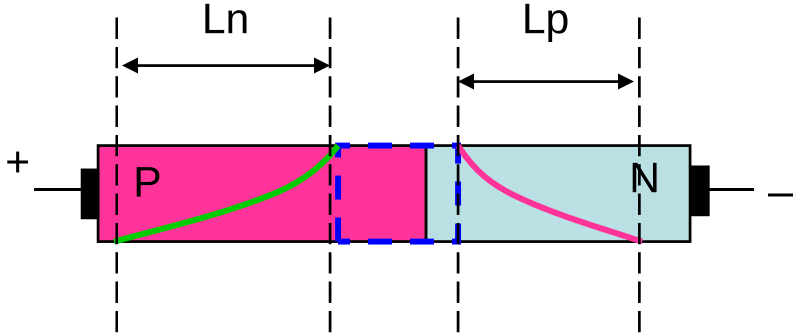
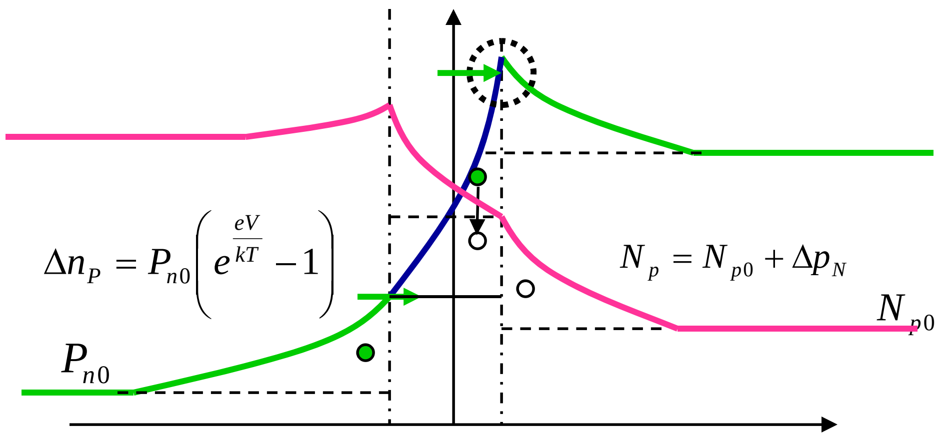


Example 3.3.1:

$$L_n = \sqrt{D_n \tau_n} \approx 13\mu m$$

$$L_p = \sqrt{D_p \tau_p} \approx 3\mu m$$

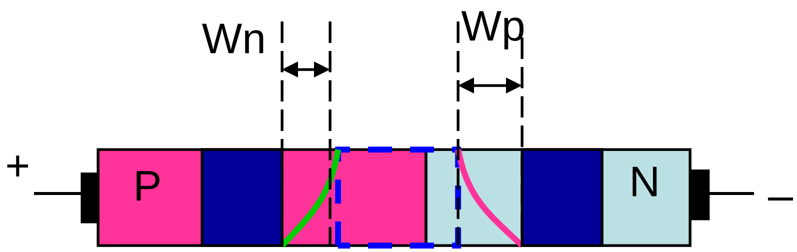
Minimum LED size? why?



$$\Delta p_N = N_{p0}\left(e^{\frac{eV}{kT}} - 1\right) \quad J_p = \frac{qD_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV}{kT}} - 1)$$

$\Delta p_p = \Delta n_p$ $\Delta p_p = \Delta n_p$

$$J_n = \frac{qD_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV}{kT}} - 1) \qquad J_r = \left(\frac{qn_iW}{2\tau_n} + \frac{qn_iW}{2\tau_p}\right)\left(e^{\frac{V}{\eta kT}} - 1\right)$$



