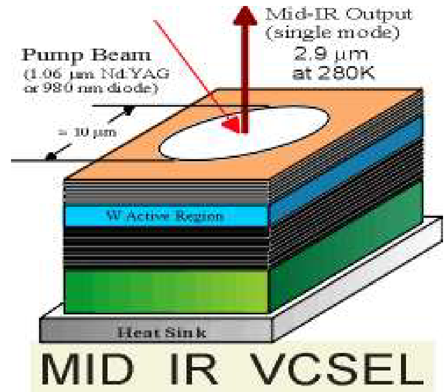
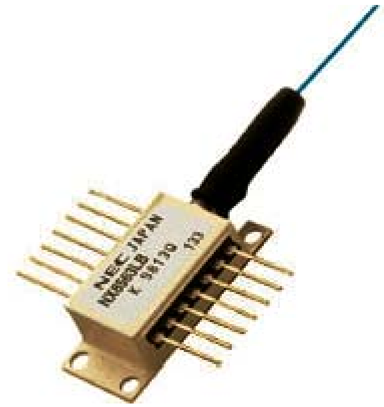
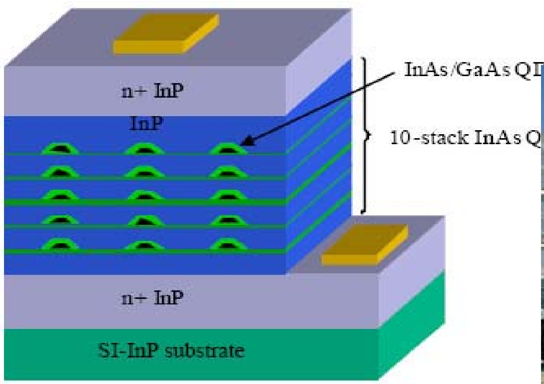
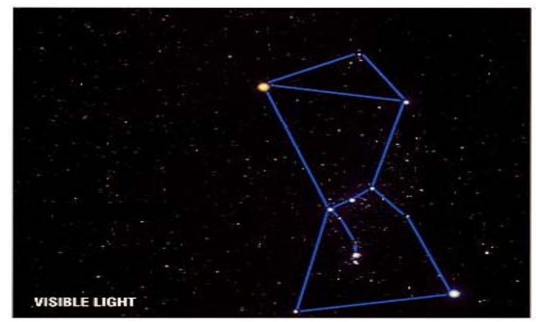
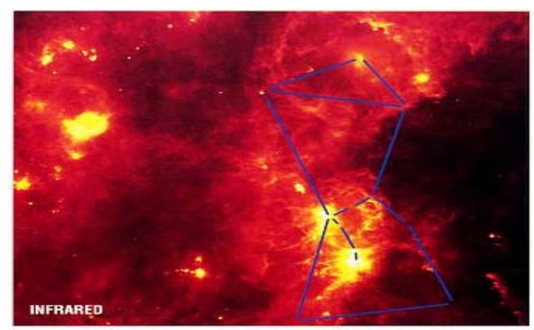
Optoelectronics:











Course Website: http://faculty.uml.edu/xlu/16.669

Office Hours: MW 1:30 – 3:00pm

Text book: *Optoelectronics and Photonic, Principles and Practices* by S. O. Kasap, Prentice Hall, 2001.

Reference book: http://ecee.colorado.edu/~bart/book/book/index.html

Class overview:

1. Review of quantum mechanics, and solid state electronics
2. Optical process in semiconductors
3. Pn junction principles and band diagrams
4. Light emitting diode (LED): principle, device structure, and materials
5. Hetero-junctions
6. Semiconductor Laser and quantum well lasers
7. Semiconductor optical amplifiers
8. Photo-detector
9. Photovoltaic devices and solar cells
10. Polarization and Electro-optical modulators

Principles of quantum mechanics:

1. Heisenberg Uncertainty Principle

$$(\Delta x)(\Delta p_x) \geq \hbar, \qquad \hbar = h/2\pi$$

$$(\Delta E)(\Delta t) \geq \hbar,$$

2. Probability and probability density function

$P(x)\Delta x:$ Probability of finding the particles in position [x, x+Δx]

$$\int_{-\infty}^{+\infty} P(x)dx = 1$$

Average position:

$$< x >= \int_{-\infty}^{+\infty} xP(x)dx$$

Average value:

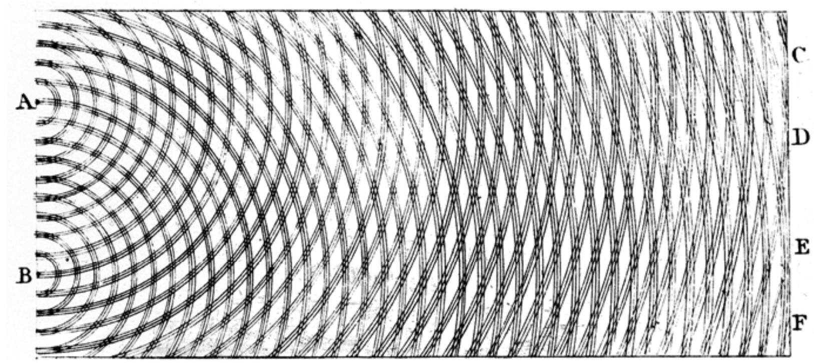
$$< f(x) >= \int_{-\infty}^{+\infty} f(x)P(x)dx$$

Principles of quantum mechanics:

3. Wave–particle duality

$$\lambda = \frac{h}{p}, \quad p = \frac{h}{\lambda} = \hbar k, \quad k = \frac{2\pi}{\lambda},$$

Interference of two electron beams



Photon:

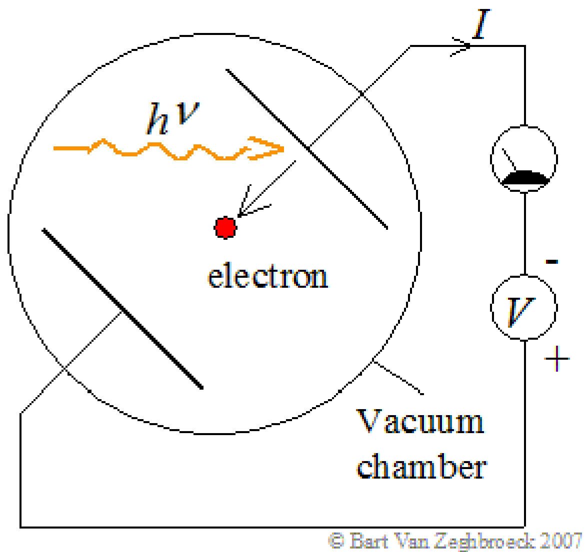
$$p = \frac{h}{\lambda} = \hbar k, \quad E = cp,$$

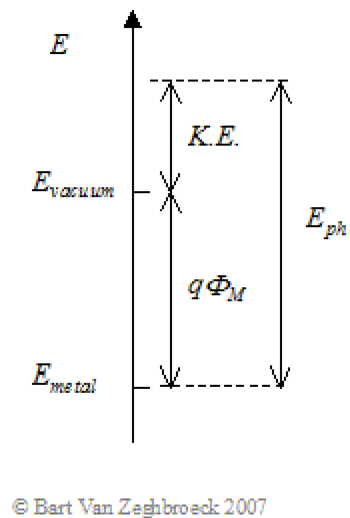
Assume 532nm light, p = 1.3×10^{-27} kg.m/s

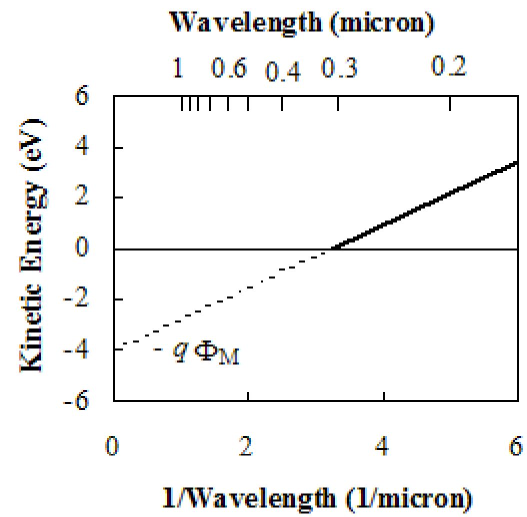
Free electron, assuming E = 1eV

$$p = mv = \sqrt{2mE}, \quad$$ p = 6.6×10^{-25} kg.m/s $$\lambda = \frac{h}{p},$$ λ= 3.2Å

4. Photoelectric effect







<u>Photon:</u>

$$p = \frac{h}{\lambda} = \hbar k, \quad E = cp, \quad E = \frac{hc}{\lambda},$$

Kenetic energy

$$K = E - \Phi_m = \frac{hc}{\lambda} - \Phi_m,$$

Workfunction

How to measure the Kenetic energy?

5. Blackbody emission– will discuss later on

Wave and particle duality

$$\lambda = \frac{h}{p}$$

Wave and propagation

$$\psi(x) = e^{ikx} \qquad k = \frac{2\pi}{\lambda}$$

Helmholtz's wave equation

$$\nabla^2\psi(x) + k^2\psi(x) = 0 \qquad \lambda = \frac{h}{p} \qquad k = p/\hbar$$

$$\hbar^2\nabla^2\psi(x) + p^2\psi(x) = 0 \qquad \frac{\hbar^2}{2m}\nabla^2\psi(x) + \frac{p^2}{2m}\psi(x) = 0$$

$$\frac{\hbar^2}{2m}\nabla^2\psi(x) = -(E\psi - V\psi)$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

Time independent Schrödinger equation

The Schrödinger wave equation and postulates

1. Each particle is described by a wave function $\psi(x, y, z, t),$

2. Quantum operators

Classical variables	Quantum operators
x	x
$f(x)$	$f(x)$
$p(x)$	$\frac{h}{i}\frac{\partial}{\partial x}$
E	$-\frac{h}{i}\frac{\partial}{\partial t}$

$$\frac{p^2}{2m} + V = E$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t) = E\psi(x,t)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

