) = (G-1) n_{sp} h\nu,$$

$$n_{sp} = \frac{N_2}{N_2 - N_1},$$

Total ASE power:

$$P_{ASE} = S(\nu)\Delta\nu = (G-1) n_{sp} h\nu\Delta\nu,$$

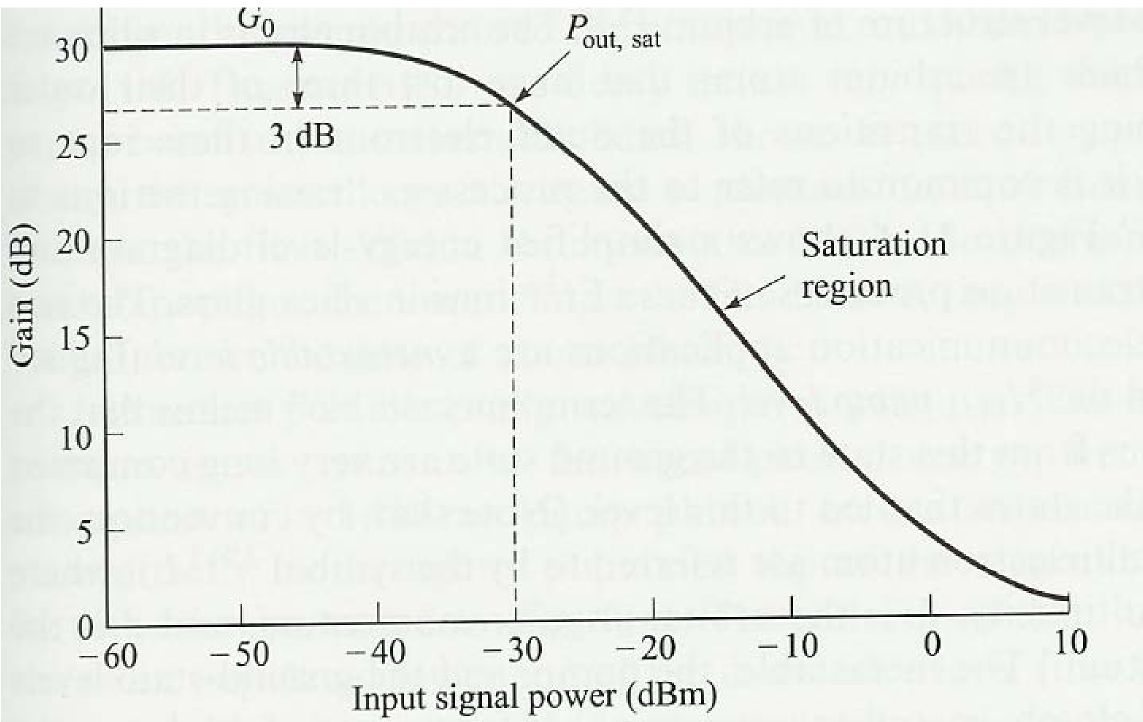
Total power:

$$P = P_{signal} + P_{ASE} = GP_{s,in} + S_{ASE}\Delta\nu,$$

Total amplifier gain

$$G = \frac{P_{s,out}}{P_{s,in}} = \exp[(\Gamma g - \alpha)L], \quad g(z) = \frac{g_0}{1 + P_{s,in}(z)/P_{s,sat}} = \frac{1}{P_{s,in}(z)} \frac{dP_{s,in}(z)}{dz},$$

$$g_0 dz = [\frac{dP_{s,in}(z)}{P_{s,in}(z)} + \frac{P_{s,in}(z)}{P_{s,sat}}], \quad \ln G = \ln G_0 + \frac{P_{s,out} - P_{s,in}}{P_{s,sat}}, \quad G = 1 + \frac{P_{s,sat}}{P_{s,in}} \ln(\frac{G_0}{G}),$$



Amplifier Example

For an optical amplifier, when the input signal is 2.0mW, the output signal is 40mW. When the input is –28dBm, the output power is -10dBm. What's the maximum power can be extracted from the amplifier?

Solution:

$$G_0 = 18dB = 63 = \exp(g_0 L), \quad \ln G = \ln G_0 + \frac{P_{s,out} - P_{s,in}}{P_{s,sat}},$$

$$g_0 L = 4.14, \quad \ln G = \ln(\frac{40}{2}) = \ln G_0 + \frac{40 - 2}{P_{s,sat}},$$

$$P_{s,sat} = 33.2 mW / cm^2,$$

$$P_{\max} = g_0 L P_{s,sat},$$

Receiver noise

Total photo-detector current:

$$i_{total} \propto (E_s + E_n)^2, \quad P = P_{signal} + P_{ASE} = GP_{s,in} + S_{ASE}\Delta\nu,$$

$$\left\langle i^2_{total,noise} \right\rangle = \sigma_T^2 + \sigma^2_{shot-S} + \sigma^2_{shot-ASE} + \sigma^2_{S-ASE} + \sigma^2_{ASE-ASE},$$

$\sigma_T^2 = \dfrac{4kT}{R}$, R is the resistance of the detector load resistor.