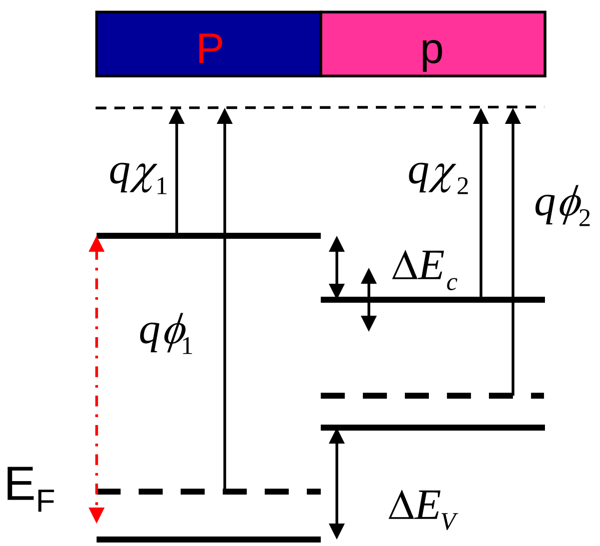
Carrier confinement

$$W_n << L_n = \sqrt{D_n \tau_n} \approx 13\mu m$$

$$W_p << L_p = \sqrt{D_p \tau_p} \approx 3\mu m$$

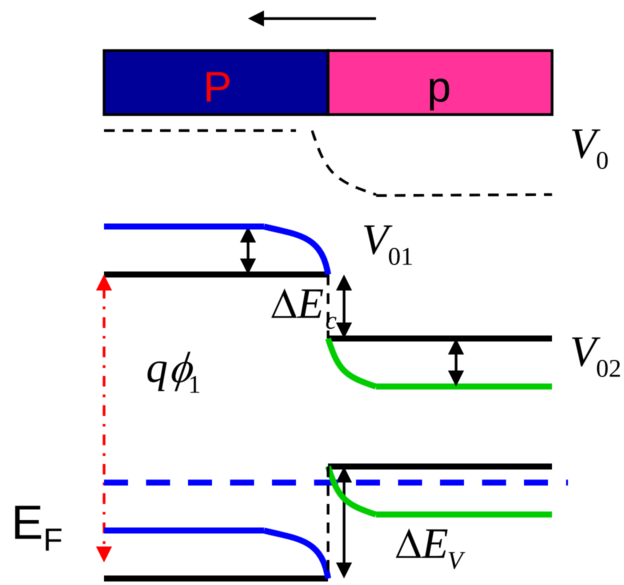
Better light emitting efficiency

Band diagram



$$\Delta E_g = \Delta E_c + \Delta E_V$$

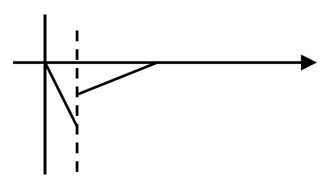
$$\Delta E_c = q\chi_2 - q\chi_1$$

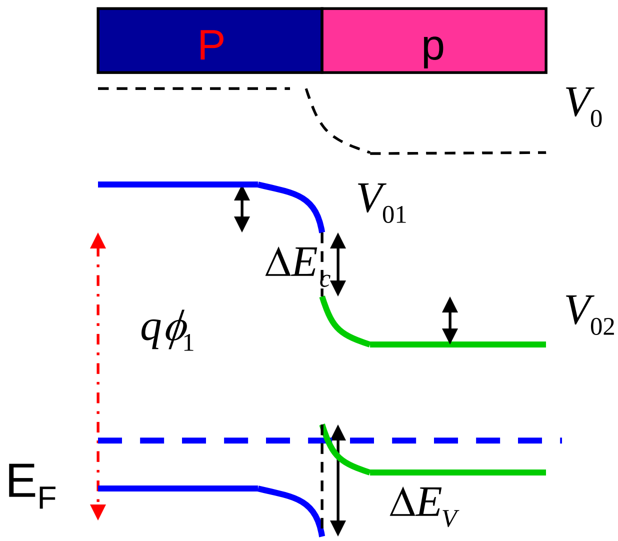


$$V_0 = V_{01} + V_{02}$$

$$\frac{V_{01}}{V_{02}} = \frac{\varepsilon_2 N_{A2}}{\varepsilon_1 N_{A1}}$$

$$\varepsilon_1 E_1 = \varepsilon_2 E_2$$

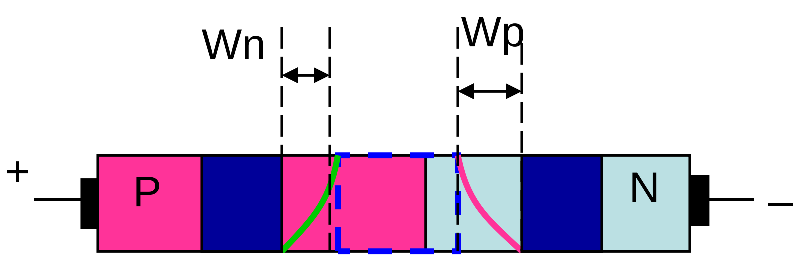


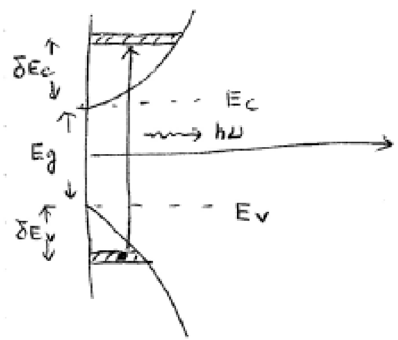


$$V_0 = V_{01} + V_{02}$$

$$n_1 = n_2 e^{-\frac{q(V_0 + \Delta E_C)}{kT}}$$

Carrier confinement:





$$E_2 - E_c = \frac{\hbar^2 k_c^2}{2m_e^*}$$

$$E_v - E_1 = \frac{\hbar^2 k_v^2}{2m_h^*}$$

$k_c = k_v = k$ no-photon transitions

$$\Rightarrow \quad E_2 - E_c = \frac{m_h^*}{m_e^*}(E_v - E_1)$$

$$h\nu = E_2 - E_1$$

$$\Rightarrow \quad E_2 - E_c = E_2 - E_1 + E_1 - E_v + E_v - E_c$$

$$= h\nu - (E_v - E_1) - E_g$$

$$= h\nu - E_g - (E_v - E_1)$$

$$\Rightarrow \quad E_2 - E_c = \frac{1}{1 + \frac{m_e^*}{m_h^*}}(h\nu - E_g) = \frac{m_h^*}{m_e^* + m_h^*}(h\nu - E_g)$$

$$E_v - E_1 = \frac{1}{1 + \frac{m_h^*}{m_e^*}}(h\nu - E_g) = \frac{m_e^*}{m_e^* + m_h^*}(h\nu - E_g)$$

$$\because \quad E_2 - E_c = \frac{m_h^*}{m_e^*}(E_v - E_1)$$