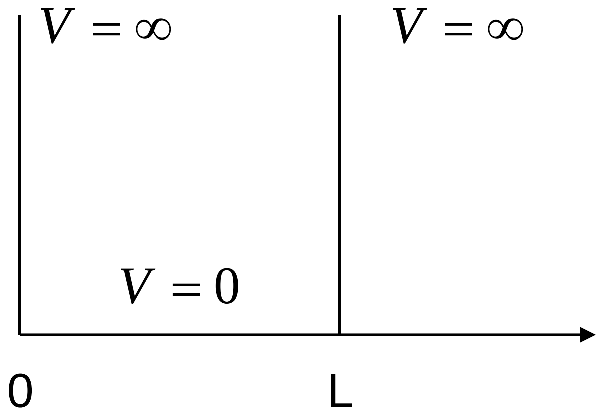
3. Expectation values

$$< Q >= \int_{-\infty}^{\infty}\psi^*(x, y, z)Q\psi(x, y, z)dxdydz$$

4. Boundary conditions

ψ $\quad \frac{\partial\psi}{\partial x}$ Continuous at boundaries

1-D potential well example



$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)+V(x)\psi(x)=E\psi(x)$$

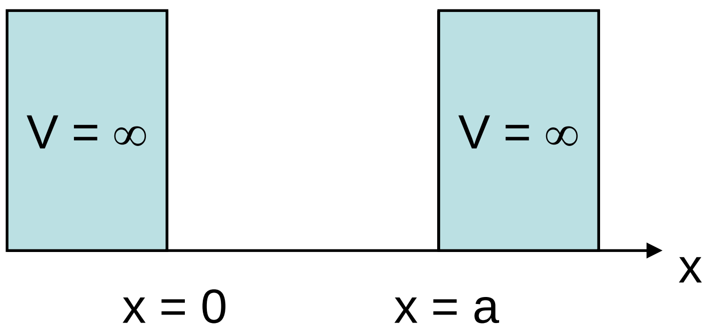
$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x)=E\psi(x)$$

$$\psi(x)=A\sin kx,\qquad k=\frac{\sqrt{2mE}}{\hbar}$$

Boundary conditions

$$\psi(x=0)=\psi(x=L)=0\qquad\qquad kL=\frac{\sqrt{2mE}}{\hbar}L=n\pi$$

Example 1, particle in infinite barriers



$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

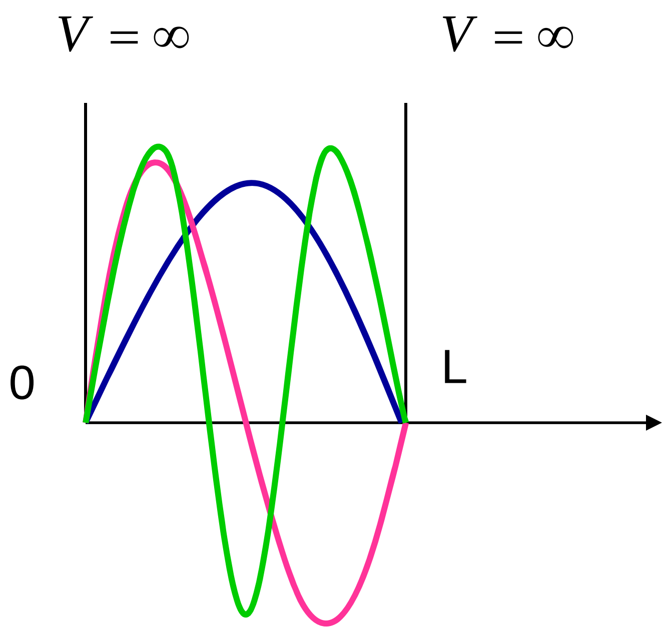
$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi \qquad -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi$$

$$\psi(x) = C_1\cos kx + C_2\sin kx \qquad k^2 = \frac{2mE}{\hbar^2}$$

Boundary conditions

$\psi(x)$ $\psi'(x)$ continuous at boundaries

1-D potential well example

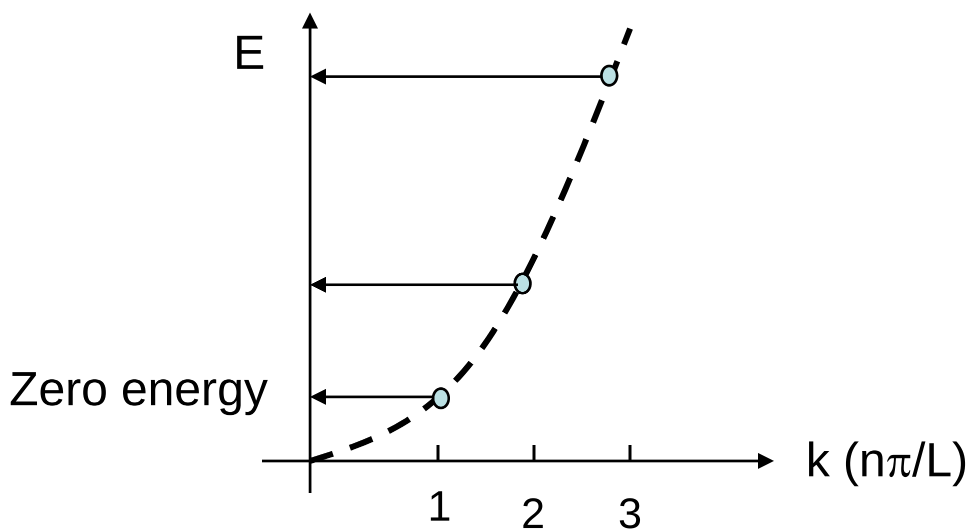


$$\psi(x) = A\sin kx, \quad \psi(x) = A\sin\left(\frac{n\pi}{L}x\right),$$

$$kL = \frac{\sqrt{2mE}}{\hbar}L = n\pi, \quad \text{n > 0} \qquad E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

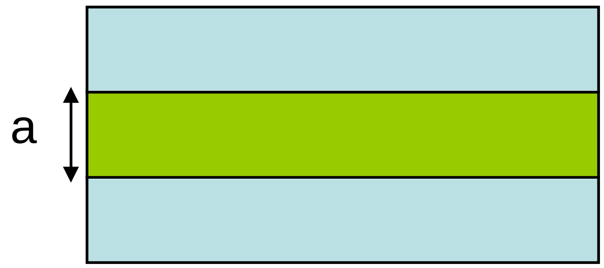
k – E dispersive relation

$$k = \frac{\sqrt{2mE}}{\hbar},$$



Lecture 1, Review-- Basics of Quantum mechanics

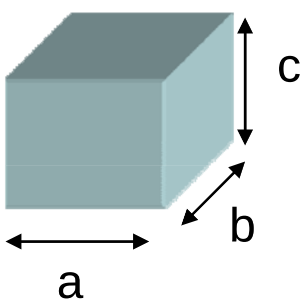
Quantum well



$$\psi(x) = e^{ik_x x} e^{ik_y y} \sin\frac{n\pi}{a} z \qquad k_z^2 = \left(\frac{n\pi}{a}\right)^2$$

$$E_n = \frac{\hbar^2}{2m}\left[k_x^2 + k_y^2 + \left(\frac{n\pi}{a}\right)^2\right]$$

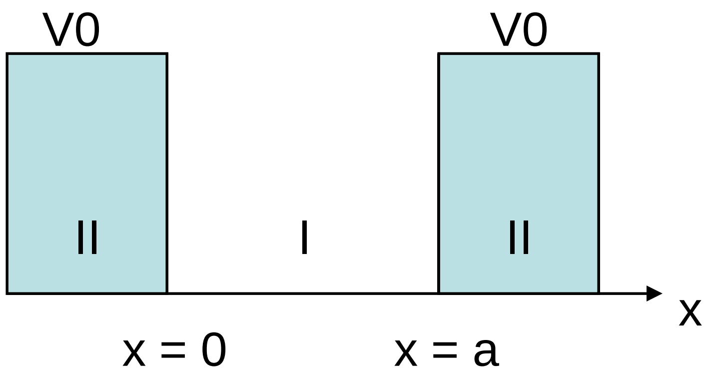
Quantum box



$$\psi(x) = \sin\frac{n\pi}{a} z \sin\frac{n\pi}{b} y \sin\frac{n\pi}{c} x$$

$$E_{n,m,l} = \frac{\hbar^2}{2m}\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2\right]$$

Example 2, particle in finite barriers



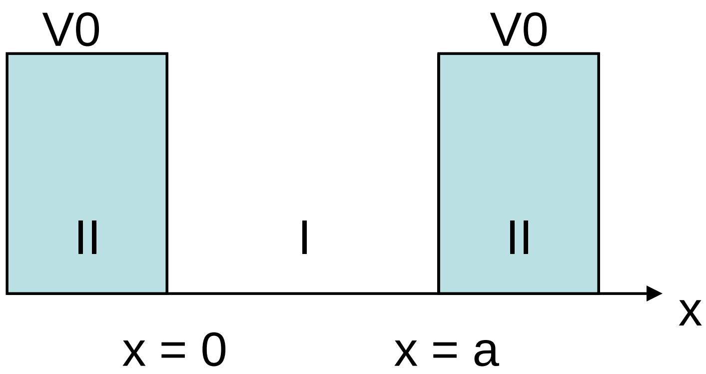
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

I: $$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi \qquad \psi_I(x) = C_1\cos k_I x + C_2\sin k_I x \qquad k_I^2 = \frac{2mE}{\hbar^2}$$

II: $$-\frac{\hbar^2}{2m}\nabla^2\psi = (E-V)\psi \qquad \psi_{II}(x) = Ae^{-k_{II}x} \qquad k_{II}^2 = k_{III}^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

III: $$-\frac{\hbar^2}{2m}\nabla^2\psi = (E-V)\psi \qquad \psi_{III}(x) = Be^{k_{III}x}$$

Example 2, particle in finite barriers



$$\left(-\frac{\hbar^2}{2m}\nabla^2+V\right)\psi=E\psi$$

$$C_1\cos k_I a+C_2\sin k_I a=Ae^{-k_{II}a}$$

$$C_1=B$$

$$-C_1k_I\sin k_I a+C_2k_I\cos k_I a=-k_{II}Ae^{-k_{II}a}$$

$$C_2k_I=k_{III}B$$

$$C_1 \cos k_I a + C_2 \sin k_I a = A e^{-k_{II} a}$$

$$-C_1 k_I \sin k_I a + C_2 k_I \cos k_I a = -k_{II} A e^{-k_{II} a}$$

$$C_1 = B$$

$$C_2 k_I = k_{III} B$$

$$\Longrightarrow \quad C_2 k_I = k_{III} C_1 \quad \Longrightarrow \quad C_2 = (k_{III} / k_I) C_1$$

$$\left.\begin{aligned} & C_1 \cos k_I a + C_1 (k_{III} / k_I) \sin k_I a = A e^{-k_{II} a} \\ & -C_1 k_I \sin k_I a + C_1 k_{III} \cos k_I a = -k_{II} A e^{-k_{II} a} \end{aligned}\right\}$$

$$\Longrightarrow \quad \frac{k_I \cos k_I a + k_{III} \sin k_I a}{-k_I \sin k_I a + k_{III} \cos k_I a} = -\frac{k_I}{k_{III}}$$

$$\Longrightarrow \quad \frac{k_I \cos k_I a + k_{III} \sin k_I a}{-k_I \sin k_I a + k_{III} \cos k_I a} = -\frac{k_I}{k_{III}}$$

$$\Longrightarrow \quad \frac{\frac{k_I}{k_{III}} + tg k_I a}{1 - \frac{k_I}{k_{III}} tg k_I a} = -\frac{k_I}{k_{III}}$$

