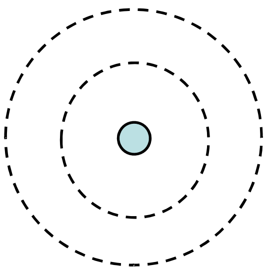
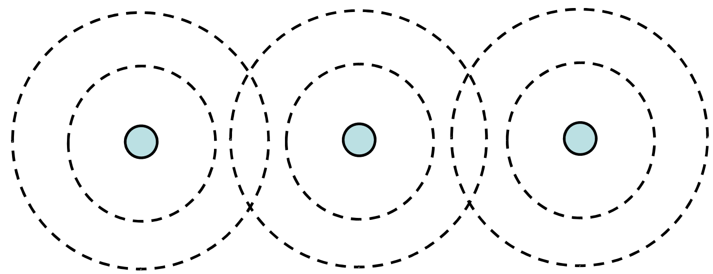
Principles of solid state electronics:

1. Energy bands

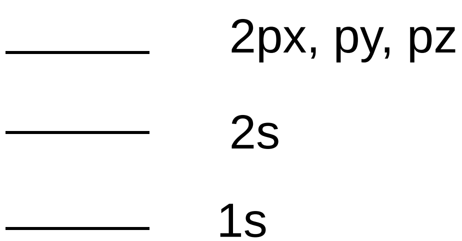
Atom



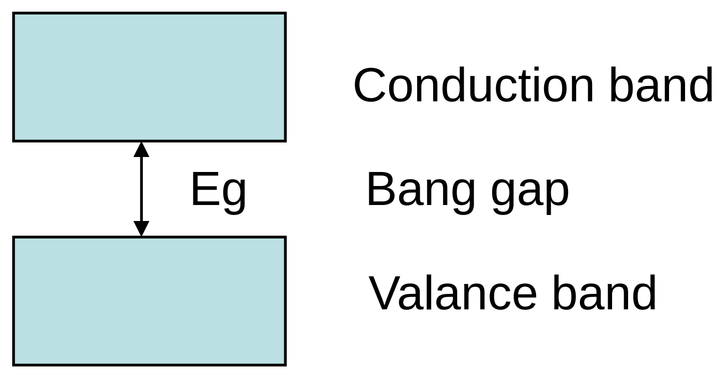
Semiconductor

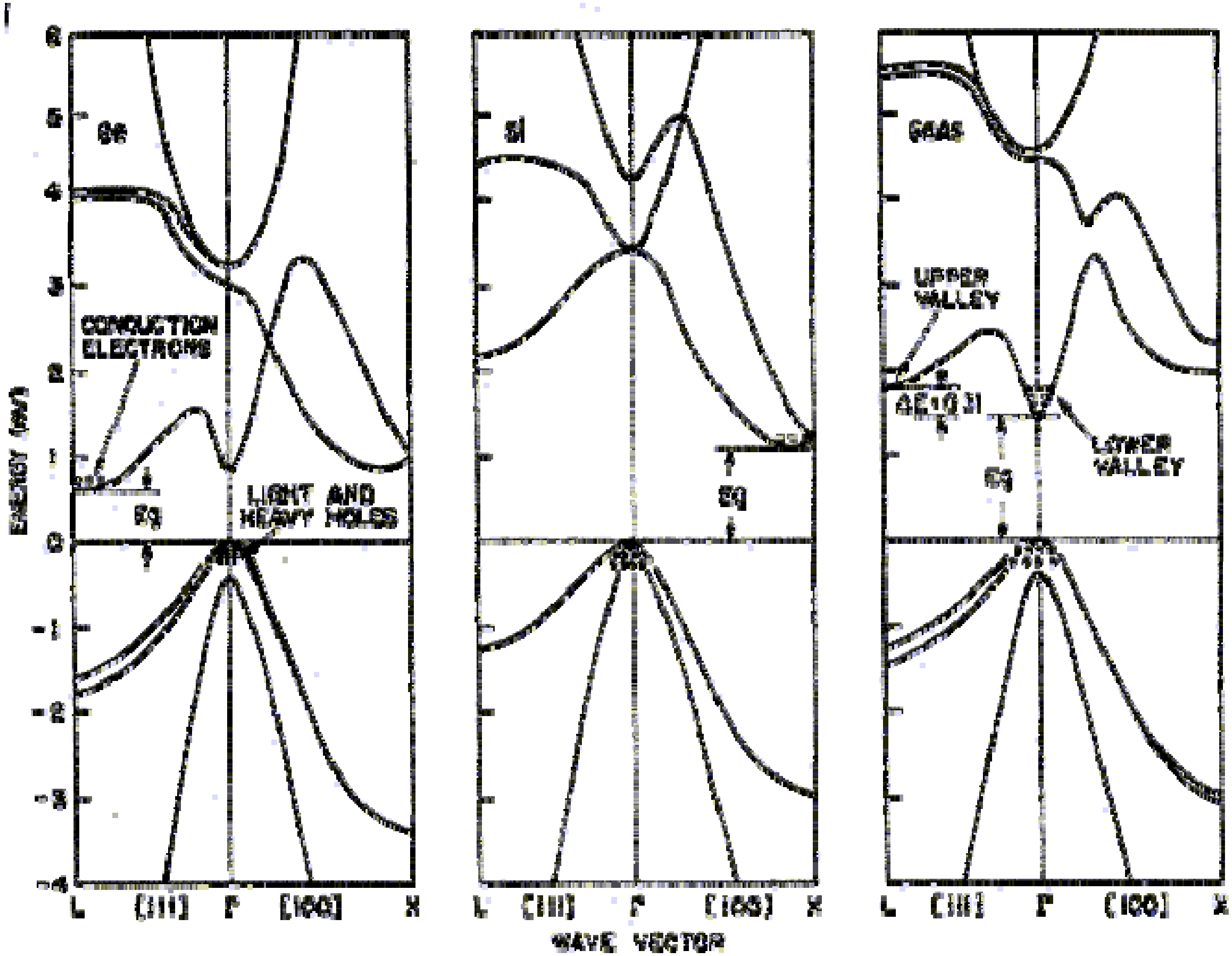


Energy levels



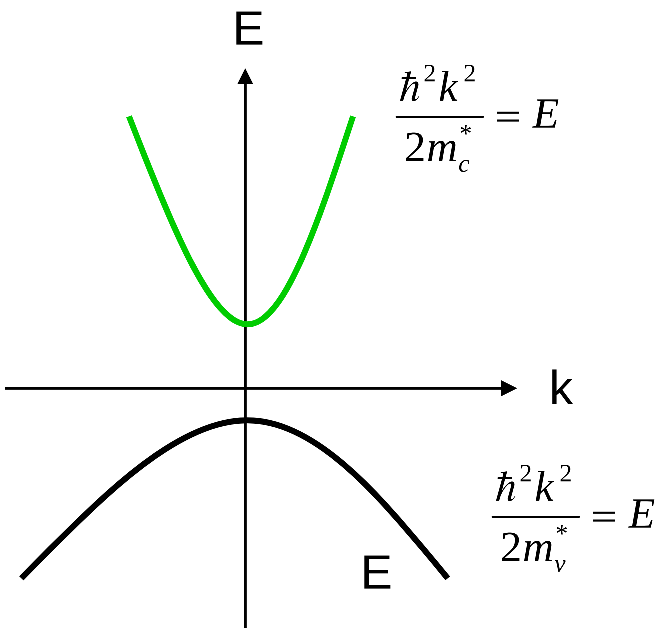
Energy bands

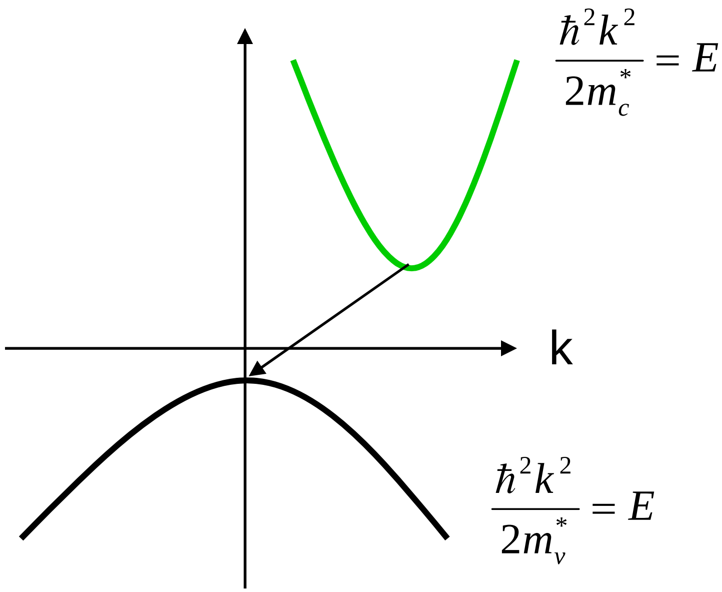




Energy band diagram of (a) germanium, (b) silicon and (c) gallium arsenide

3. Direct and indirect materials





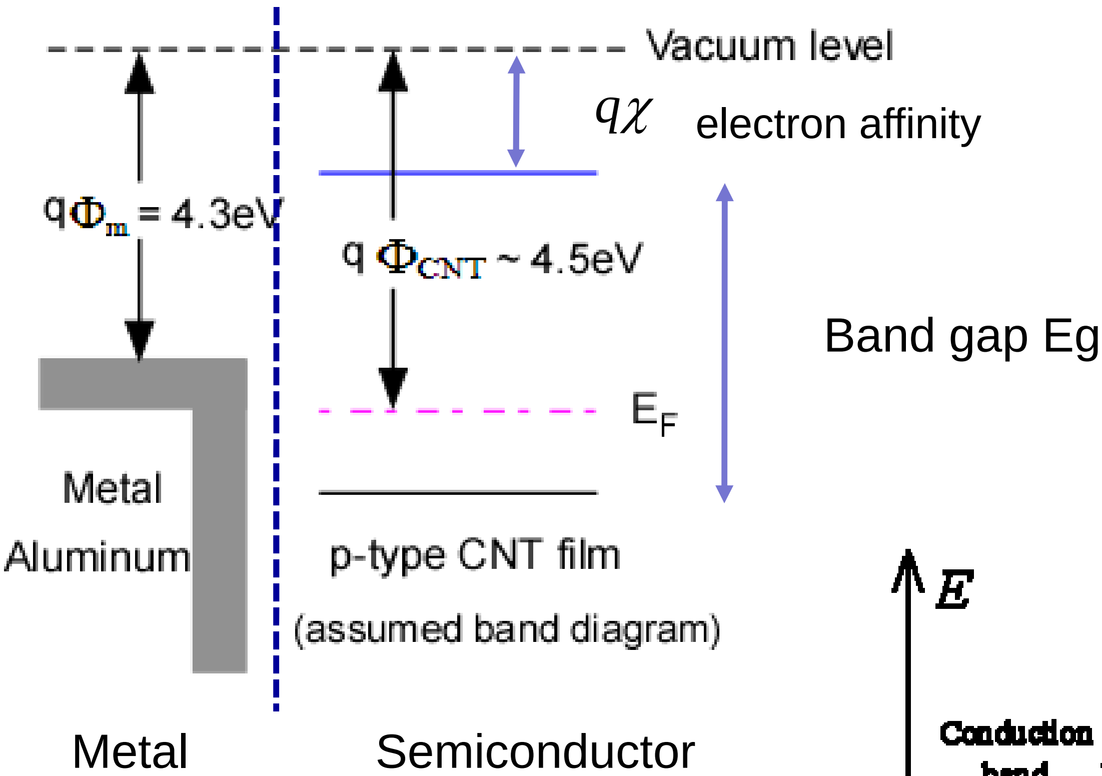
direct material, GaAs, AlxGa1-xAs, x<42%