$$\Rightarrow \quad dE_2 = -\frac{m_h^*}{m_e^*} dE_1$$

<u>Reduced density of states</u>

$$\rho(h\nu)\,dE = \rho(h\nu)\, d(E_2 - E_1) = \rho(h\nu)(dE_2 - dE_1)$$

$$dE = \hbar^2\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right) k\, dk$$

$$\Rightarrow \quad \frac{dk}{dE} = \frac{1}{\hbar^2\left[\frac{1}{m_e^*} + \frac{1}{m_h^*}\right] k}$$

$$\Rightarrow \rho(h\nu) = \frac{4\pi k^2}{(2\pi)^3} \cdot \frac{dk}{dE} = \frac{1}{2\pi^2\hbar^2} \left(\frac{m_e^* m_h^*}{m_e^* + m_h^*}\right) k \quad \uparrow m_r^*$$

$$\hbar k = \left[2 m_e^* (E_c - E_c)\right]^{1/2}$$

$$\Rightarrow \rho(h\nu) = \frac{1}{4\pi^2} \left(\frac{2 m_r^*}{\hbar^2}\right)^{\frac{3}{2}} (h\nu - E_g)$$

$$m_r^* = \frac{m_e^* m_h^*}{m_e^* + m_h^*}$$

$\rho(h\nu)$ ← reduced density of states. and reflects the fact the condition are placed on the recombination.

Photon density of State (DOS)

Optical mode volume:

$$k_x = m_x \frac{\pi}{L_x}$$

$$V_{mode} = \frac{2\pi}{L_x}\frac{2\pi}{L_y}\frac{2\pi}{L_z}$$

Optical mode density from k to k+dk, # of optical modes / volume (m^3) /dk

$$\left(\frac{1}{2\pi}\right)^3 4\pi k^2 dk$$

$$E = c\hbar k \quad dk = \frac{EdE}{c\hbar}$$

Optical mode density from E to E+dE, #/ of optical modes / volume (m^3) /dE

$$\left(\frac{1}{2\pi}\right)^3 4\left(\frac{E}{\hbar c}\right)^3 dE$$

Photon density of State (DOS)