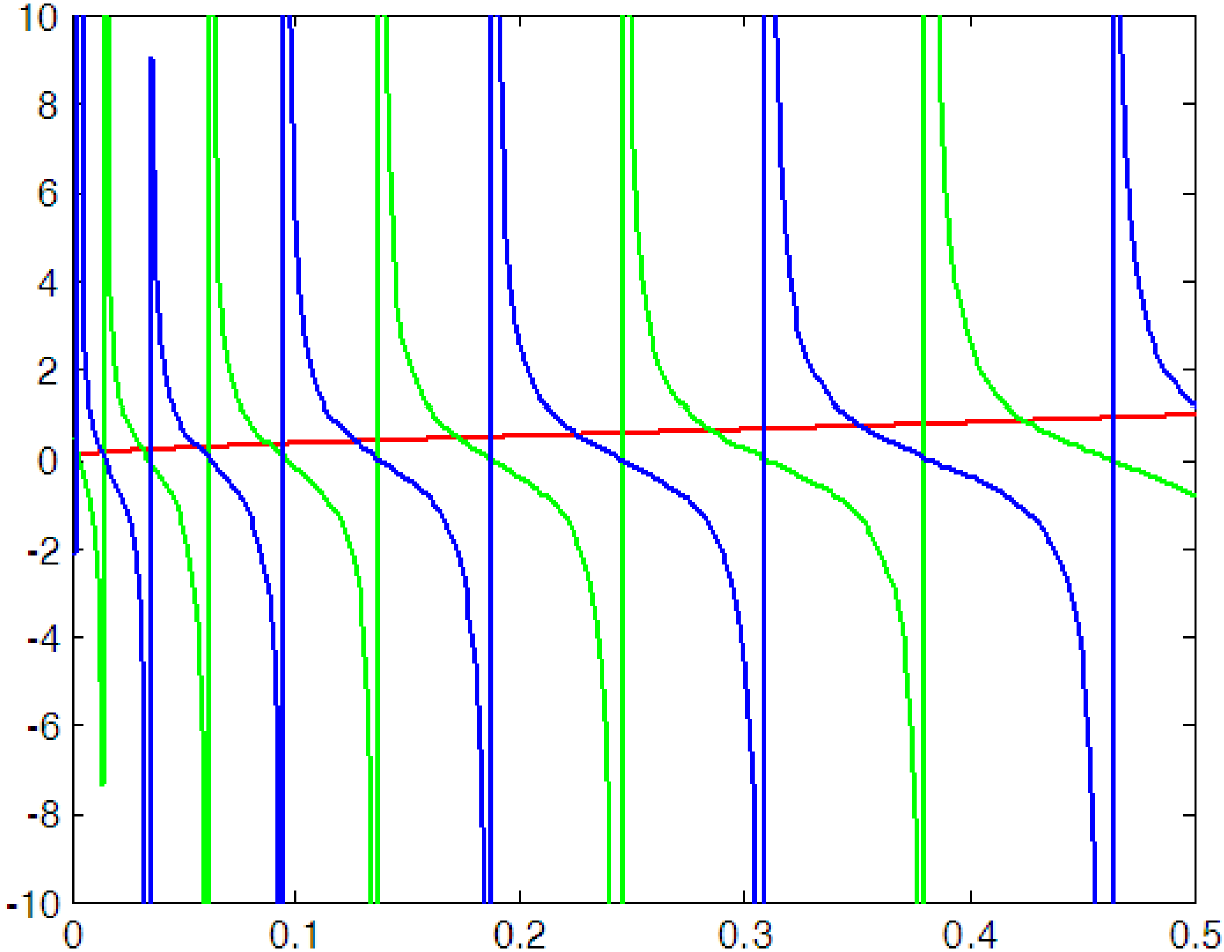
$$tg(\alpha + \beta) = -tg(\beta) \qquad \alpha = k_I a \qquad tg\,\beta = \frac{k_I}{k_{III}}$$

$$\alpha + 2\beta = n\pi \qquad tg\beta = tg\left(\frac{n\pi}{2} - \frac{\alpha}{2}\right)$$

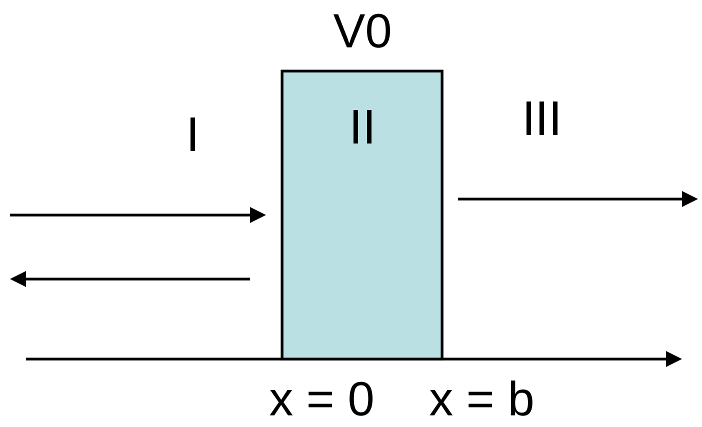
$$\frac{k_I}{k_{III}} = tg\left(\frac{n\pi}{2} - \frac{k_1 a}{2}\right)$$ Graphic solution

$$\frac{k_I}{k_{III}} = tg\left(\frac{n\pi}{2} - \frac{k_1 a}{2}\right)$$

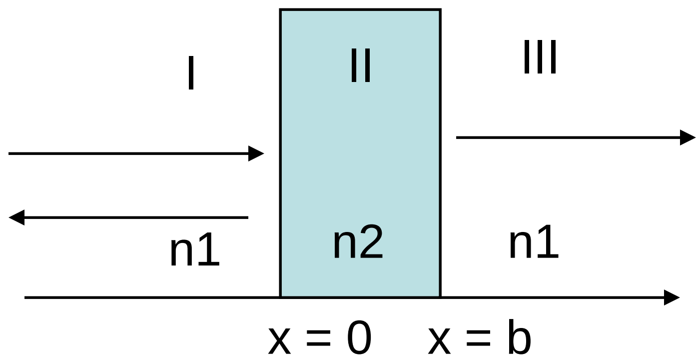
Graphic solution



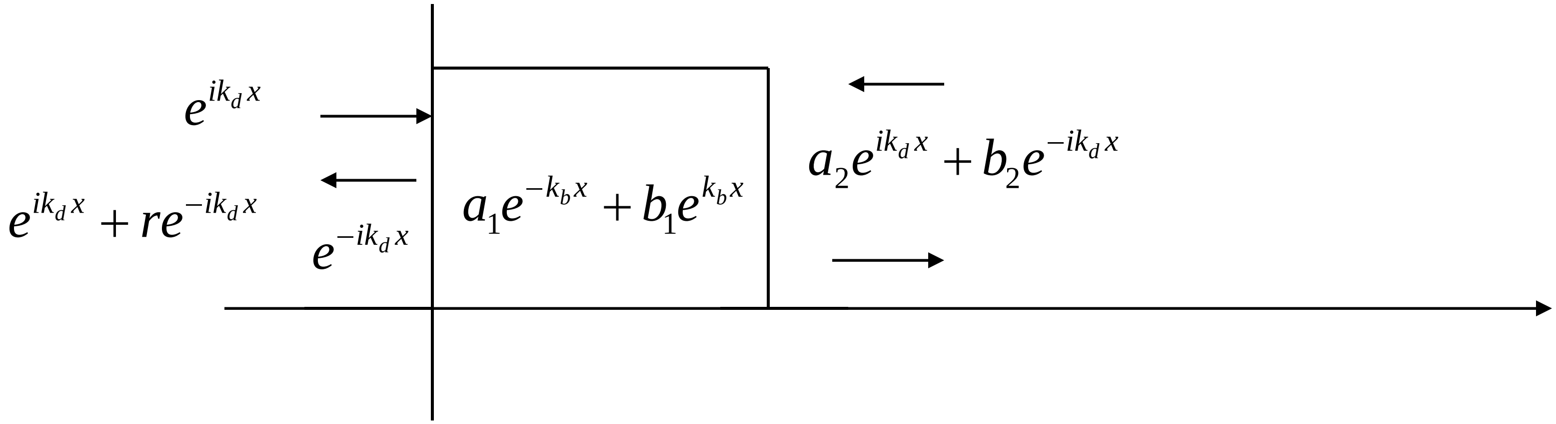
Example 3, Electron reflection and tunneling



Optical beam



$$r = \frac{n_1 - n_2}{n_1 + n_2} \qquad t = \frac{2n_1}{n_1 + n_2}$$



At this interface

$$e^{ik_d x} + re^{-ik_d x} = a_1 e^{-k_b x} + b_1 e^{k_b x} \,|_{x=0}$$

$$1 + r = a_1 + b_1$$

$$ik_d - ik_d r = -k_b a_1 + k_b b_1$$

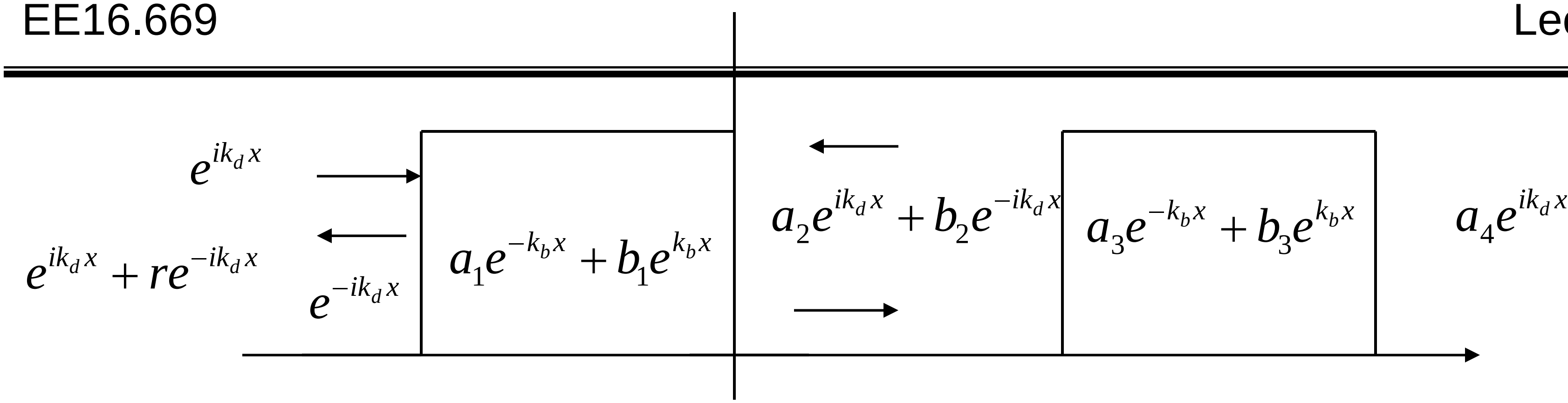
$$k_b + k_b r = k_b a_1 + k_b b_1$$

$$2k_b b_1 = ik_d(1-r) + k_b(1+r)$$

$$b_1 = \frac{ik_d(1-r) + k_b(1+r)}{2k_b}$$

$$k_b(1+r) - ik_d(1-r) = 2k_b a_1$$

$$a_1 = \frac{k_b(1+r) - ik_d(1-r)}{2k_b}$$



At this interface

$$a_1 e^{-k_b x} + b_1 e^{k_b x} = a_2 e^{ik_d x} + b_2 e^{-ik_d x} \big|_{x=b}$$

$$a_1 e^{-k_b b} + b_1 e^{k_b b} = a_2 e^{ik_d b} + b_2 e^{-ik_d b}$$

$$-k_b a_1 e^{-k_b b} + k_b b_1 e^{k_b b} = ik_d a_2 e^{ik_d b} - ik_d b_2 e^{-ik_d b}$$