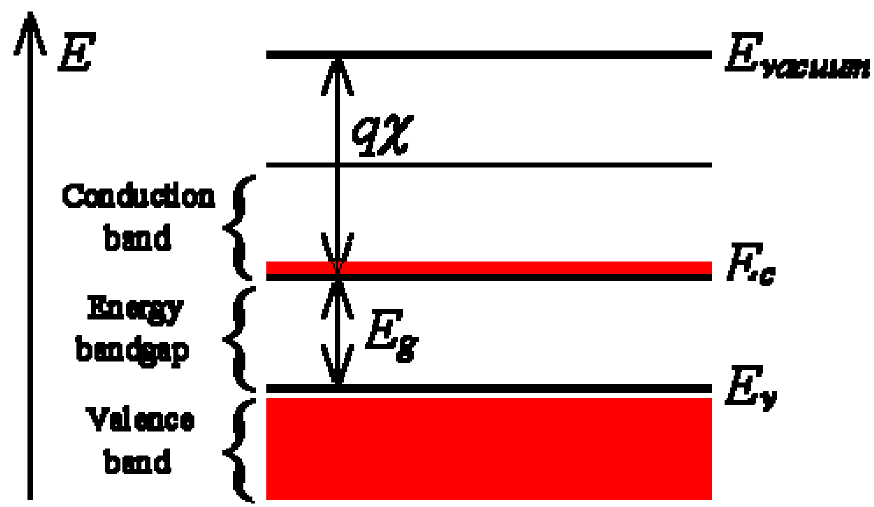
Direct transition allowed, efficient transition, good light emitter

Indirect material, Si, AlxGa1-xAs, x>42%, GaAsxP1-x, x>55%

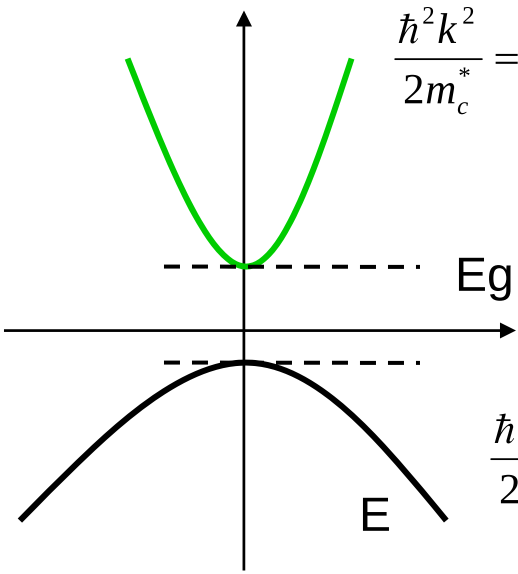
Transition involves phonons (lattice vibration) to keep momentum conservation. Inefficient transition, poor light emitter

Energy band diagram





Temperature dependent energy gap

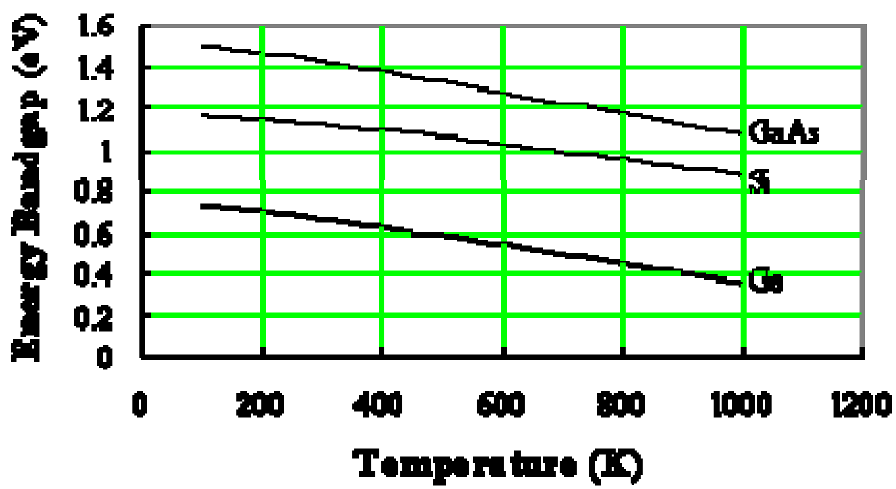


$$\frac{\hbar^2 k^2}{2m_c^*} = E$$

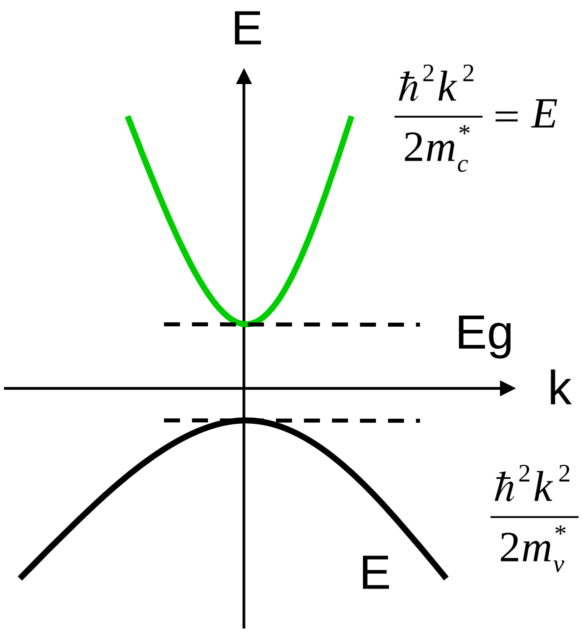
$$\frac{\hbar^2 k^2}{2m_v^*} = E$$

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{T + \beta}$$

	Germanium	Silicon	GaAs
$E_g(0)$ (eV)	0.7437	1.166	1.519
α (meV/K)	0.477	0.473	0.541
β (K)	235	636	204



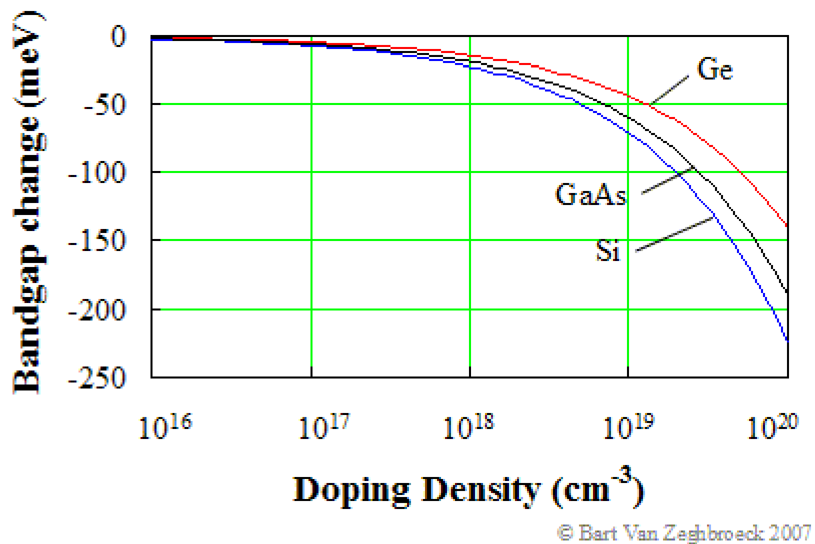
Doping dependence of the energy bandgap



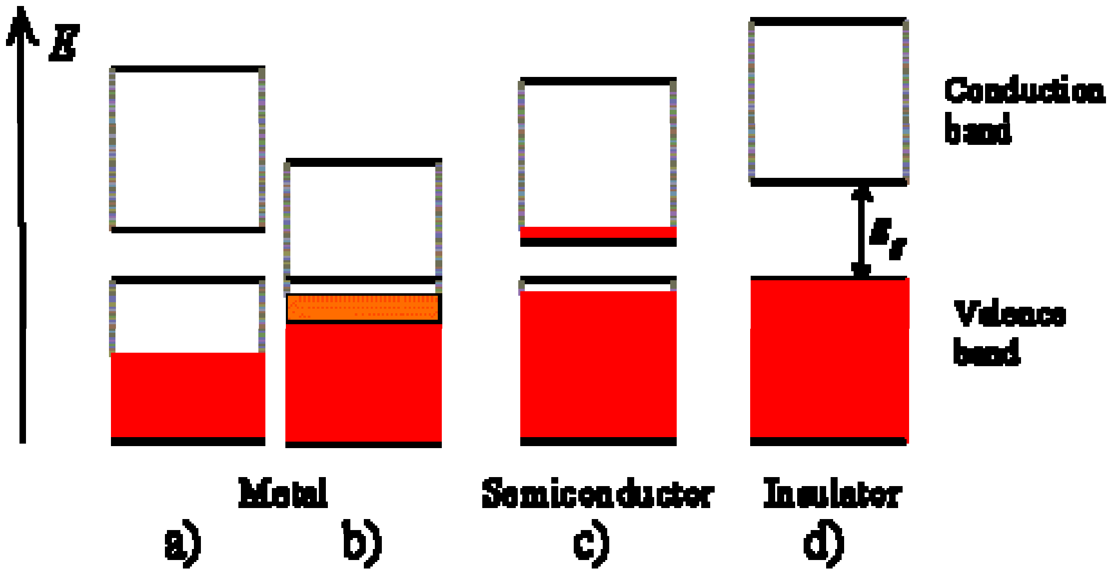
$$\frac{\hbar^2 k^2}{2m_c^*} = E$$

$$\frac{\hbar^2 k^2}{2m_v^*} = E$$

$$\Delta E_g(N) = -\frac{3q^2}{16\pi\varepsilon_s}\sqrt{\frac{q^2 N}{\varepsilon_s kT}}$$