$$\sigma^2_{S-ASE} = 4\Re^2 P_{in} n_{sp} h\nu G(G-1)B_e,$$

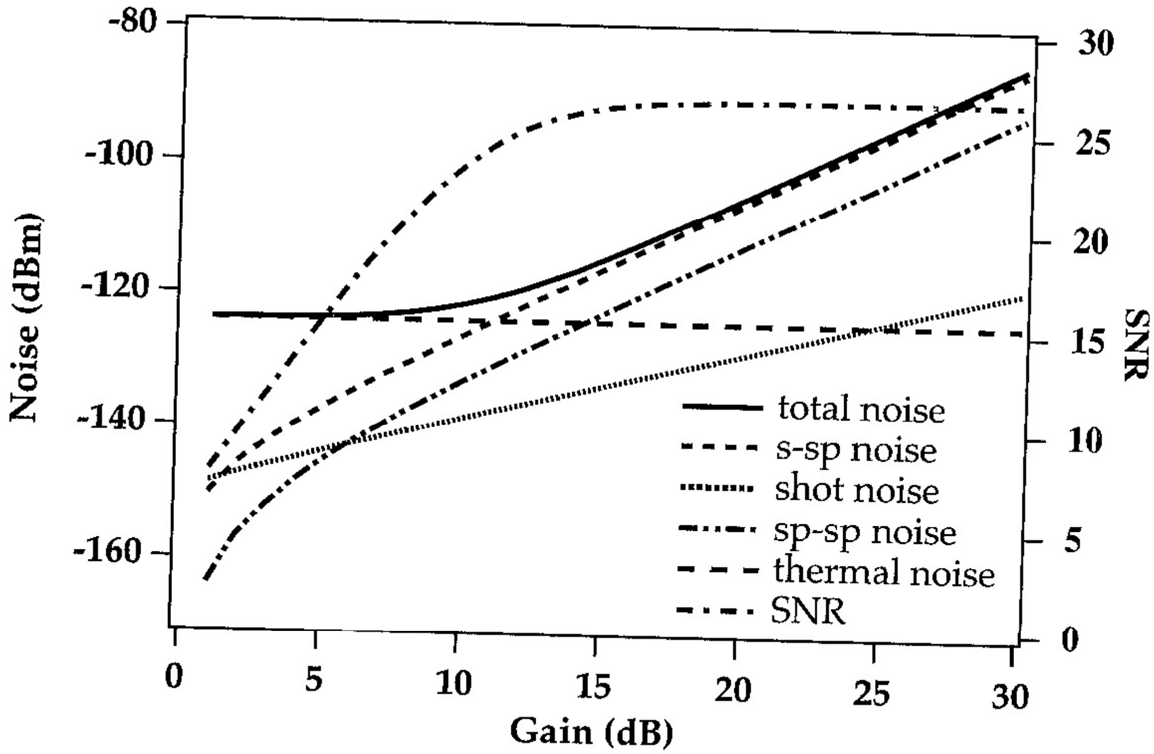
$$\sigma^2_{short-S} = 2e\Re G P_{in} B_e,$$

$$\left(\frac{S}{N}\right)_{out} = \frac{i^2_{ph}}{i^2_{noise}} = \frac{\Re P_{s,in}}{2eB_e}\frac{G}{1+2n_{sp}(G-1)}, \qquad \left(\frac{S}{N}\right)_{in} = \frac{\Re P_{s,in}}{2eB_e},$$

$$NF = \frac{(S/N)_{in}}{(S/N)_{out}} = \frac{1+2n_{sp}(G-1)}{G} \approx 2n_{sp},$$