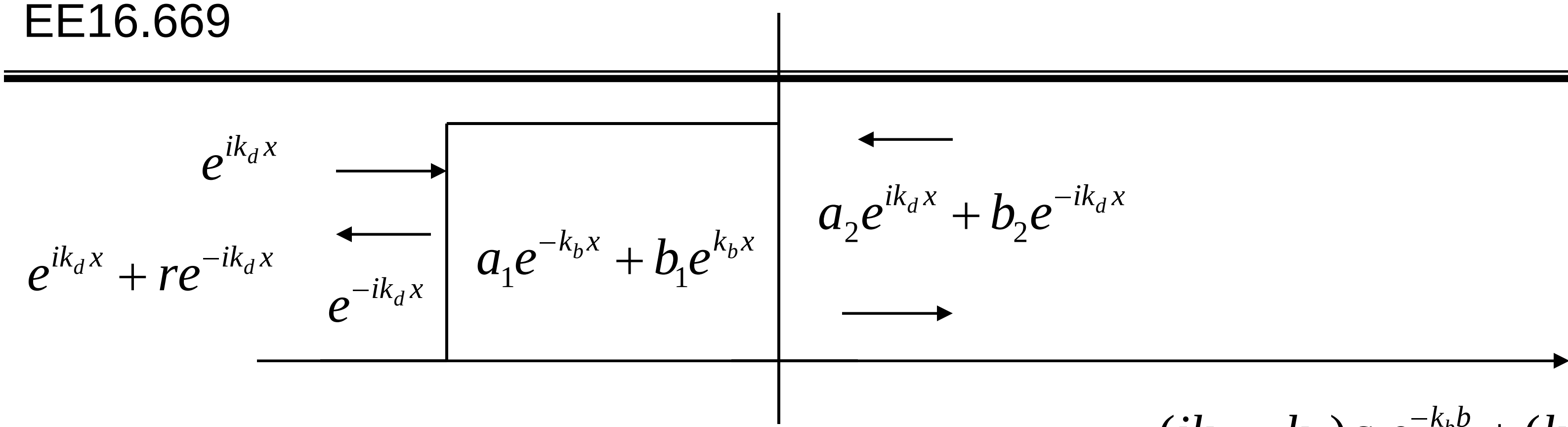
$$ik_d a_1 e^{-k_b b} + ik_d b_1 e^{k_b b} = ik_d a_2 e^{ik_d b} + ik_d b_2 e^{-ik_d b}$$

$$a_1 = \frac{(k_b - ik_d) + r(k_b + ik_d)}{2k_b}$$

$$b_1 = \frac{(k_b + ik_d) + r(k_b - ik_d)}{2k_b}$$

$$a_2 = \frac{(ik_d - k_b) a_1 e^{-k_b b} + (k_b + ik_d) b_1 e^{k_b b}}{2ik_d e^{ik_d b}}$$

$$b_2 = \frac{(ik_d + k_b) a_1 e^{-k_b b} + (ik_d - k_b) b_1 e^{k_b b}}{2ik_d e^{ik_d b}}$$



$$a_1 = \frac{(k_b - ik_d) + r(k_b + ik_d)}{2k_b}$$

At this interface

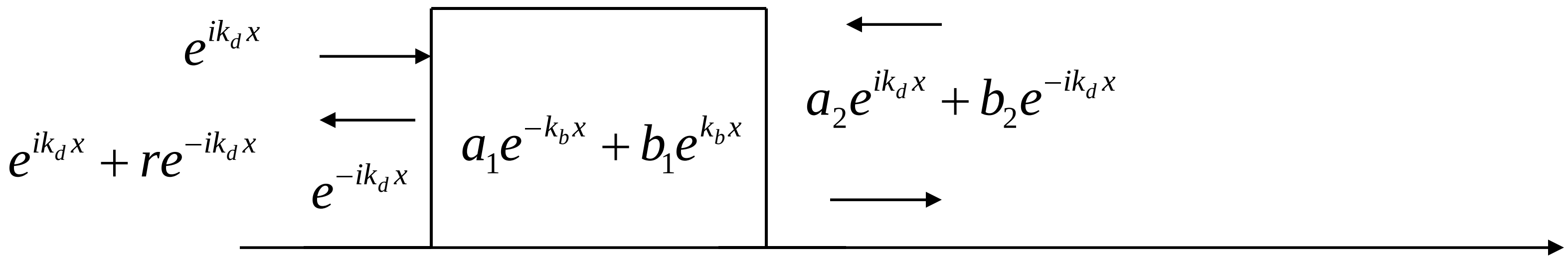
$$a_2 = \frac{(ik_d - k_b)a_1 e^{-k_b b} + (k_b + ik_d) b_1 e^{k_b b}}{2ik_d e^{ik_d b}}$$

$$b_1 = \frac{(k_b + ik_d) + r(k_b - ik_d)}{2k_b}$$

$$b_2 = \frac{(ik_d + k_b)a_1 e^{-k_b b} + (ik_d - k_b) b_1 e^{k_b b}}{2ik_d e^{ik_d b}}$$

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \frac{1}{2k_b} \begin{pmatrix} (k_b - ik_d) & (k_b + ik_d) \\ (k_b + ik_d) & (k_b - ik_d) \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix}$$

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \frac{1}{2ik_d e^{ik_d b}} \begin{pmatrix} -(k_b - ik_d) e^{-k_b b} & (k_b + ik_d) e^{k_b b} \\ (k_b + ik_d) e^{-k_b b} & -(k_b - ik_d) e^{k_b b} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$



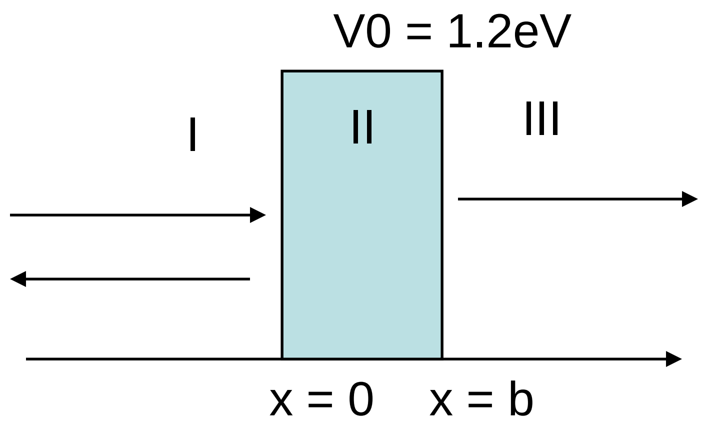
$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \frac{1}{2k_b} \begin{pmatrix} (k_b - ik_d) & (k_b + ik_d) \\ (k_b + ik_d) & (k_b - ik_d) \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix}$$

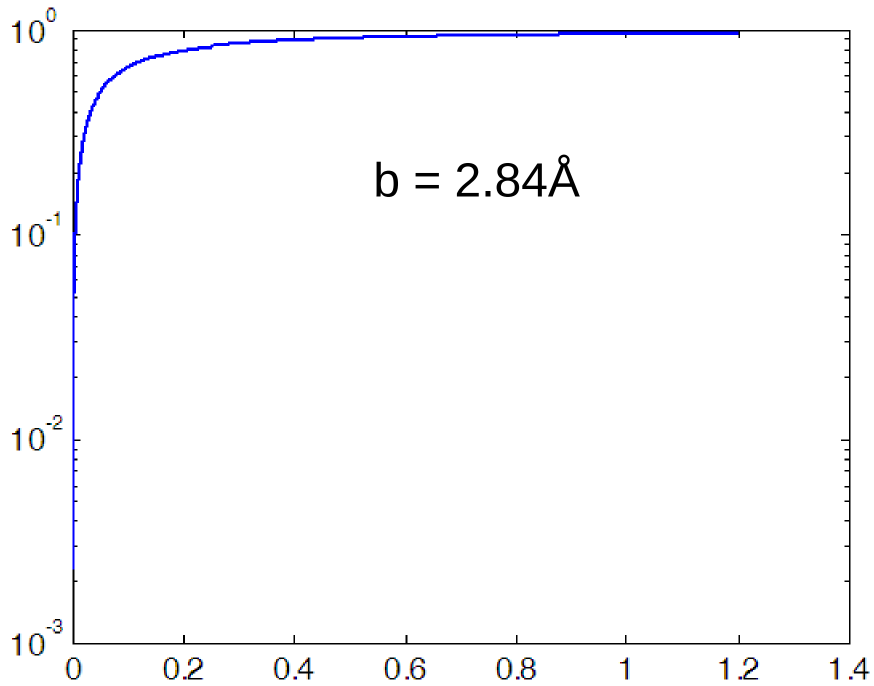
$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \frac{1}{2ik_d} \begin{pmatrix} -(k_b - ik_d)e^{-k_b b} & (k_b + ik_d)e^{k_b b} \\ (k_b + ik_d)e^{-k_b b} & -(k_b - ik_d)e^{k_b b} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

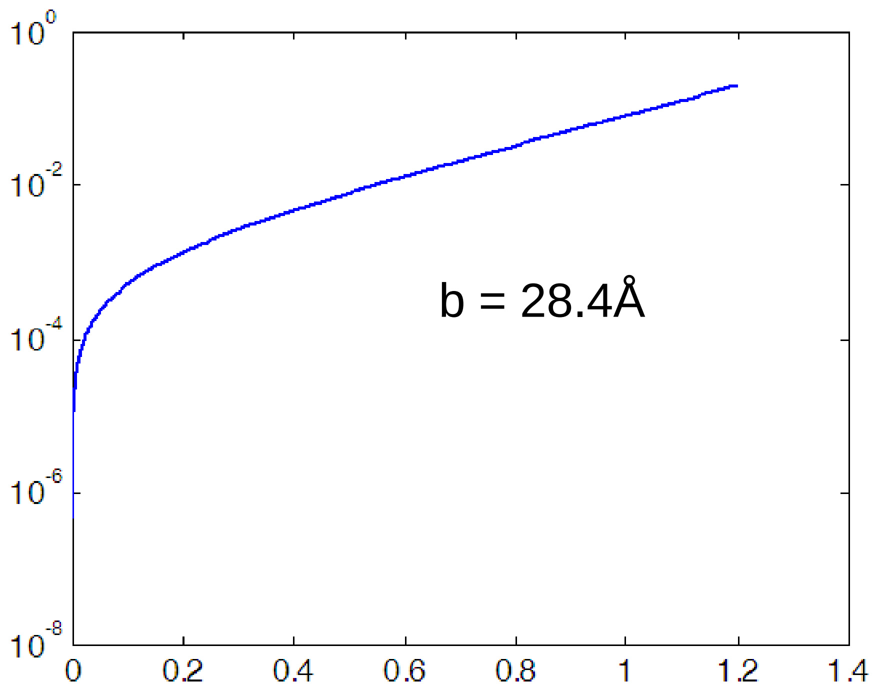
$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix}$$