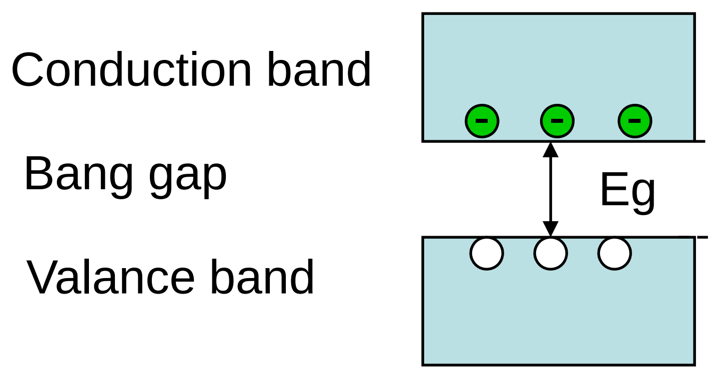


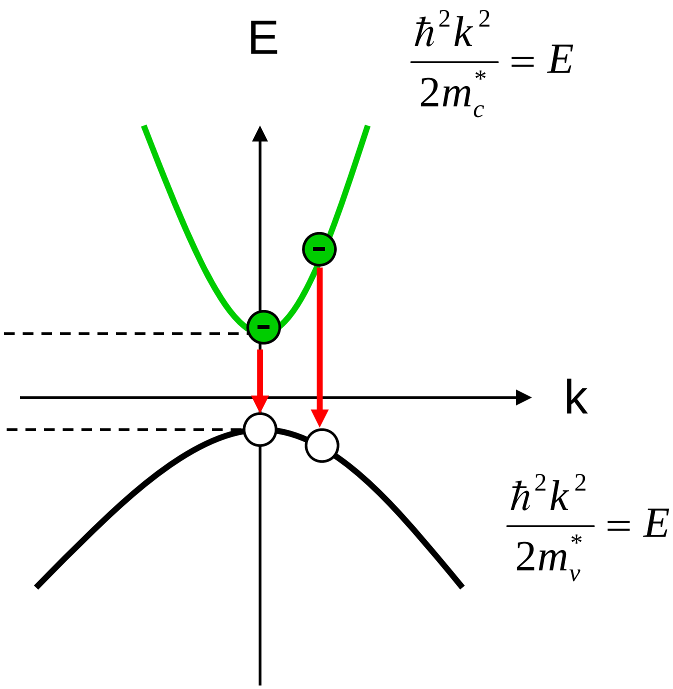
Metal, semiconductor and insulator



2. Effective mass

$$\frac{\hbar^2 k^2}{2m_c^*} = E \qquad m_c^* \cong 0.067 m_0 \qquad m_v^* \cong 1.6 m_0$$





		Germanium	Silicon	GaAs
Smallest energy bandgap at 300 K	E_g (eV)	0.66	1.12	1.424
Electron effective mass for density of states calculations	$\frac{m^*_{e,dos}}{m_0}$	0.55	1.08	0.067
Hole effective mass for density of states calculations	$\frac{m^*_{h,dos}}{m_0}$	0.37	0.811	0.45
Electron effective mass for conductivity calculations	$\frac{m^*_{e,cond}}{m_0}$	0.12	0.26	0.067
Hole effective mass for conductivity calculations	$\frac{m^*_{h,cond}}{m_0}$	0.21	0.386	0.34

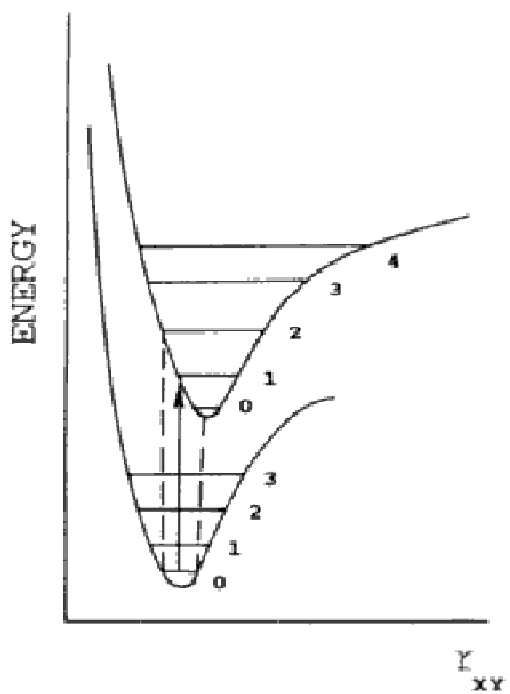
3. Vertical transition

$$p = \frac{h}{\lambda} = \hbar k,$$ Assume 532nm light, p = 1.3×10^{-27} kg.m/s

Free electron, E = 1eV $$p = mv = \sqrt{2mE},$$ p = 6.6×10^{-27} kg.m/s

Momentum conservation requires vertical transition.

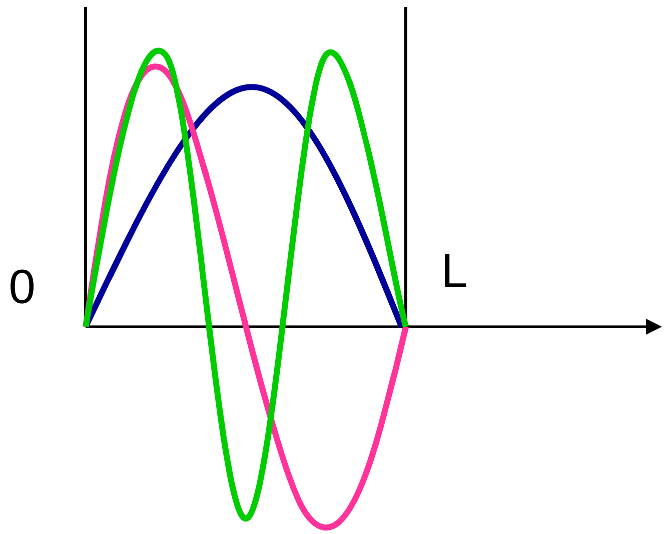
Frank-Condon Principle Molecular electron transitions

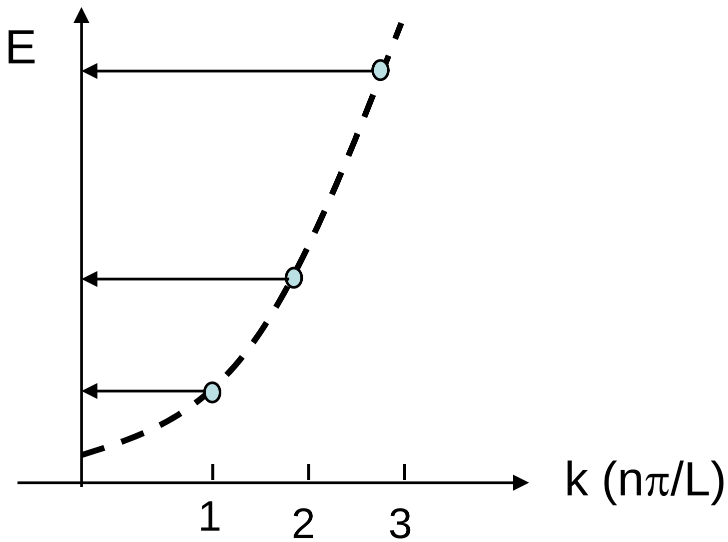


Atom coordinate

3. Density of states (DOS)

Recall 1-D potential well example:





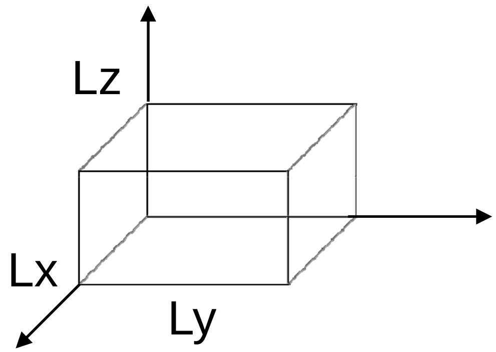
$$\psi(x) = A\sin kx, \quad \psi(x) = A\sin\left(\frac{n\pi}{L}x\right), \quad E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Each k corresponding to a wave function ⟹ Two states with opposite spins

At specific E, more than two states. $\rho(E)$ Density of states (DOS)

States from E to E+dE: $\rho(E)dE$

3. Density of states (DOS), bulk material

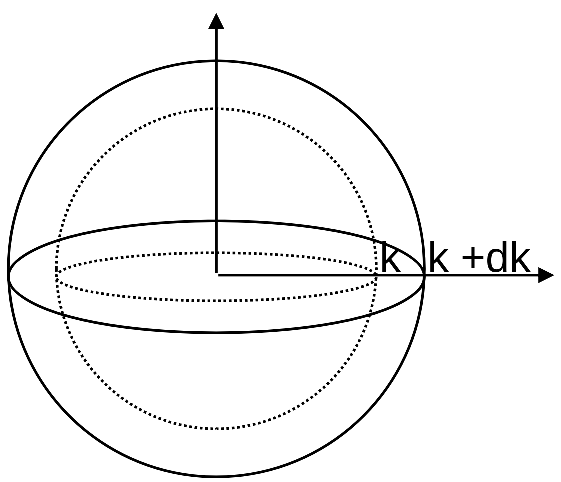


$$\psi(x) = A \sin k_x x \sin k_y y \sin k_z z,$$

$$k_x = 2\frac{n_x \pi}{L_x} \quad k_y = \frac{2n_y \pi}{L_y} \quad k_z = \frac{2n_z \pi}{L_z}$$

Ignore electron spin first:

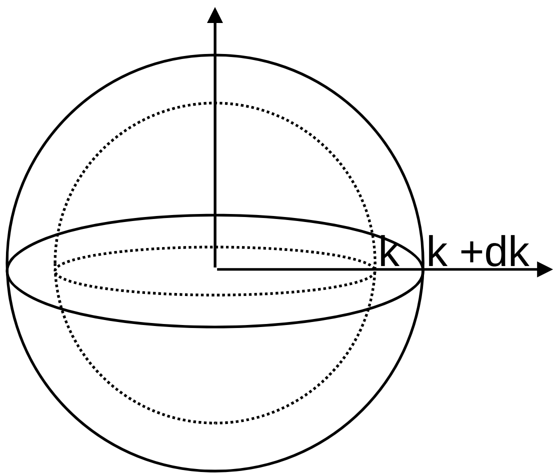
Each state takes $\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right) = \frac{8\pi^3}{V}$ volume in k-space



Total k-space volume between k to k+dk: $4\pi k^2 dk$

Total states between k to k+dk: $\frac{4\pi k^2 dk}{8\pi^3 / V}$

3. Density of states (DOS), bulk material

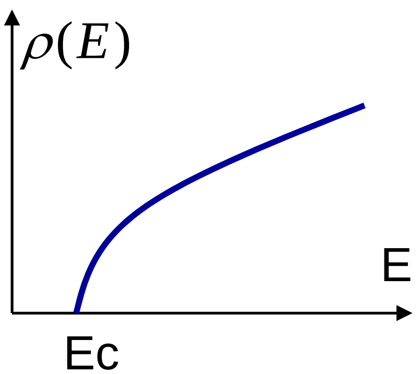


Total states between k to k+dk: $\dfrac{4\pi k^2 dk}{8\pi^3/V}$

$$\frac{\hbar^2 k^2}{2m_c^*} = E \Rightarrow \frac{\hbar^2 2kdk}{2m_c^*} = dE \Rightarrow kdk = \frac{2m_c^*}{\hbar^2} dE$$