Receiver noise

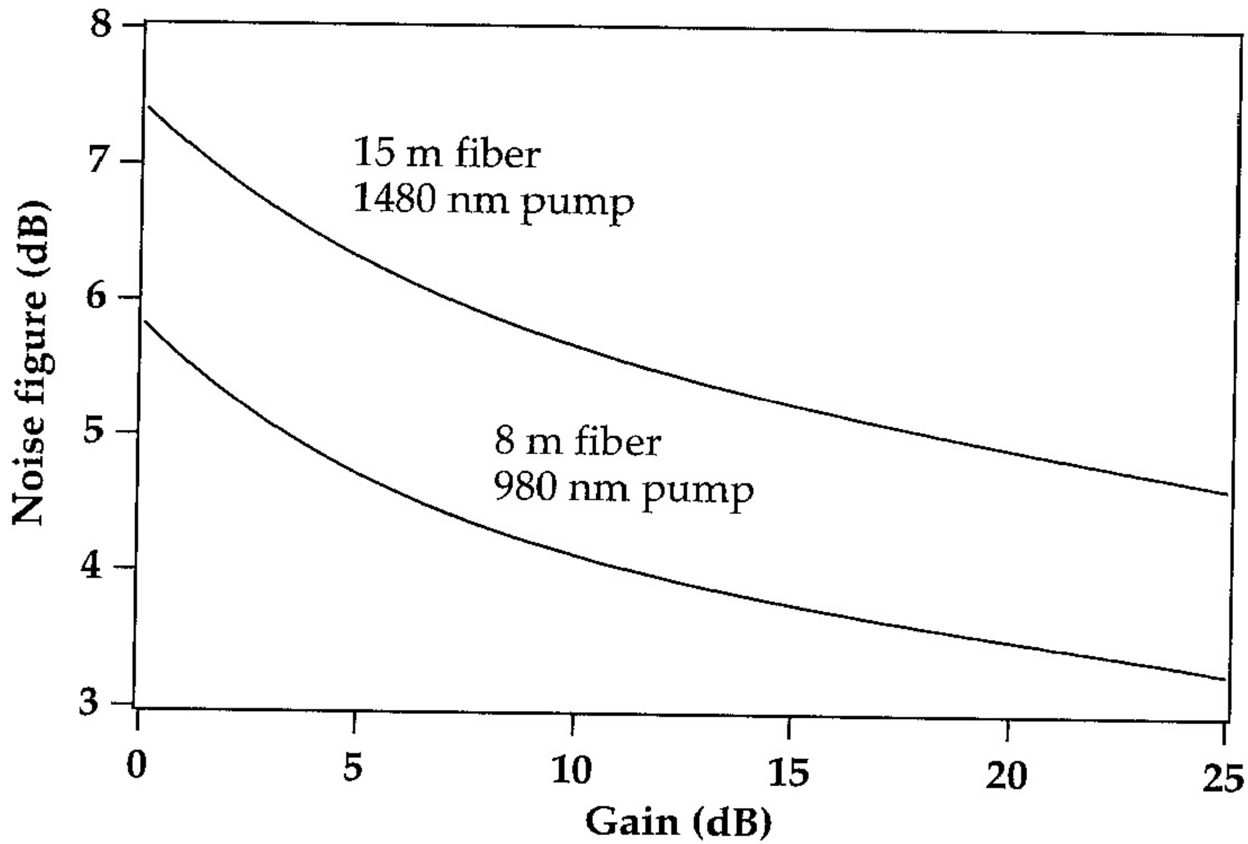
Total photo-detector current:



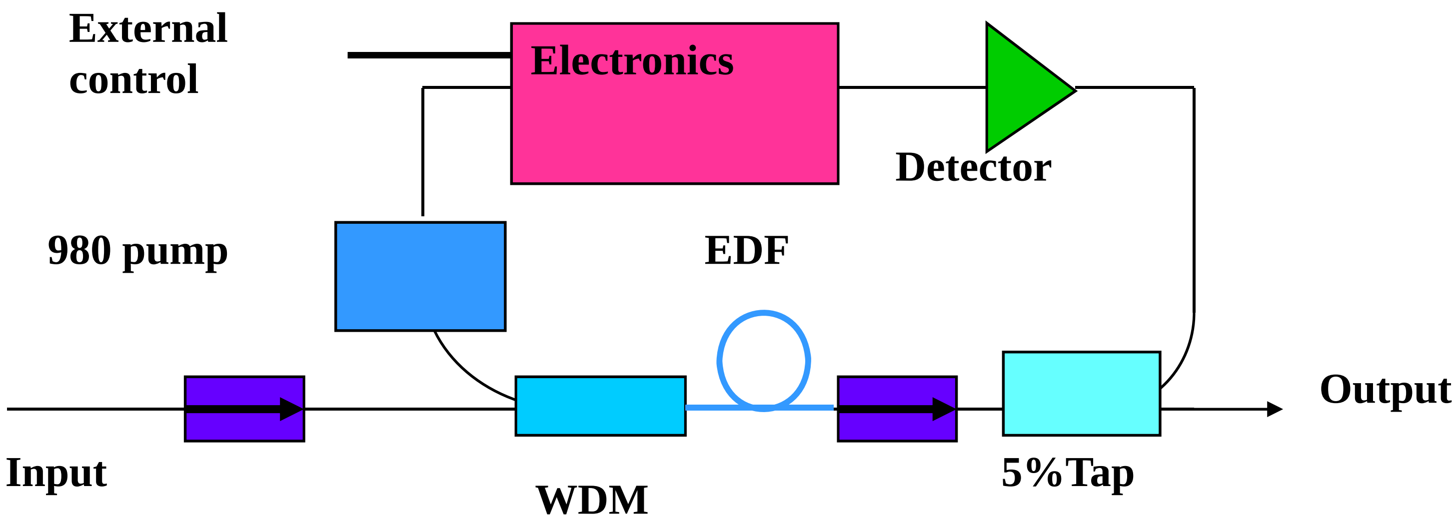
$$\left\langle i_{total,noise}^{2} \right\rangle = \sigma_{T}^{2} + \sigma_{short-S}^{2} + \sigma_{short-ASE}^{2} + \sigma_{S-ASE}^{2} + \sigma_{ASE-ASE}^{2},$$

Gain and Noise Figure

Total photo-detector current:



Amplifier System Noise figure (NF)



$$NF_{total}(dB) = NF_{OAP}(dB) + Loss_{input}(dB),$$