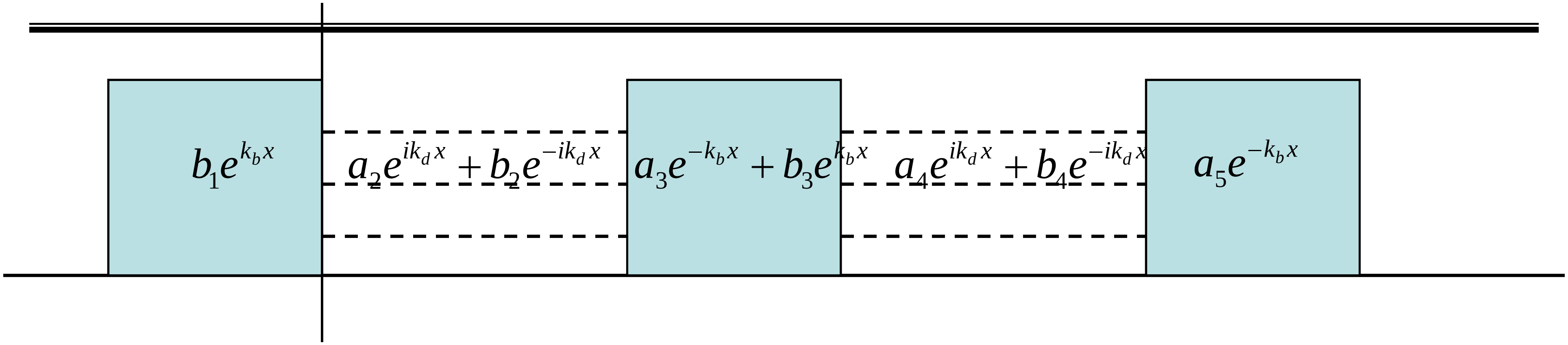
$$T = M_{11} + rM_{12} = M_{11} - \frac{M_{21} M_{12}}{M_{22}}$$

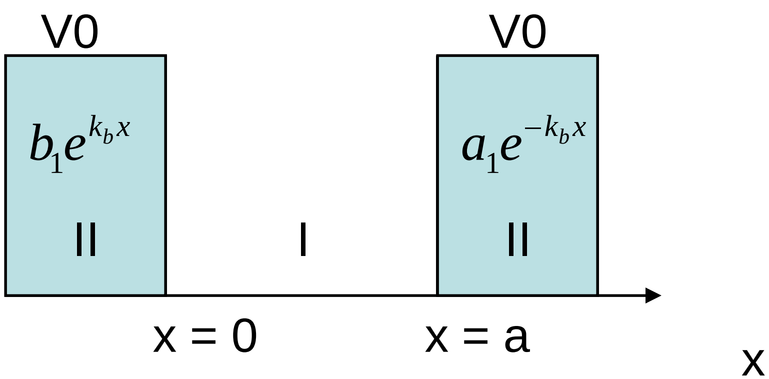
Example 3, Electron reflection and tunneling











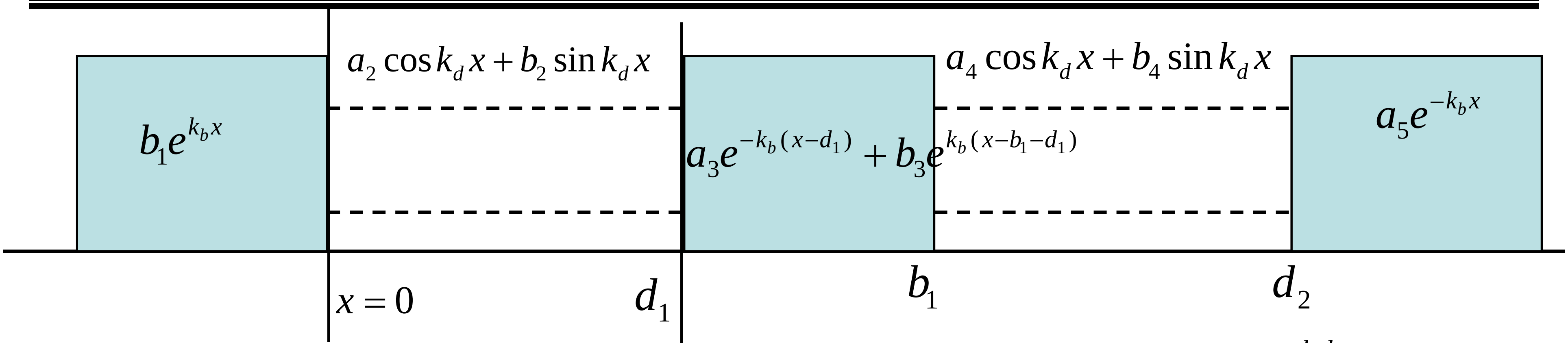
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

$$C_1 \cos k_I a + C_2 \sin k_I a = Ae^{-k_{II} a}$$

$$C_1 = B$$

$$-C_1 k_I \sin k_I a + C_2 k_I \cos k_I a = -k_{II} Ae^{-k_{II} a}$$

$$C_2 k_I = k_{III} B$$



$$a_2 \cos k_d d_1 + b_2 \sin k_d d_1 = a_3 + b_3 e^{-k_b b_1}$$

$$b_1 = a_2$$

$$-a_2 k_d \sin k_d d_1 + b_2 k_d \cos k_d d_1 = -k_b a_3 + k_b b_3 e^{-k_b b_1}$$

$$k_b b_1 = k_d b_2$$

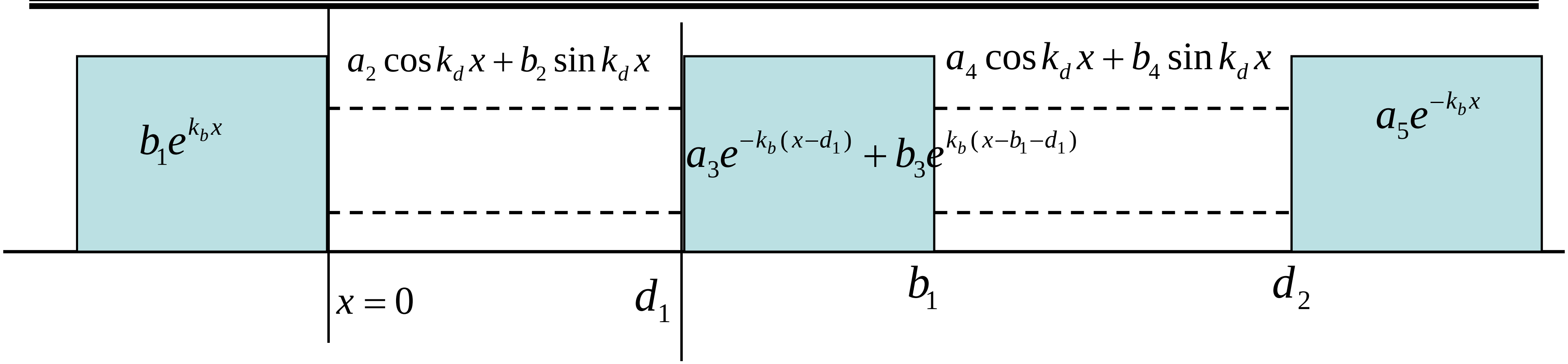
Look at b3 item, and see the effect of this item

$$a_2 \cos k_d d_1 + b_2 \sin k_d d_1 - b_3 e^{-k_b b_1} = a_3$$

$$a_2 = \frac{k_d}{k_b} b_2$$

$$a_2 k_d \sin k_d d_1 - b_2 k_d \cos k_d d_1 + k_b b_3 e^{-k_b b_1} = k_b a_3$$

$$\frac{\frac{k_d}{k_b} \cos k_d d_1 + \sin k_d d_1 - \frac{b_3}{b_2} e^{-k_b b_1}}{\frac{k_d}{k_b} k_d \sin k_d d_1 - k_d \cos k_d d_1 + k_b \frac{b_3}{b_2} e^{-k_b b_1}} = \frac{1}{k_b}$$



$$b_1 = a_2$$

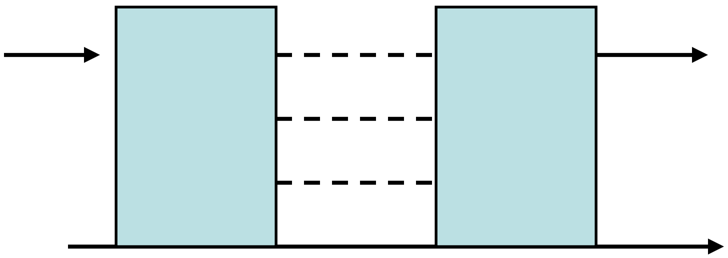
$$k_b b_1 = k_d b_2$$

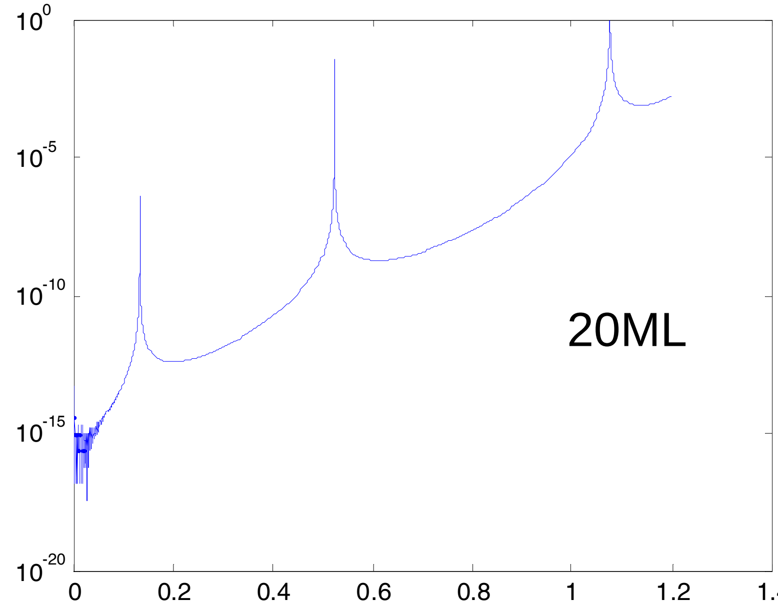
$$a_2 = \frac{k_d}{k_b} b_2$$

Look at b3 item, and see the effect of this item

$$\frac{\frac{k_d}{k_b}\cos k_d d_1 + \sin k_d d_1 - \frac{b_3}{b_2} e^{-k_b b_1}}{\frac{k_d}{k_b} k_d \sin k_d d_1 - k_d \cos k_d d_1 + k_b \frac{b_3}{b_2} e^{-k_b b_1}} = \frac{1}{k_b}$$

$$\frac{\frac{k_d}{k_b}\cos k_d d_1 + \sin k_d d_1 - \frac{b_3}{b_2} e^{-k_b b_1}}{\left(\frac{k_d}{k_b}\right)^2 \sin k_d d_1 - \frac{k_d}{k_b}\cos k_d d_1 + \frac{b_3}{b_2} e^{-k_b b_1}} = 1$$