Optical mode density from E to E+dE, # of optical modes / volume (m^3) /dE

$$\left(\frac{1}{2\pi}\right)^3 4\pi\left(\frac{E}{\hbar c}\right)^3 dE = \left(\frac{1}{2\pi}\right)^3 4\pi\left(\frac{\omega}{c}\right)^3 dE$$

# of photons per optical mode

$$\frac{1}{e^{\hbar\omega/kT}-1}$$

# of photons from E to E+dE

$$n(E) = \left(\frac{1}{2\pi}\right)^3 4\pi\left(\frac{\omega}{c}\right)^3 \frac{1}{e^{\hbar\omega/kT}-1} dE$$

$$n(E) = 4\pi\left(\frac{v}{c}\right)^3 \frac{h}{e^{hv/kT}-1} dv = \frac{4\pi h v^3}{c^3}\frac{dv}{e^{hv/kT}-1}$$

$$\rho(hv) = hv\frac{4\pi h v^3}{c^3}\frac{1}{e^{hv/kT}-1} dv$$

In $J/m^3/Hz$: Photon energy density per Hz

$$\rho(hv) = hv\frac{8\pi h v^3}{c^3}\frac{dv}{e^{hv/kT}-1}$$

2 polarizations

Photon density of State (DOS)

Optical mode volume:

$$k_x = m_x \frac{\pi}{L_x}$$

$$V_{\text{mod}e} = \frac{2\pi}{L_x}\frac{2\pi}{L_y}\frac{2\pi}{L_z}$$

Optical mode density per dk, #/ of optical modes / volume (m^3) /dk

$$\left(\frac{1}{2\pi}\right)^3 4\pi k^2 dk$$

$$\rho(hv)$$

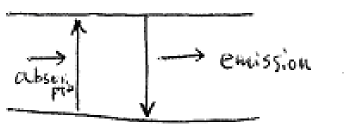
In m^3/J/s/Hz,
stimulated emission coefficient

$$E = c\hbar k \qquad dk = \frac{EdE}{c\hbar}$$

Optical mode density per dE, #/ of optical modes / volume (m^3) /dE

1. Emission, Absorption & Amplification

- two level system model



Note: (1) this is a simplified model,

(2) $E_2 - E_1 = h\nu$

a). absorption rate:

$$R_{ab} = B_{12} N_1 \rho(h\nu)$$

$\rho(h\nu)$ In J/m^3/Hz: Photon energy density per Hz

R_{ab} In #/m^3 s, absorption rate per unit volum

b). emission:

① spontaneous

$$R_{sp} = A_{21} N_2$$

① random phase
② random polarization

A_{21} In /s, spontaneous emission rate

② Stimulate emission

$$R_{st} = B_{21} N_2 \rho(h\nu).$$

① fixed phase
② fixed polarization.

B In m^3/J/s/Hz, stimulated emission coefficient

$$N_2 = N_1 \exp -(E_2 - E_1)/kT.$$

At equilibrium, absorption = emission rate

$$B_{12} N_1 \rho(h\nu) = A_{21} N_2 + B_{21} N_2 \rho(h\nu) \cdots ①$$

In thermal equilibrium, $N_2 = N_1 e^{-(E_2-E_1)/kT}$

$$\rho_{eq}(h\nu) = \frac{8\pi h\nu^3}{c^3\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]}$$

← plank's black body distribution Law

$$\left[B_{12} N_1 - B_{21} N_1 e^{-(E_2-E_1)/kT}\right] \cdot \frac{8\pi h\nu^3}{c^3\left[e^{h\nu/kT}-1\right]} = A_{21} N_2$$

$\Rightarrow$ $B_{12} = B_{21}$, $\quad A_{21} = B_{21} \cdot \dfrac{8\pi h\nu^3}{c^3}$

A21 = B21x # optical modes

Einstein coefficients ← valid in any condition.

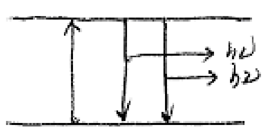
- stimulated emission rate to spontaneous emission rate

$$\frac{R_{st}}{R_{sp}} = \frac{B_{21} N_2 \rho(h\nu)}{A_{21} N_2} = \rho(h\nu) \cdot \frac{c^3}{8\pi h\nu^3} = N(hv)$$

- stimulated emission rate to absorption

$$\frac{R_{st}}{R_{ab}} = \frac{B_{21} N_2 \rho(h\nu)}{B_{12} N_1 \rho(h\nu)} = \frac{N_2}{N_1}$$

what stimulated emission rate $R_{st} > R_{ab}$.