Total states between k to k+dk: $\dfrac{4\pi k}{8\pi^3/V}\left(\dfrac{2m_c^*}{\hbar^2}\right)dE$

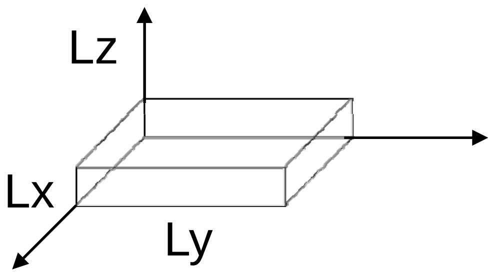
$$\frac{\hbar^2 k^2}{2m_c^*} = E \Rightarrow k = \left(\frac{2m_c^*}{\hbar^2}E\right)^{1/2}$$



Total states between E to E+dE: $\dfrac{4V}{8\pi^2}\left(\dfrac{2m_c^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E}\,dE$

$$\rho(E) = \frac{\sqrt{2}V}{\pi^2}\left(\frac{m_c^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E} \qquad \rho(E) = \frac{\sqrt{2}V}{\pi^2}\left(\frac{m_c^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E - E_c}$$

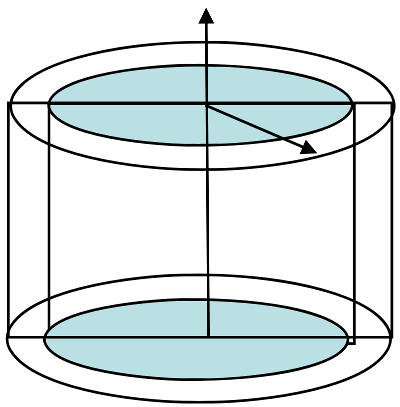
3. Density of states (DOS), quantum well



$$\psi(x) = A\sin k_x x \sin k_y y \sin k_z z,$$

$$k_x = \frac{n_x \pi}{L_x} \qquad k_y = \frac{n_y \pi}{L_y} \qquad k_z = \frac{n_z \pi}{L_z} \qquad E = E_z + \frac{\hbar^2\left(k_x^2 + k_y^2\right)}{2m_c^*}$$

Each state takes $\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right) = \frac{8\pi^3}{V}$ volume in k-space

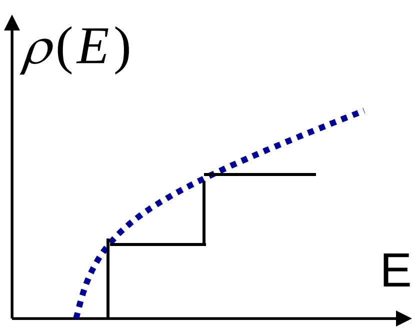


Total k-space volume between k to k+dk: $2\pi k dk$

Total states between k to k+dk: $\frac{2\pi k dk}{8\pi^3 / V}$

$$E = E_z + \frac{\hbar^2\left(k_x^2 + k_y^2\right)}{2m_c^*} \Longrightarrow dE = \frac{\hbar^2}{2m_c^*} 2k dk$$

Total states between E to E+dE: $\frac{\pi}{8\pi^3 / V}\left(\frac{2m_c^*}{\hbar^2}\right) dE$

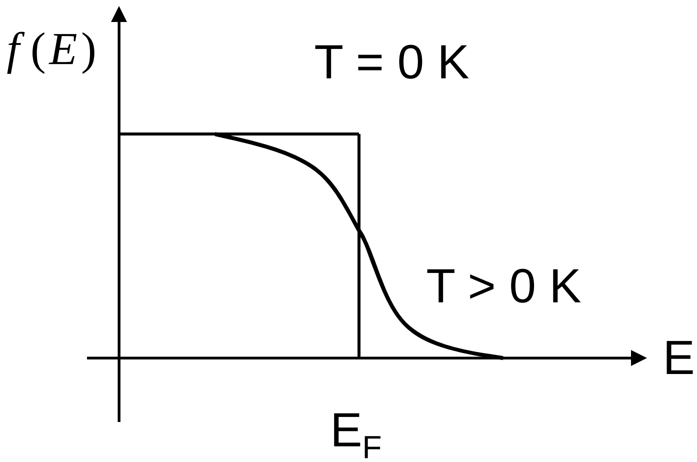


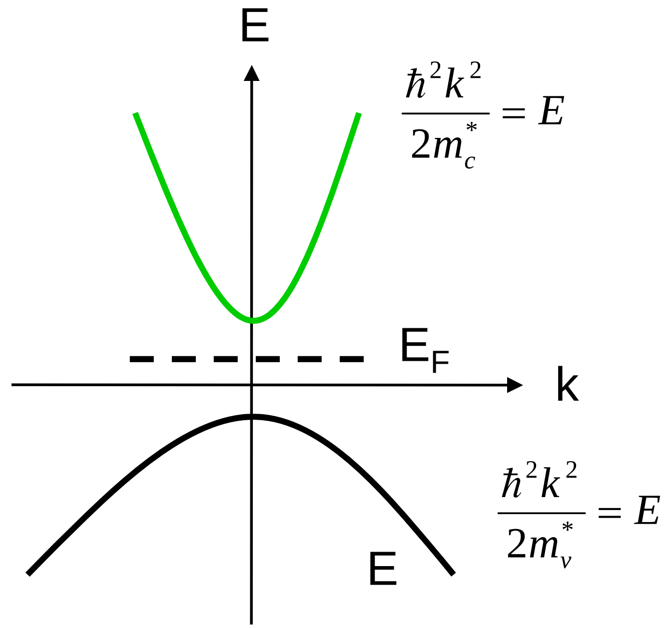
$$\rho(E) = \frac{1}{4\pi^2}\left(\frac{m_c^*}{\hbar^2}\right) dE$$ DOS per volume

4. Electron distribution, Fermi level

Probability of occupying the energy level E

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$





5. Electron population

$$n(E) = \rho(E) f(E)$$

5. Electron population

Total electrons in conduction band

$$n = \int_{E_c}^{\infty} n(E)dE = \int_{E_c}^{\infty} \rho(E) f(E) dE$$

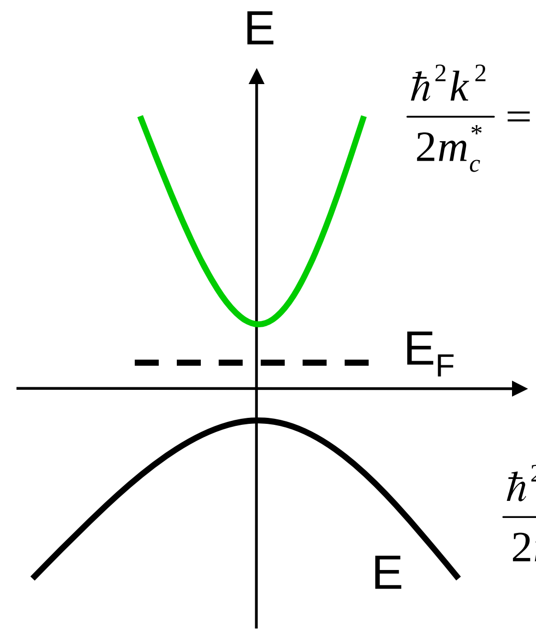
$$n = \int_{E_c}^{\infty} \frac{\sqrt{2}}{\pi^2}\left(\frac{m_c^*}{\hbar^2}\right)^{\frac{3}{2}} \sqrt{E}\, \frac{1}{1+e^{(E-E_F)/kT}}\, dE$$

$$n = \int_{E_c}^{\infty} \frac{\sqrt{2}}{\pi^2}\left(\frac{m_c^*}{\hbar^2}\right)^{\frac{3}{2}} \sqrt{E}\, e^{-(E-E_F)/kT} dE$$

$$\int_0^{\infty} \sqrt{x} e^{-ax} dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$$

$$n = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_c^*}{\hbar^2}\right)^{\frac{3}{2}} e^{(E_F-E_c)/kT}\, \frac{(kT)^{3/2}\sqrt{\pi}}{2}$$

$$N_C = 2\left(\frac{2\pi m_c^* kT}{h^2}\right)^{\frac{3}{2}} \qquad N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{\frac{3}{2}}$$



$$\frac{\hbar^2 k^2}{2m_c^*} = E$$

$$\frac{\hbar^2 k^2}{2m_v^*} = E$$

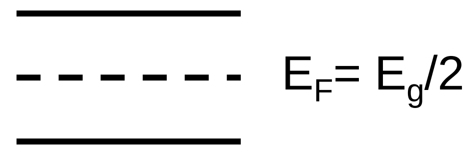
Electron spin

$$n = 2\left(\frac{2\pi m_c^* kT}{h^2}\right)^{\frac{3}{2}} e^{(E_F-E_c)/kT}$$

$$n = N_C e^{-(E_c-E_F)/kT} \qquad p = N_V e^{-(E_F-E_V)/kT}$$

5. Carrier concentration at thermal equilibrium, no Fermi level splitting

$$n = N_C e^{-(E_c - E_F)/kT} \quad p = N_V e^{-(E_F - E_V)/kT}$$



$$n_i^2 = np = N_C N_V e^{-(E_c - E_V)/kT} = N_C N_V e^{-E_g/kT}$$

Si, Eg = 1.1 eV,

$$N_C = 2\left(\frac{2\pi m_c^* kT}{h^2}\right)^{\frac{3}{2}} \quad N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{\frac{3}{2}}$$

300K

$Nc = 4.3 \times 10^{23}$

$Nv = 4.3 \times 10^{30}$

$ni = 2.1 \times 10^{15}$

GaAs Eg = 1.46eV

300K

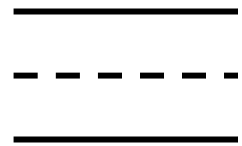
$Nc = 4.3 \times 10^{23}$

$Nv = 4.3 \times 10^{30}$

$ni = 1.9 \times 10^{12}$

E_F level for intrinsic material at thermal equilibrium

$$n = N_C e^{-(E_c - E_F)/kT} \quad p = N_V e^{-(E_F - E_V)/kT}$$



$E_F \cong E_g/2$

Si, Eg = 1.1 eV,

300K

Nc = $4.3\text{x}10^{23}$

Nv = $4.3\text{x}10^{30}$

ni = $2.1\text{x}10^{15}$

Ec-E_F = 0.496eV

E_F-Ev = 0.603eV

6. Carrier concentration in extrinsic materials

Charge neutrality:

$$p_0 + N_D^+ = n_0 + N_A^-$$

n_0 Electron concentration at thermal equilibrium

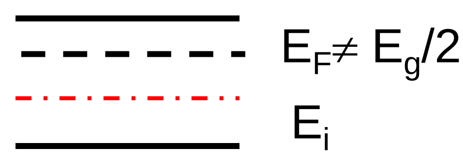
p_0 hole concentration at thermal equilibrium

$$p_0 = n_0 + N_A^- - N_D^+$$

$$n_0 = N_C e^{-(E_c - E_F)/kT} \quad p_0 = N_V e^{-(E_F - E_V)/kT} \quad n_0 p_0 = N_C N_V e^{-(E_c - E_V)/kT} = N_C N_V e^{-E_g/kT} = n_i^2$$