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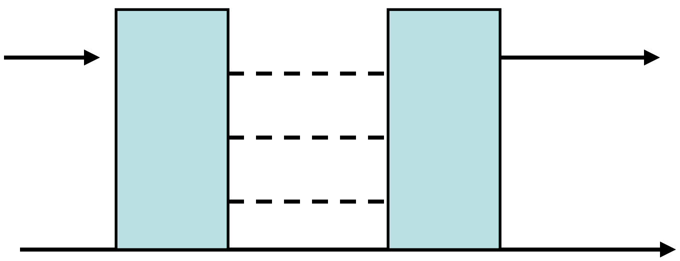
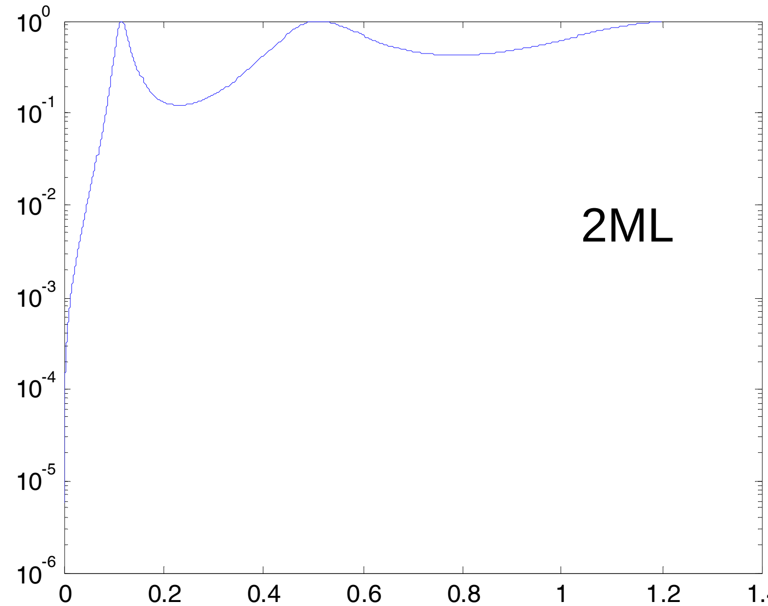
10^{-5}
10^{-10}
10^{-15}
10^{-20}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
20ML

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10^{-1}
10^{-2}
10^{-3}
10^{-4}
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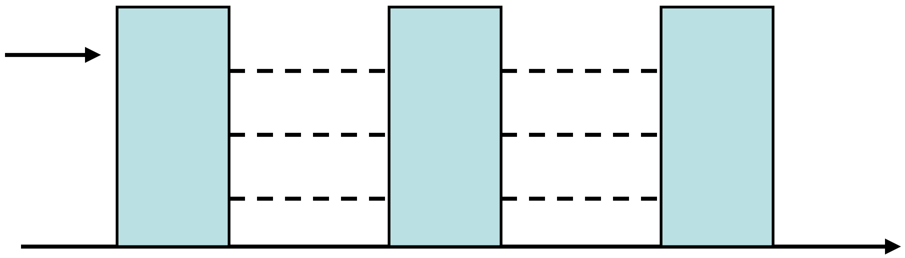
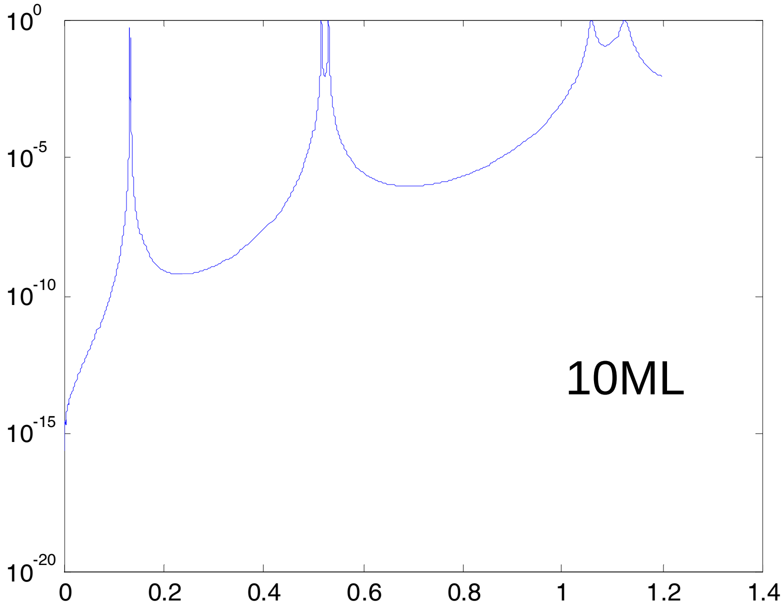
0.2
0.4
0.6
0.8
1
1.2
1.4
2ML

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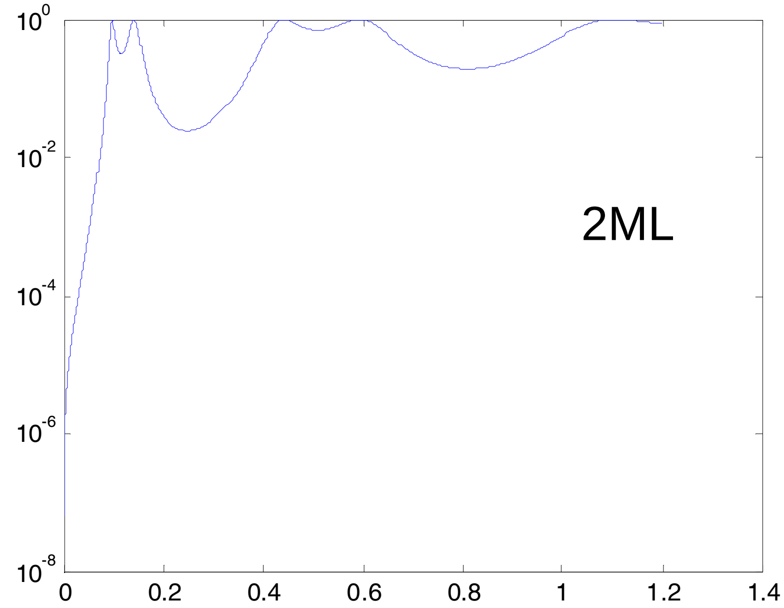
10^-5
10^-10
10^-15
10^-20
0
0.2
0.4
0.6
0.8
1
1.2
1.4
10ML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10^-2
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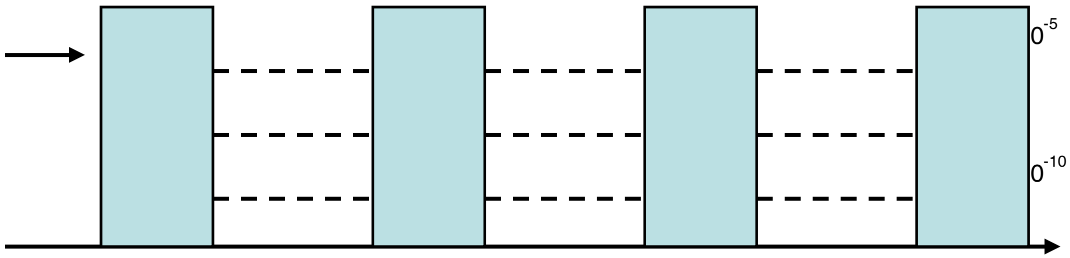
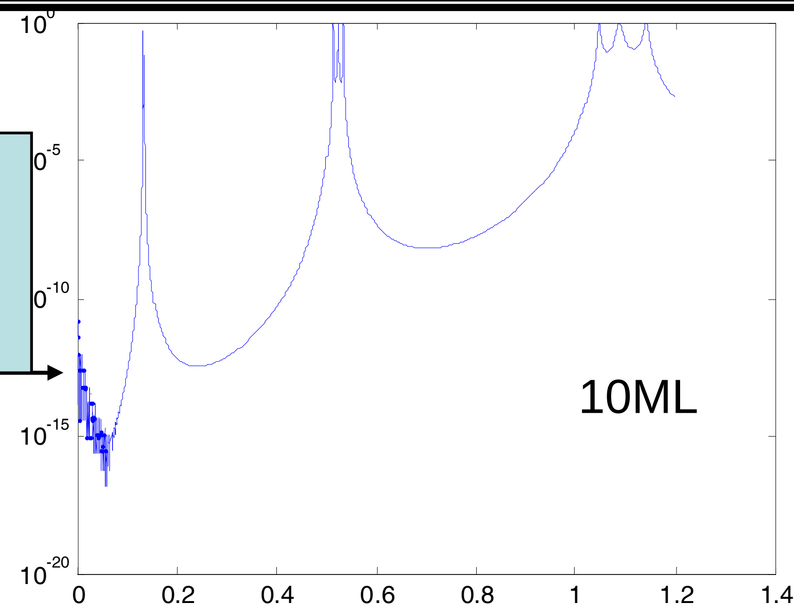
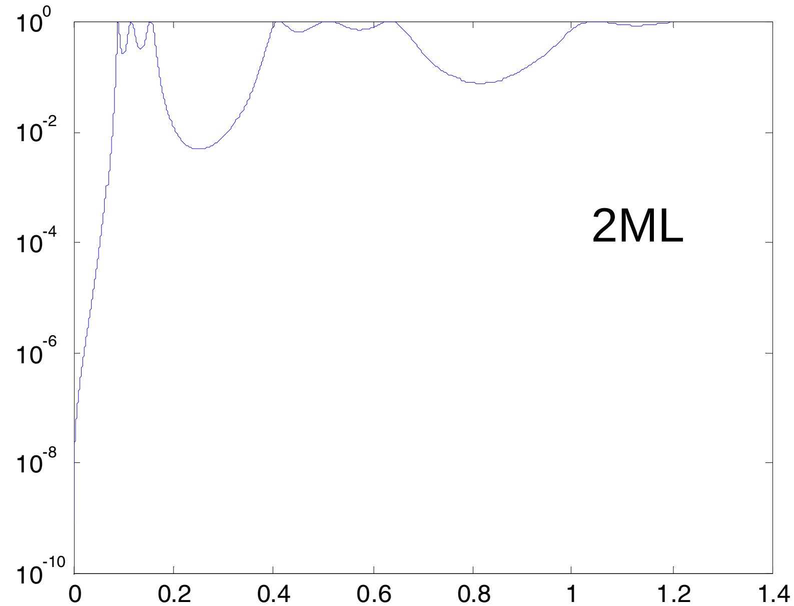
10^-6
10^-8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
2ML