$\Rightarrow$ $N_2 > N_1$, population inversion.

$N_2 = N_2$, optical transparency.

Example: Calculate the spontaneous to stimulate emission ratio at frequency 5×10^{14} Hz @ 1300 K.

$$\frac{R_{sp}}{R_{st}} = \frac{A_{21} N_2}{B_{21} N_2 \rho(h\nu)} = \frac{A_{21}}{B_{21}\ \rho(h\nu)}$$

$$= \frac{\frac{8\pi h\nu^3}{c^3}}{\rho(h\nu)}$$

$\rho(h\nu)$ at thermal equilibrium

$$\rho(h\nu) = \frac{8\pi h\nu^3}{c^3\left(e^{\frac{h\nu}{kT}}-1\right)}$$

$$\Rightarrow \frac{R_{sp}}{R_{st}} \simeq e^{h\nu/kT} = e^{18.4}$$

Time-independent perturbation theory

$$H_0|n^{(0)}\rangle = E_n^{(0)}|n^{(0)}\rangle \quad , \quad n = 1, 2, 3, \cdots$$

$$(H_0 + \lambda V)\,|n\rangle = E_n|n\rangle.$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \cdots$$

$$|n\rangle = |n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \lambda^2|n^{(2)}\rangle + \cdots$$

$$(H_0 + \lambda V)\left(|n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \cdots\right) = \left(E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \cdots\right)\left(|n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \cdots\right)$$

$$H_0|n^{(1)}\rangle + V|n^{(0)}\rangle = E_n^{(0)}|n^{(1)}\rangle + E_n^{(1)}|n^{(0)}\rangle$$

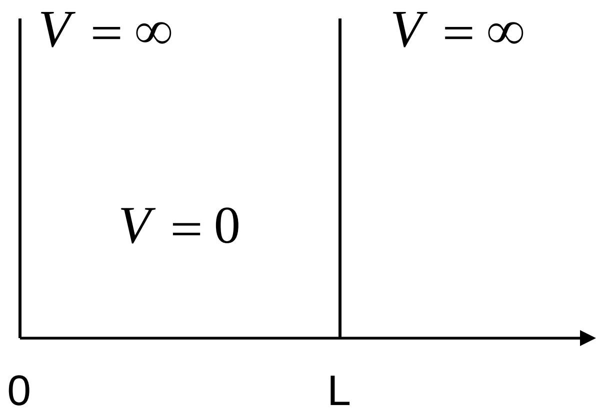
$$E_n^{(1)} = \langle n^{(0)}|V|n^{(0)}\rangle$$

$$\left(E_n^{(0)} - H_0\right)|n^{(1)}\rangle = \sum_{k\neq n}|k^{(0)}\rangle\langle k^{(0)}|V|n^{(0)}\rangle$$

$$|n^{(1)}\rangle = \sum_{k\neq n}\frac{\langle k^{(0)}|V|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}}|k^{(0)}\rangle$$

$$E_n = E_n^{(0)} + \langle n^{(0)}|V|n^{(0)}\rangle + \sum_{k\neq n}\frac{|\langle k^{(0)}|V|n^{(0)}\rangle|^2}{E_n^{(0)} - E_k^{(0)}} + \cdots$$