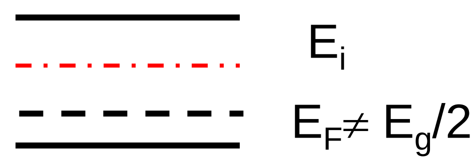
$$n_0 = n_i e^{-(E_i - E_F)/kT} \quad p_0 = n_i e^{-(E_F - E_i)/kT}$$

Extrinsic, N doped: $n = N_D \quad p = \frac{n_i^2}{N_D}$



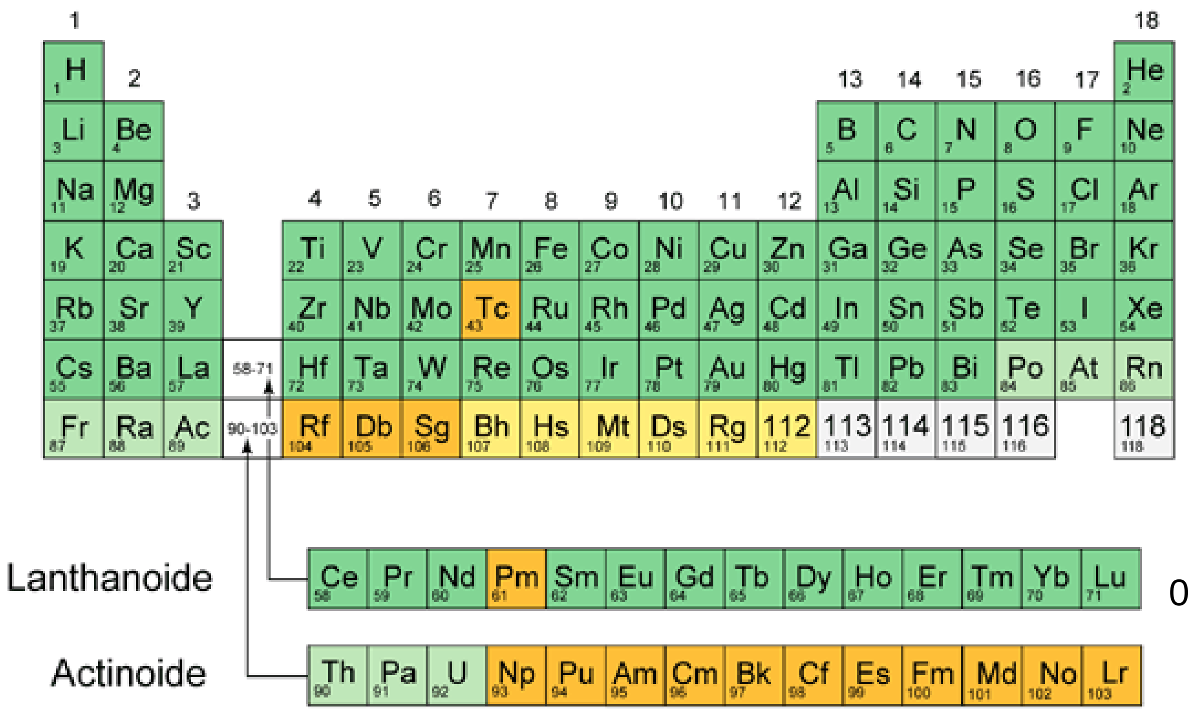
$$p_0 = n_i e^{-(E_F - E_i)/kT} = \frac{n_i^2}{N_D} \quad E_F - E_i = -kT \ln\left(\frac{n_i}{N_D}\right)$$

Extrinsic, P doped: $p = N_A \quad n = \frac{n_i^2}{N_A}$



$$n_0 = n_i e^{-(E_i - E_F)/kT} = \frac{n_i^2}{N_A} \quad E_i - E_F = kT \ln\left(\frac{n_i}{N_A}\right)$$

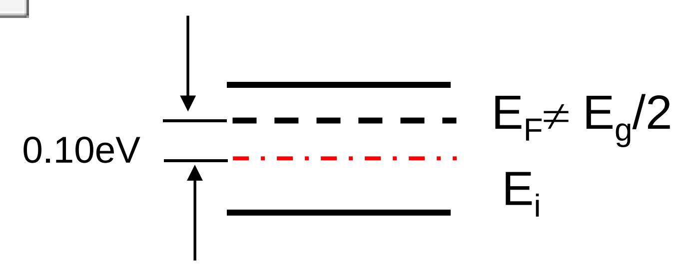
Example: a Si sample is doped with 10^{17}cm^{-3} As atoms, what's the equilibrium concentration of holes at 300K, where the E_F relative to E_i assuming $n_i = 2.0\times10^{15}$cm^{-3}



$$p_0 = n_i e^{-(E_i - E_F)/kT} = \frac{n_i^2}{N_D}$$

p = 4.0×10^{13} cm^{-3}

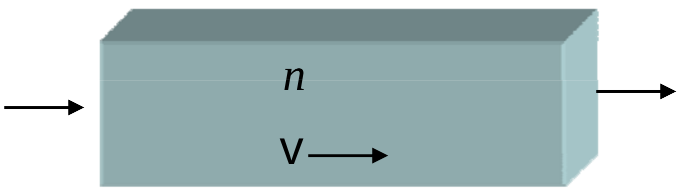
$$E_F - E_i = -kT\ln\left(\frac{n_i}{N_D}\right) = 0.10eV$$



7. Conductivity and mobility

$$J = \sigma E$$ Ohm's law

$$\sigma = nq\mu_n + pq\mu_p$$ Drift mobility



$$J = vnq$$

$$v = \mu E$$ Before velocity saturation

Diffusion coefficient, D

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}}$$ Einstein relation

8. Current continuous equations

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} \qquad J_{drift} = -q\mu \frac{dn}{dx}$$

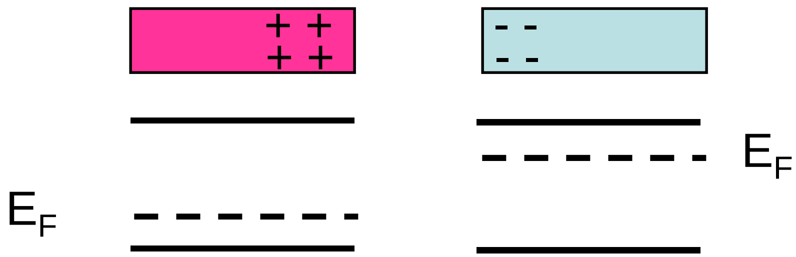
Drift mobility

$$q\mu \frac{d^2 n}{dx^2} = \frac{\partial \rho}{\partial t} \qquad \mu \frac{d^2 n}{dx^2} = \frac{\partial n}{\partial t}$$

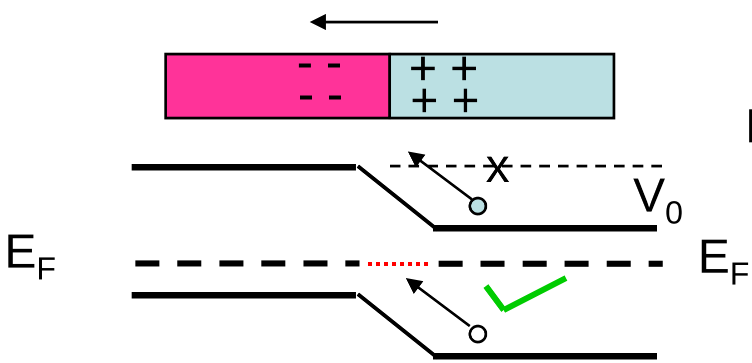
Steady state, with no E

$$\frac{d^2 n}{dx^2} = 0$$

1-D Laplace’s equation



E Build-in electric field



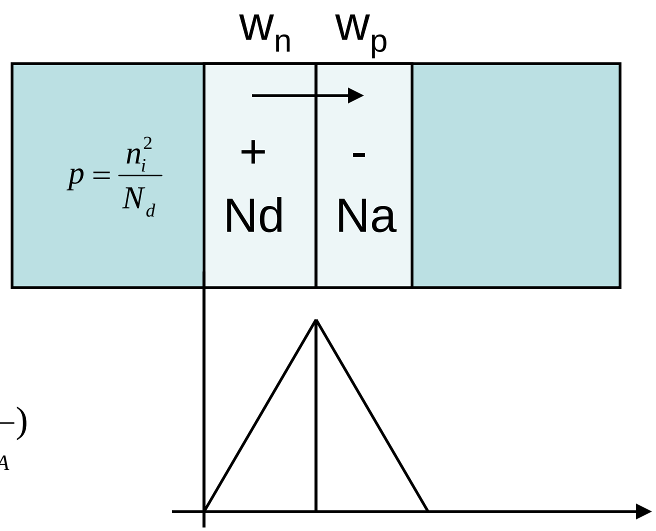
Fermi level aligned

$$E_F - E_i = -kT \ln\left(\frac{n_i}{N_D}\right)$$

$$E_i - E_F = kT \ln\left(\frac{n_i}{N_A}\right)$$

$$V_0 = kT \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$E_F - E_i = -\frac{kT}{q}\ln(\frac{n_i}{N_d})$$



$$E_F - E_i = -kT\ln(\frac{n_i}{N_A})$$

$$V_0 = -\frac{kT}{q}\ln(\frac{n_i^2}{N_d N_A})$$

$$\nabla \cdot D = qN_D$$

$$\frac{dE}{dx} = \frac{q}{\varepsilon}N_d \qquad \frac{dE}{dx} = -\frac{9}{\varepsilon}N_A \qquad N_d W_n = N_A W_p$$

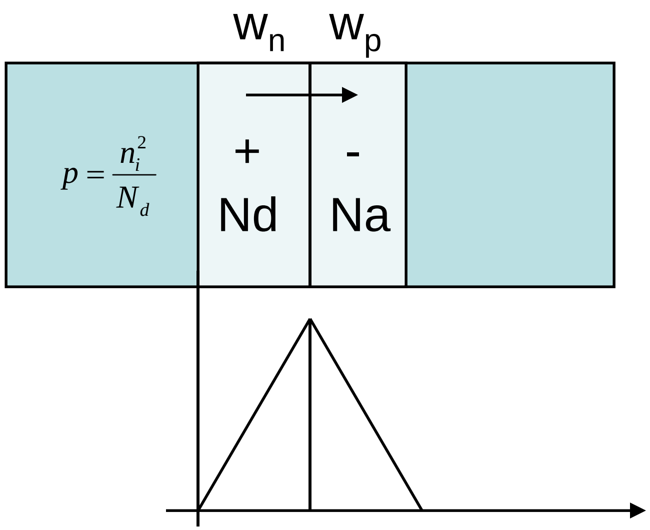
$$V_0 = \frac{q}{2}(W_p + W_n)\frac{N_d}{\varepsilon}W_n \qquad V_0 = \frac{q}{2}(1+\frac{N_d}{N_A})\frac{N_d}{\varepsilon}W_n^2$$

$$W_n = \sqrt{\frac{2\varepsilon V_0 / q}{N_A + N_d}\frac{N_A}{N_d}}$$

$$W = (1+\frac{N_d}{N_A})W_n = \sqrt{\frac{2\varepsilon V_0 / q}{N_A}\frac{N_A + N_d}{N_d}}$$