10^0
10^-10
10^-15
10^-20
0
0.2
0.4
0.6
0.8
1
1.2
1.4
10ML


10^0
10^-2
10^-4
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10^-8
10^-10
0
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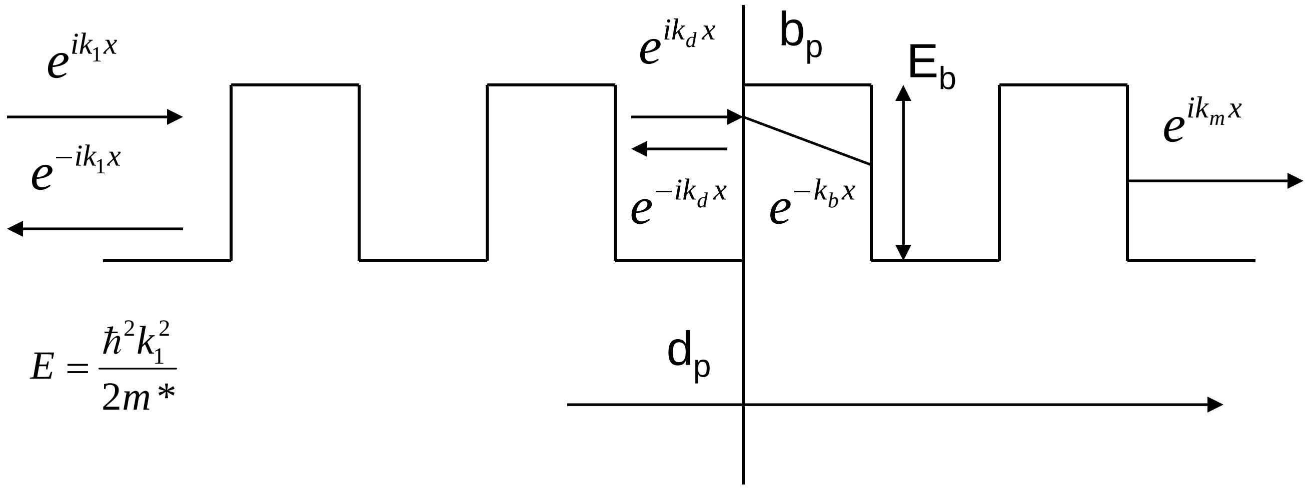
0.6
0.8
1
1.2
1.4
2ML



$$E = \frac{\hbar^2 k_1^2}{2m^*}$$

$$-\frac{\hbar^2\nabla^2}{2m^*} = (E-V)\psi$$

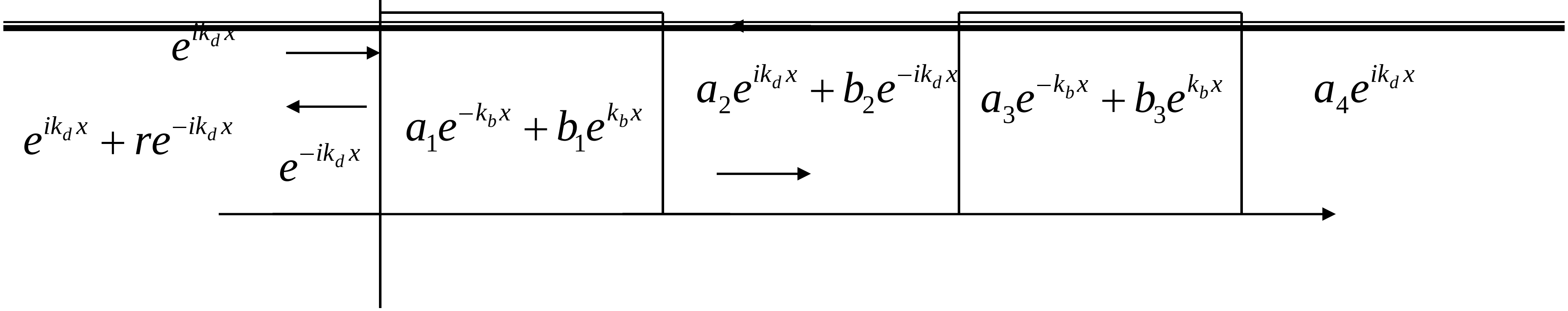
$$i\hbar\frac{\partial}{\partial t}\psi = E\psi$$

$$\frac{\hbar^2 k_b^2}{2m^*} = V_b - \frac{\hbar^2 k_d^2}{2m^*}$$

$$(E-V)\psi = \frac{p^2}{2m^*} = -\frac{\hbar^2\nabla^2}{2m^*}$$

$$k_b = \sqrt{\frac{2m^*V_b}{\hbar^2} - k_d^2}$$

$$-\frac{\hbar^2\nabla^2}{2m^*} = (E-V)\psi$$



At this interface

$$e^{ik_d x} + re^{-ik_d x} = a_1 e^{-k_b x} + b_1 e^{k_b x} \big|_{x=0}$$

$$1 + r = a_1 + b_1$$

$$ik_d - ik_d r = -k_b a_1 + k_b b_1$$

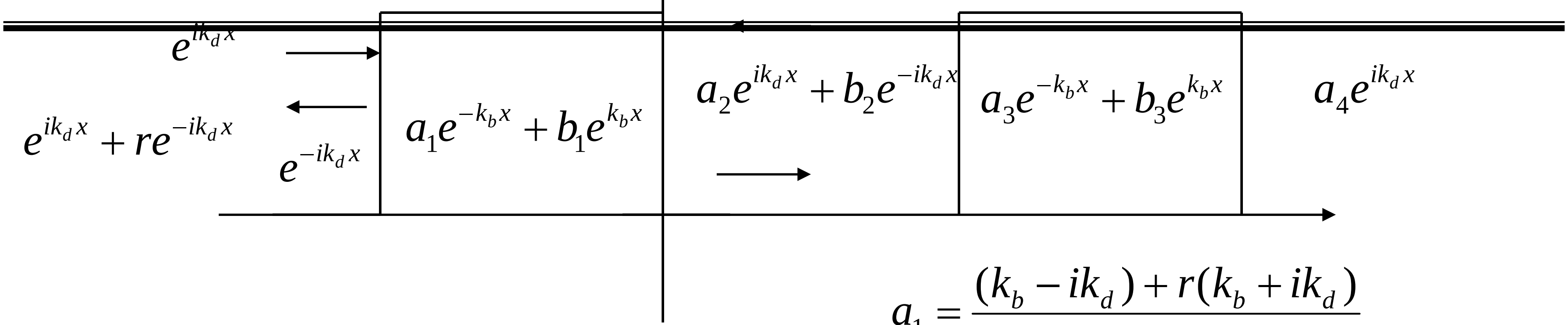
$$k_b + k_b r = k_b a_1 + k_b b_1$$

$$2k_b b_1 = ik_d (1-r) + k_b (1+r)$$

$$b_1 = \frac{ik_d (1-r) + k_b (1+r)}{2k_b}$$

$$k_b (1+r) - ik_d (1-r) = 2k_b a_1$$

$$a_1 = \frac{k_b (1+r) - ik_d (1-r)}{2k_b}$$



At this interface

$$a_1 e^{-k_b x} + b_1 e^{k_b x} = a_2 e^{ik_d x} + b_2 e^{-ik_d x} \big|_{x=b}$$

$$a_1 e^{-k_b b} + b_1 e^{k_b b} = a_2 e^{ik_d b} + b_2 e^{-ik_d b}$$

$$-k_b a_1 e^{-k_b b} + k_b b_1 e^{k_b b} = ik_d a_2 e^{ik_d b} - ik_d b_2 e^{-ik_d b}$$

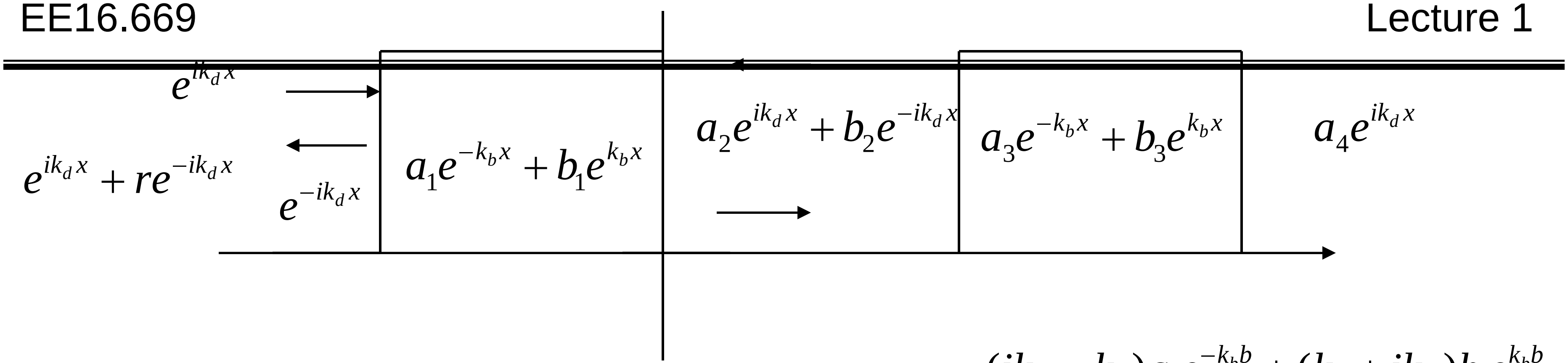
$$ik_d a_1 e^{-k_b b} + ik_d b_1 e^{k_b b} = ik_d a_2 e^{ik_d b} + ik_d b_2 e^{-ik_d b}$$

$$a_1 = \frac{(k_b - ik_d) + r(k_b + ik_d)}{2k_b}$$

$$b_1 = \frac{(k_b + ik_d) + r(k_b - ik_d)}{2k_b}$$

$$a_2 = \frac{(ik_d - k_b)a_1 e^{-k_b b} + (k_b + ik_d) b_1 e^{k_b b}}{2ik_d e^{ik_d b}}$$

$$b_2 = \frac{(ik_d + k_b)a_1 e^{-k_b b} + (ik_d - k_b) b_1 e^{k_b b}}{2ik_d e^{ik_d b}}$$



$$a_1 = \frac{(k_b - ik_d) + r(k_b + ik_d)}{2k_b}$$

At this interface

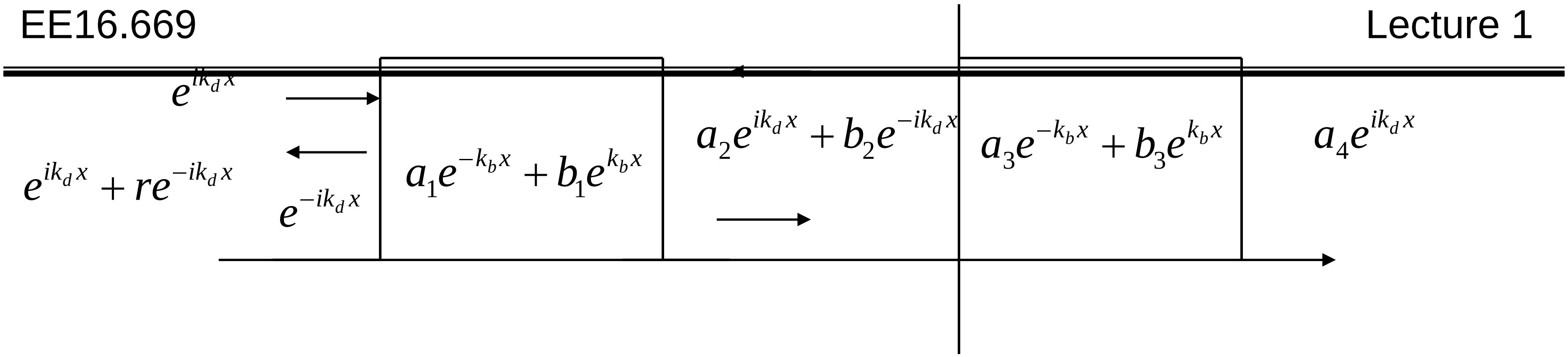
$$a_2 = \frac{(ik_d - k_b)a_1 e^{-k_b b} + (k_b + ik_d)b_1 e^{k_b b}}{2ik_d e^{ik_d b}}$$

$$b_1 = \frac{(k_b + ik_d) + r(k_b - ik_d)}{2k_b}$$

$$b_2 = \frac{(ik_d + k_b)a_1 e^{-k_b b} + (ik_d - k_b)b_1 e^{k_b b}}{2ik_d e^{ik_d b}}$$

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \frac{1}{2k_b}\begin{pmatrix} (k_b - ik_d) & (k_b + ik_d) \\ (k_b + ik_d) & (k_b - ik_d) \end{pmatrix}\begin{pmatrix} 1 \\ r \end{pmatrix}$$

$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \frac{1}{2ik_d e^{ik_d b}}\begin{pmatrix} -(k_b - ik_d)e^{-k_b b} & (k_b + ik_d)e^{k_b b} \\ (k_b + ik_d)e^{-k_b b} & -(k_b - ik_d)e^{k_b b} \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$



At this interface

$$a_2 e^{ik_d x} + b_2 e^{-ik_d x} = a_3 e^{-k_b (x-b-d)} + b_3 e^{k_b (x-b-d)} \big|_{x=b+d}$$

$$a_2 e^{ik_d (b+d)} + b_2 e^{-ik_d (b+d)} = a_3 + b_3$$

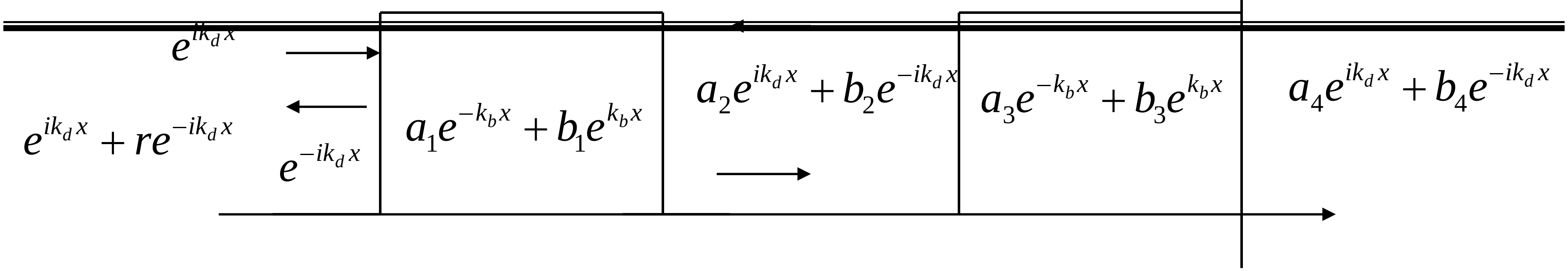
$$ik_d a_2 e^{ik_d (b+d)} - ik_d b_2 e^{-ik_d (b+d)} = -k_b a_3 + k_b b_3$$

$$k_b a_2 e^{ik_d (b+d)} + k_b b_2 e^{-ik_d (b+d)} = k_b a_3 + k_b b_3$$

$$a_3 = \frac{(k_b - ik_d) a_2 e^{ik_d (b+d)} + (k_b + ik_d) b}{2k_b}$$

$$b_3 = \frac{(k_b + ik_d) a_2 e^{ik_d (b+d)} + (k_b - ik_d) b_2}{2k_b}$$

$$\begin{pmatrix} a_3 \\ b_3 \end{pmatrix} = \frac{1}{2k_b} \begin{pmatrix} (k_b - ik_d) e^{ik_d (b+d)} & (k_b + \\ (k_b + ik_d) e^{ik_d (b+d)} & (k_b - \end{pmatrix}$$



At this interface

$$a_3 e^{-k_b(x-b-d)} + b_3 e^{k_b(x-b-d)} = a_4 e^{ik_d x} + b_4 e^{-ik_d x} =\Big|_{x=b+b+d}$$

$$a_3 e^{-k_b b} + b_3 e^{k_b b} = a_4 e^{ik_d(2b+d)} + b_4 e^{-ik_d(2b+d)}$$

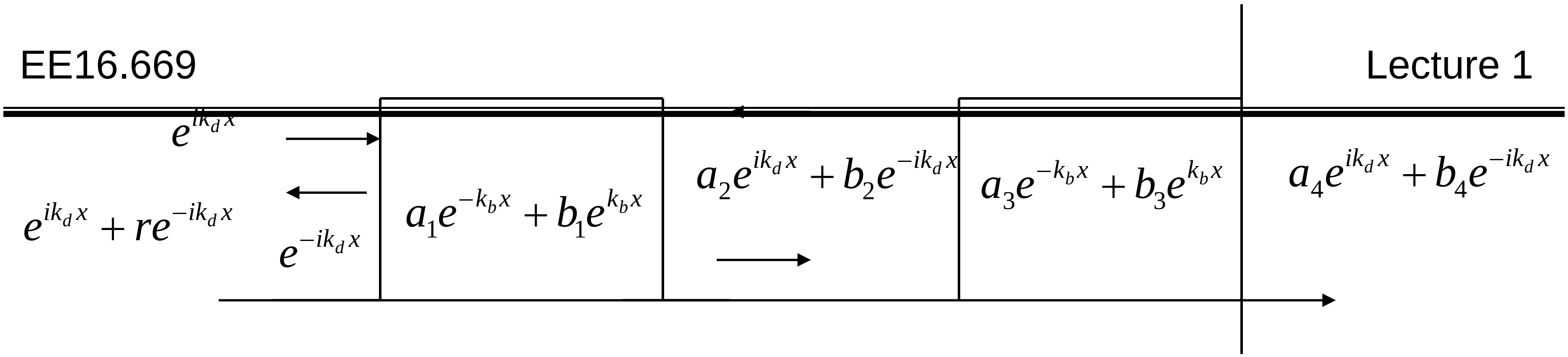
$$-k_b a_3 e^{-k_b b} + k_b b_3 e^{k_b b} = ik_d a_4 e^{ik_d(2b+d)} - ik_d b_4 e^{-ik_d(2b+d)}$$

$$ik_d a_3 e^{-k_b b} + ik_d b_3 e^{k_b b} = ik_d a_4 e^{ik_d(2b+d)} + ik_d b_4 e^{-ik_d(2b+d)}$$

$$a_4 = \frac{(ik_d - k_b) a_3 e^{-k_b b} + (k_b + ik}{2ik_d e^{ik_d(2b+d)}}$$

$$b_4 = \frac{(ik_d + k_b) a_3 e^{-k_b b} + (-k_b +}{2ik_d e^{ik_d(2b+d)}}$$

$$\begin{pmatrix} a_4 \\ b_4 \end{pmatrix} = \frac{1}{2ik_d e^{ik_d(2b+d)}} \begin{pmatrix} -(k_b - ik_d) \\ (k_b + ik_d)e \end{pmatrix}$$



$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \frac{1}{2k_b} \begin{pmatrix} (k_b - ik_d) & (k_b + ik_d) \\ (k_b + ik_d) & (k_b - ik_d) \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix}$$

$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix}$$

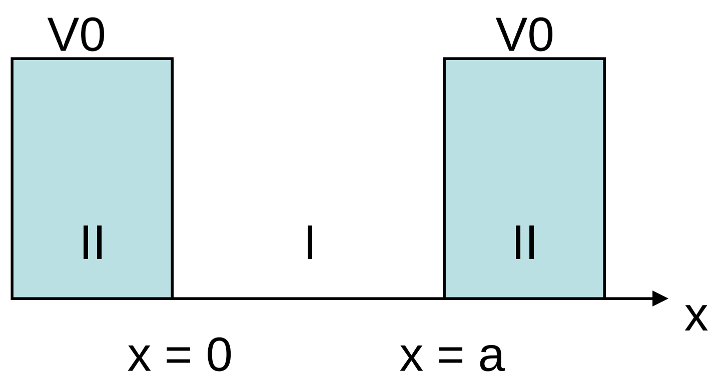
$$\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \frac{1}{2ik_d} \begin{pmatrix} -(k_b - ik_d)e^{-k_b b} & (k_b + ik_d)e^{k_b b} \\ (k_b + ik_d)e^{-k_b b} & -(k_b - ik_d)e^{k_b b} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

$$T = M_{11} + rM_{12} = M_{11} - \frac{M}{}$$

$$\begin{pmatrix} a_3 \\ b_3 \end{pmatrix} = \frac{1}{2k_b} \begin{pmatrix} (k_b - ik_d)e^{ik_d d} & (k_b + ik_d)e^{-ik_d d} \\ (k_b + ik_d)e^{ik_d d} & (k_b - ik_d)e^{-ik_d d} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

$$\begin{pmatrix} a_4 \\ b_4 \end{pmatrix} = \frac{1}{2ik_d} \begin{pmatrix} -(k_b - ik_d)e^{-k_b b} & (k_b + ik_d)e^{k_b b} \\ (k_b + ik_d)e^{-k_b b} & -(k_b - ik_d)e^{-k_b b} \end{pmatrix} \begin{pmatrix} a_3 \\ b_3 \end{pmatrix}$$

Hw#1

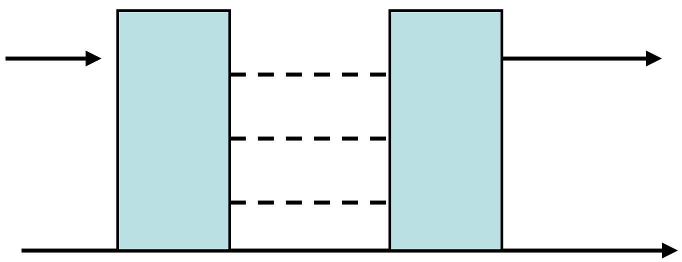


V0 = 1eV, a = 10nm

(1) calculate the first three energy levels and the corresponding wave functions.

(2) What’s the barrier thickness to effectively confine these waves?

Hw#2, Matlab simulation of the tunneling probility.



Reading assignment: online book: Ch. 1.2

http://ecee.colorado.edu/~bart/book/book/index.html