Depletion layer capacitance

$$E_F - E_i = -\frac{kT}{q}\ln(\frac{n_i}{N_d})$$



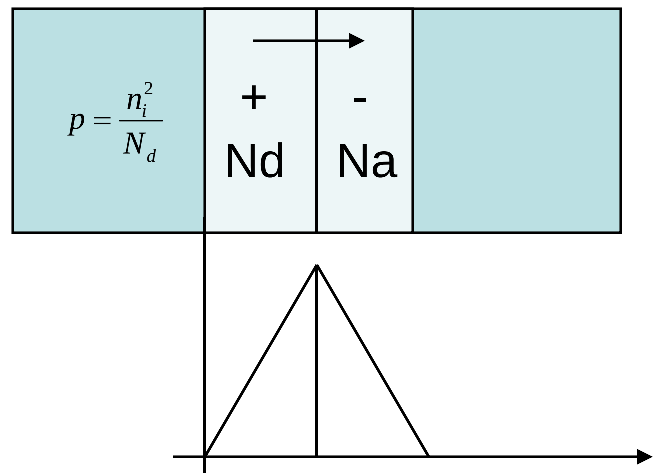
$$E_F - E_i = -kT\ln(\frac{n_i}{N_A})$$

$$\frac{dE}{dx} = \frac{1}{\varepsilon}N_d \qquad \frac{dE}{dx} = -\frac{1}{\varepsilon}N_A \qquad N_d W_n = N_A W_p$$

$$C_{dep} = \frac{\varepsilon A}{W} = \frac{\varepsilon A}{\sqrt{\frac{2\varepsilon V_0}{q}\frac{N_A + N_d}{N_d N_A}}} = \frac{A}{V_0}\sqrt{\frac{q\varepsilon}{2}\frac{N_d N_A}{N_d + N_A}}$$

Example: An abrupt p-n junction formed by n-type doped $N_d = 10^{16} cm^{-3}$, Na = $4x10^{18} cm^{-3}$, calculate: V_0, W, X_p, X_n , and E_0

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.85V$$



$$W = \sqrt{\frac{2\varepsilon V_0}{q} \frac{N_d + N_A}{N_d N_A}} = 0.33\mu m$$

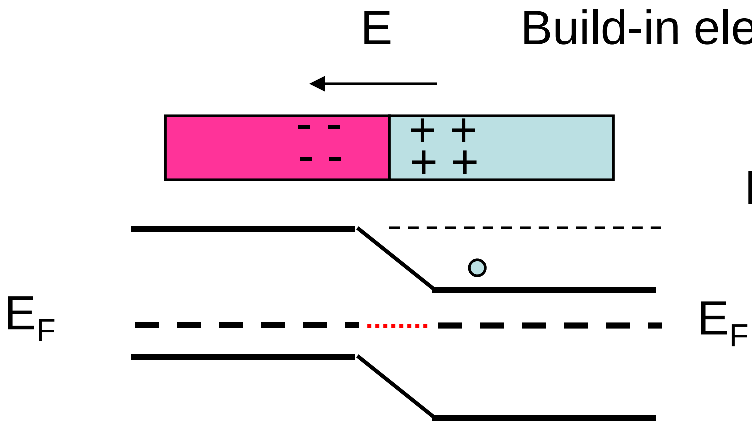
$$X_p = \frac{W N_d}{N_A + N_d} = 0.83nm$$

$$X_n = \frac{W N_A}{N_A + N_d} = 0.33\mu m$$

$$\frac{1}{2} E_0 W = V_0$$

$$E_0 = 5 \times 10^4 V / cm$$

P-n junction at equilibrium

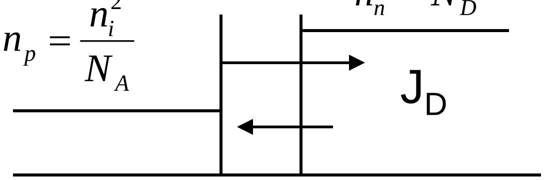


Fermi level aligned

$$E_F - E_i = -kT \ln\left(\frac{n_i}{N_D}\right)$$

$$E_i - E_F = kT \ln\left(\frac{n_i}{N_A}\right)$$

$$qV_0 = kT \ln\left(\frac{N_A N_D}{n_i^2}\right)$$



$$n_p = \frac{n_i^2}{N_A} \qquad n_n = N_D$$

$$n_p = n_n e^{-\frac{qV_0}{kT}} = N_D e^{-\ln\frac{N_A N_D}{n_i^2}} = \frac{n_i^2}{N_A}$$

Diffusion current $J_{diffusion} = qD_n \frac{dn}{dx}$

Drift current $J_{drift} = -q\mu_n E$

Total: $0 = J_{total} = J_{diffusion} + J_{drift} = qD_n \frac{dn}{dx} - q\mu_n E$

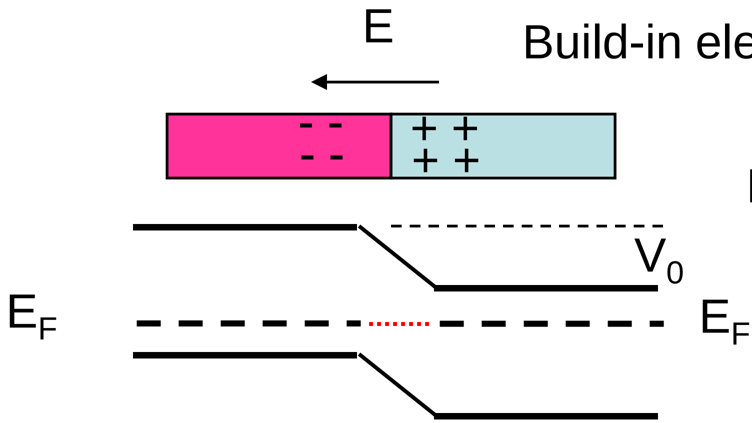
$$D_n \frac{dn}{dx} = \mu_n E$$

$$\frac{D_n}{\mu_n}\frac{dn}{dx} = -\frac{dV}{dx}$$

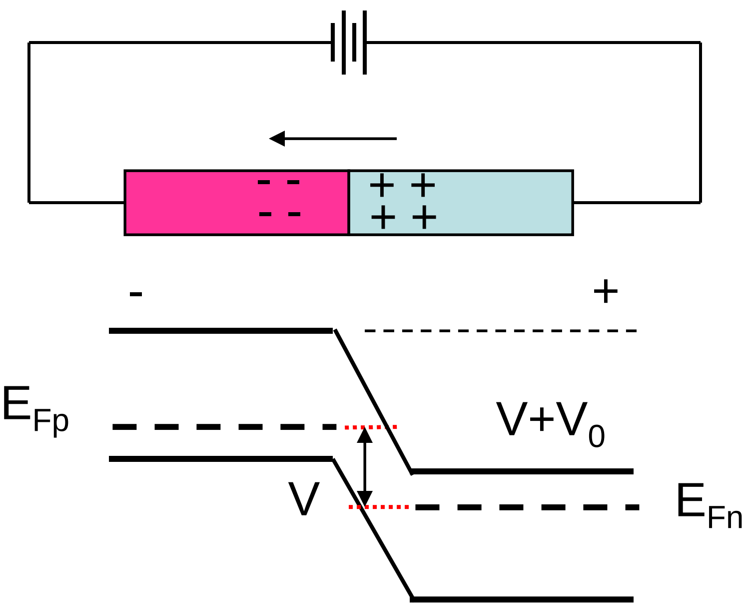
$$\frac{kT}{q}\ln n = V_0$$

Build-in electric field

Fermi level aligned



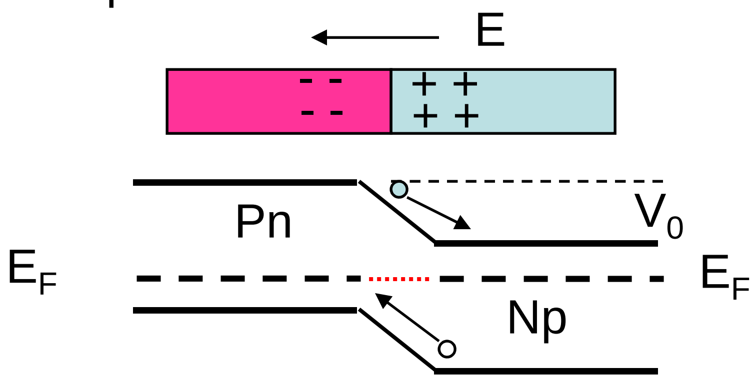
Reverse bias



Fermi level splitting, qusi-Fermi levels

P-n junction at non-equilibrium

Carrier profile



$$J = J_p + J_n$$

$$J_n = qD\frac{dn}{dx} + q\mu_n E \qquad qD\frac{dn_0}{dx} + q\mu_n E = 0$$

$$J_p = -nqD\frac{dp}{dx} + q\mu_p E$$

Excess carriers

$$\frac{\partial n(x,t)}{\partial x} = \frac{1}{q}\frac{\partial J_n}{\partial x} + \frac{\partial \Delta n}{\partial t}$$

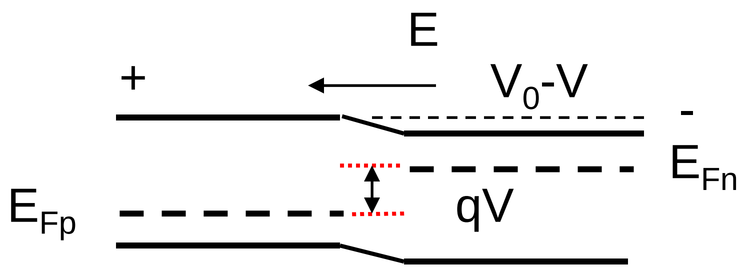
Steady state

$$\frac{\partial J_n}{\partial x} = q\frac{\Delta n}{\tau_p} \qquad D\frac{d^2\Delta n}{dx^2} = \frac{\Delta n}{\tau_p}$$

$$\Delta n(x,t) = \Delta n(x=0,t)e^{-x/L_n} \qquad L_n = \sqrt{D_n \tau_n}$$

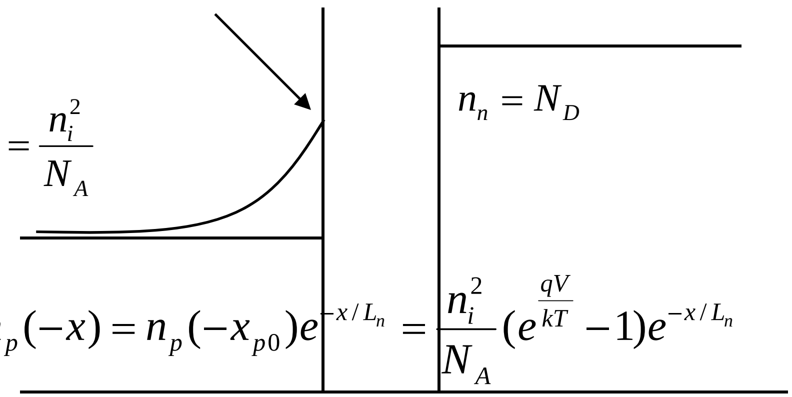
$$\Delta p(x,t) = \Delta p(x=0,t)e^{-x/L_p}$$

Excess carrier



$$n_p(-x_{p0}) = \frac{n_i^2}{N_A}(e^{\frac{qV}{kT}} - 1)$$

Excess carriers



$$n_n = N_D$$

$$n_p = \frac{n_i^2}{N_A}$$

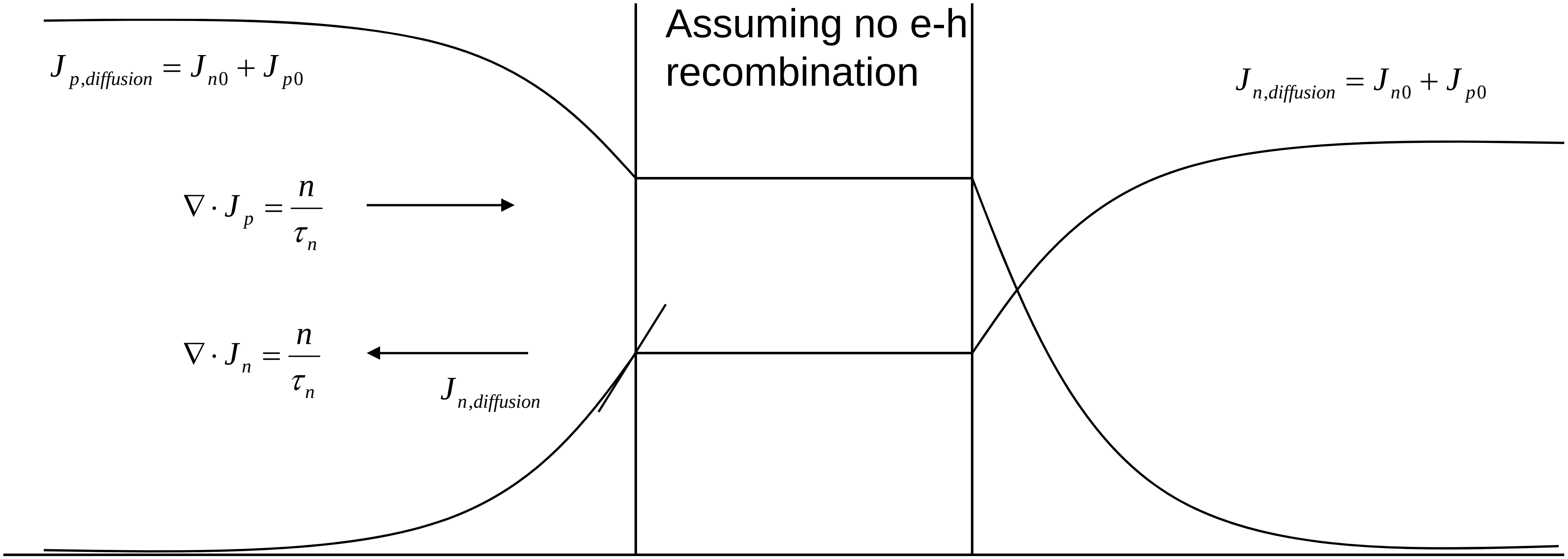
$$n_p(-x) = n_p(-x_{p0})e^{-x/L_n} = \frac{n_i^2}{N_A}(e^{\frac{qV}{kT}} - 1)e^{-x/L_n}$$

$$L_n = \sqrt{D_n \tau_n}$$

$$\nabla \cdot J = -q\frac{dn}{dt}$$

$$qD_n\frac{d^2n}{dx^2} = q\frac{dn}{dt} = q\frac{n}{\tau_n}$$

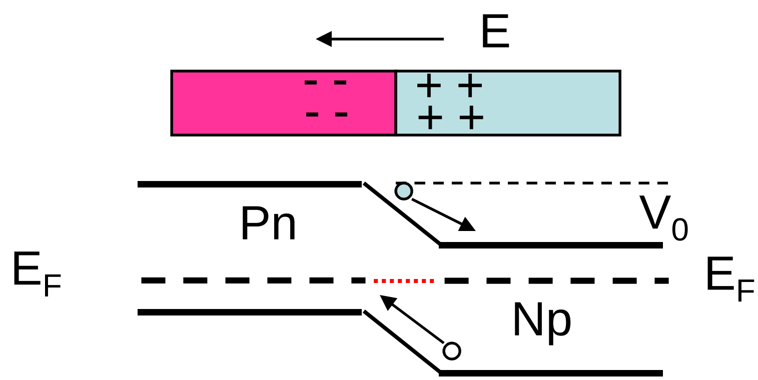
P-n junction at non-equilibrium



$$J_{p0} = qD_p \frac{d\Delta n_p(x=-x_{p0})e^{-x/L_p}}{dx} = q\frac{D_p}{L_p}\Delta n_p(x=-x_{p0}) = \frac{qD_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV}{kT}}-1)$$

$$J_{n0} = qD_n \frac{d\Delta n_n(x=x_{n0})e^{-x/L_n}}{dx} = q\frac{D_n}{L_n}\Delta p_n(x=-x_{p0}) = \frac{qD_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV}{kT}}-1)$$

Current



$$J = J_p + J_n$$

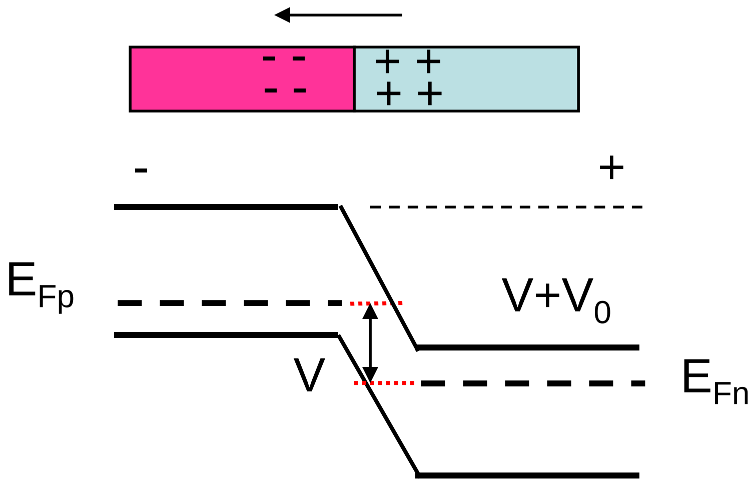
$$qD\frac{dn_0}{dx} + q\mu_n E = 0$$

$$J_p = -nqD\frac{dp}{dx} + q\mu_p E$$

$$J_n = qD\frac{dn}{dx} + q\mu_n E$$

$$J_n|_{x=0} = qD_n\frac{d\Delta n}{dx}|_{x=0} = \frac{qD_n}{L_n}n_{p,0}(e^{\frac{qV}{kT}} - 1) = \frac{qD_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV}{kT}} - 1)$$

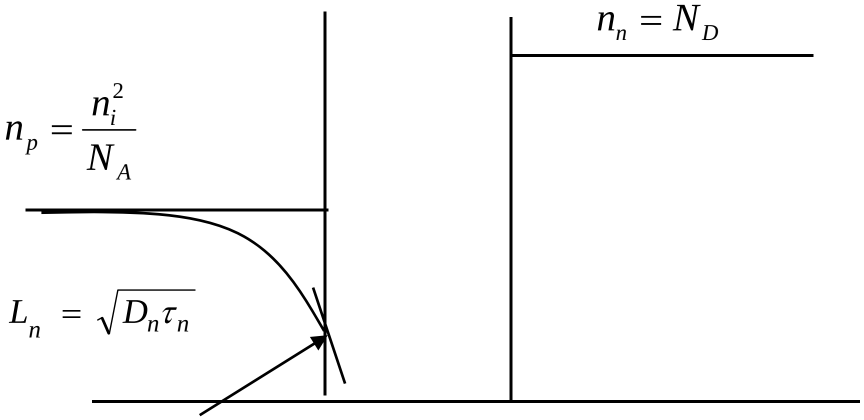
$$J_p|_{x=0} = qD_p\frac{d\Delta p}{dx}|_{x=0} = \frac{qD_p}{L_p}n_{p,0}(e^{\frac{qV}{kT}} - 1) = \frac{qD_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV}{kT}} - 1)$$

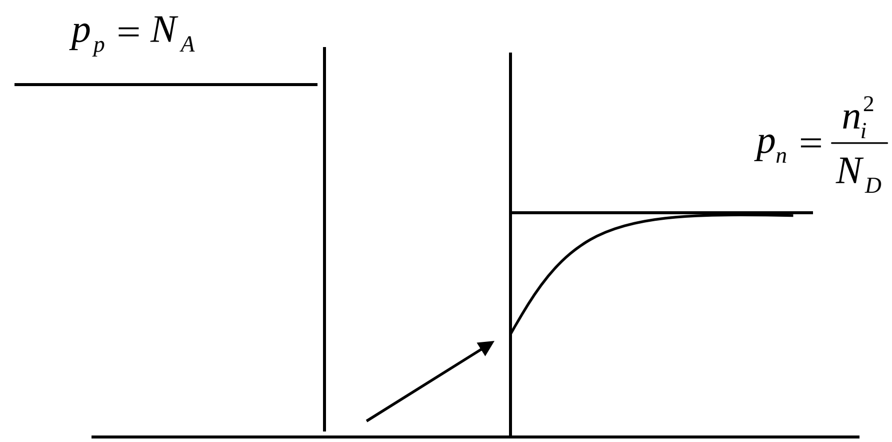


Reverse bias

Fermi level splitting, qusi-Fermi levels

Excess carriers





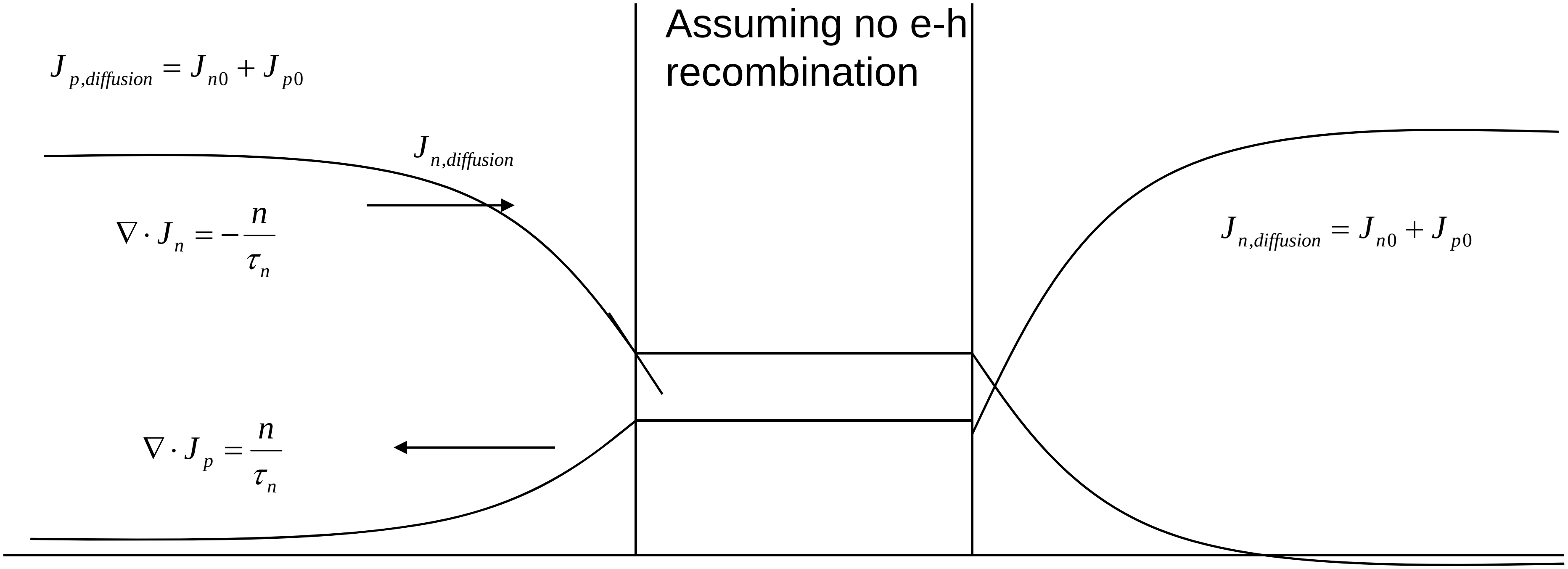
$$n_p(-x_{p0}) = \frac{n_i^2}{N_A}(e^{\frac{-qV}{kT}} - 1)$$

$$n_p(-x) = \Delta n_p(-x_{p0})e^{-x/L_n} = \frac{n_i^2}{N_A}(1 - e^{\frac{-qV}{kT}})e^{-x/L_n}$$

$$p_n(x_{n0}) = \frac{n_i^2}{N_D}(e^{\frac{-qV}{kT}} - 1)$$

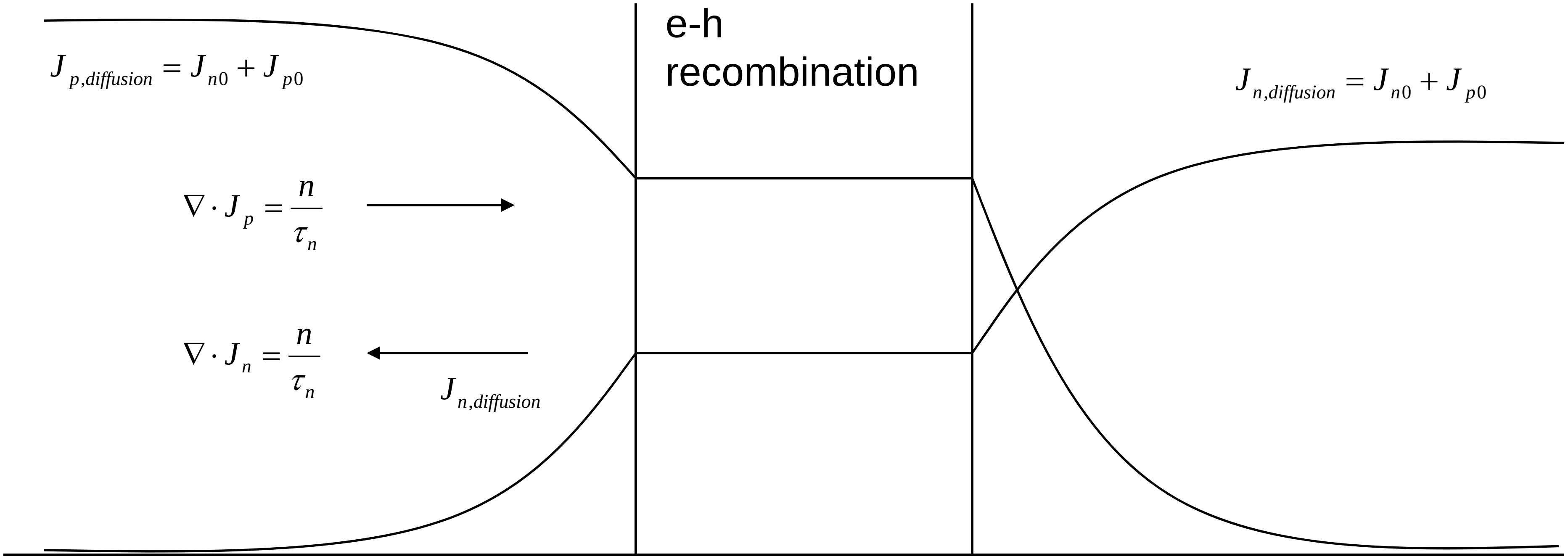
$$p_n(x) = \Delta p_n(x_{n0})e^{-x/L_n} = \frac{n_i^2}{N_D}(1 - e^{\frac{-qV}{kT}})e^{-x/L_p}$$

P-n junction at non-equilibrium



$$J_{p0} = qD_p \frac{d\Delta n_p(x=-x_{p0})e^{-x/L_p}}{dx} = q\frac{D_p}{L_p}\Delta n_p(x=-x_{p0}) = \frac{qD_p}{L_p}\frac{n_i^2}{N_D}(1-e^{-\frac{qV}{kT}})$$

$$J_{n0} = qD_n \frac{dn_n(x=x_{n0})e^{-x/L_n}}{dx} = q\frac{D_n}{L_n}\Delta p_n(x=-x_{p0}) = \frac{qD_n}{L_n}\frac{n_i^2}{N_A}(1-e^{-\frac{qV}{kT}})$$

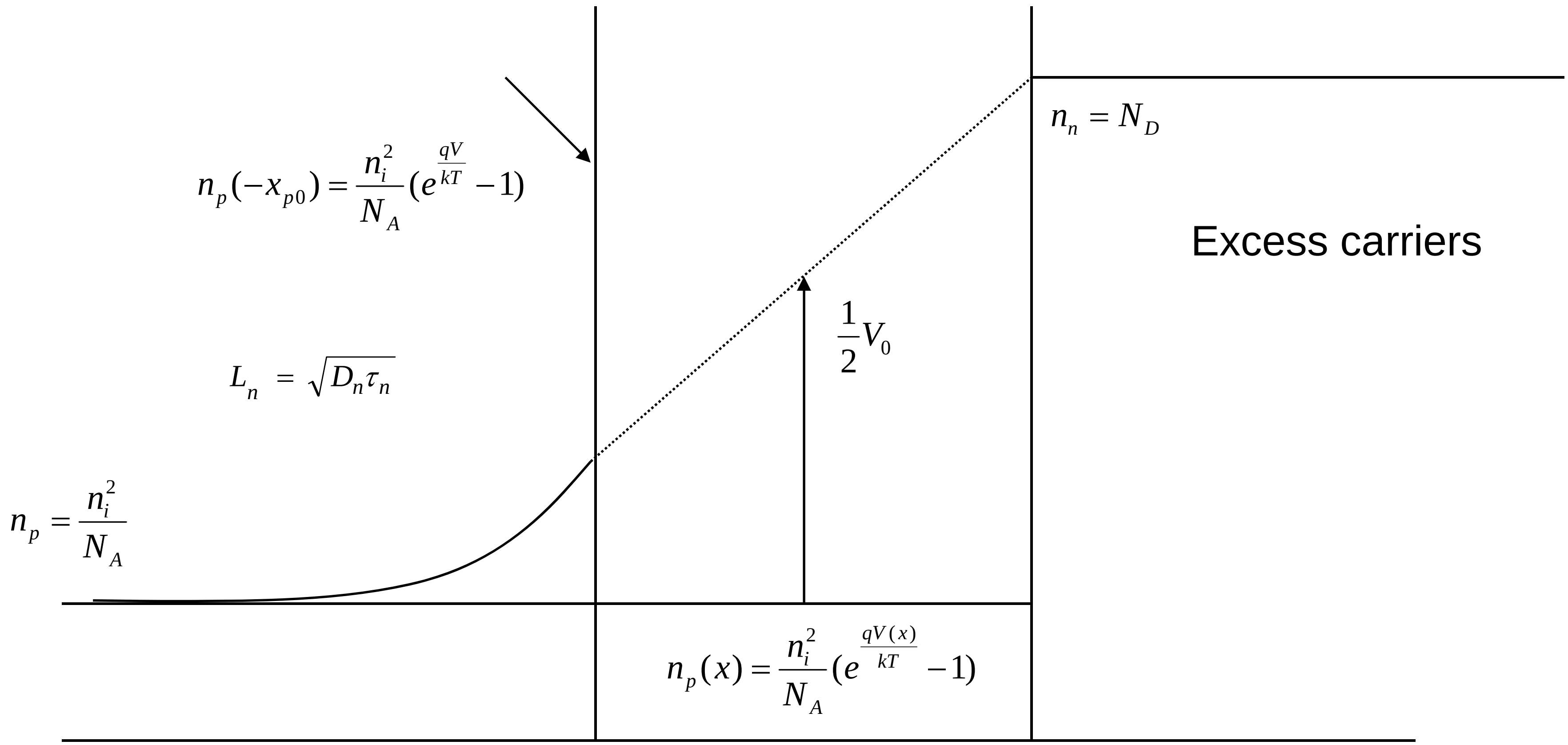


$$J_{p0} = \frac{qD_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV}{kT}} - 1) + \frac{p_{total}}{\tau_p}$$

$$J_{n0} = \frac{qD_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV}{kT}} - 1) + \frac{n_{total}}{\tau_n}$$

P-n junction at non-equilibrium

Excess carrier



$$n_{total} = \frac{1}{2} n_i W_p e^{\frac{qV}{2kT}}$$

$$J_{p0} = \frac{qD_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV}{kT}} - 1) + \frac{qn_i W_p}{2\tau_p} e^{\frac{qV}{2kT}}$$

$$J_{n0} = \frac{qD_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV}{kT}} - 1) + \frac{n_{total}}{\tau_n}$$

Hw#1, Draw the band diagram of the conduction band and the Fermi levels for GaAs with n-type doping of 10^{15} cm^{-3}, 10^{17} cm^{-3} and 10^{18} cm^{-3}. Ignore the doping dependant bandgap change.

Reading assignment: online book: Ch. 2.1-2.5

http://ecee.colorado.edu/~bart/book/book/index.html