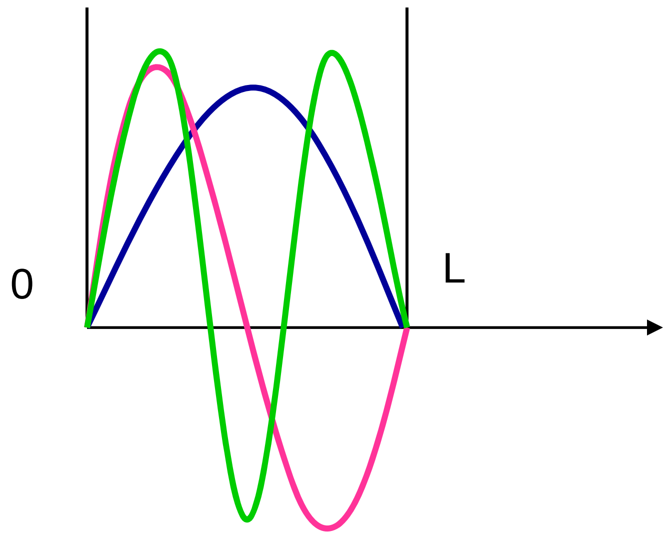
Example

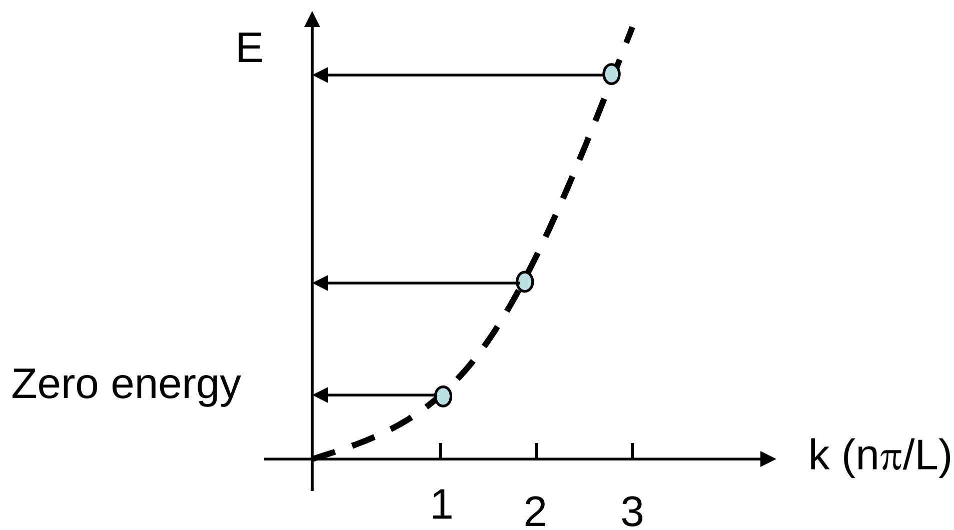


$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$$

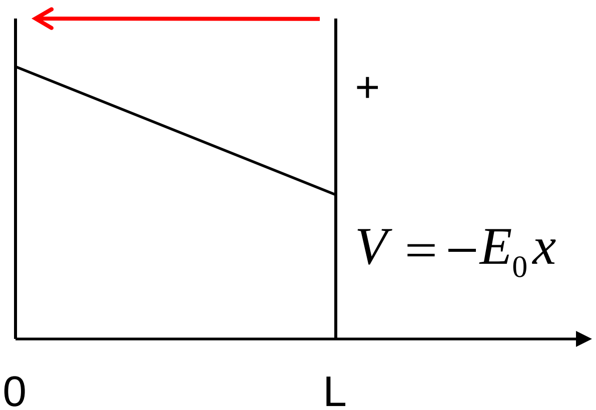
$$\psi(x) = A\sin kx, \qquad \psi(x) = A\sin\left(\frac{n\pi}{L}x\right),$$

$$kL = \frac{\sqrt{2mE}}{\hbar}L = n\pi, \quad \text{n > 0} \qquad E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$





Example



$$E_n^0 = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

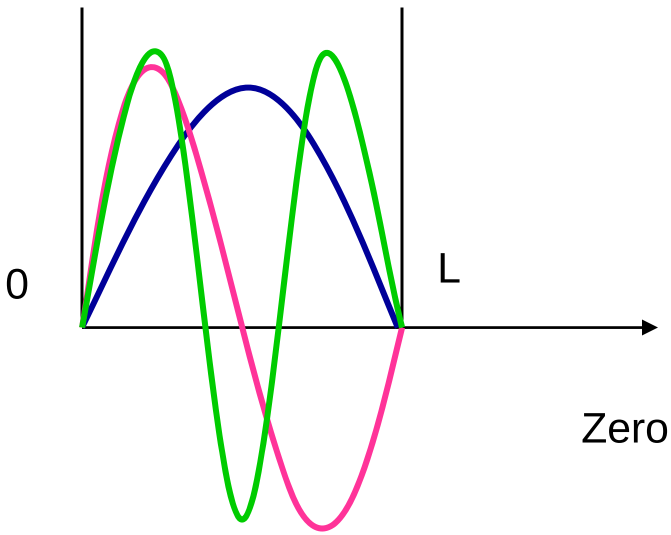
$$E_n^1 = \left\langle \psi_n^0 \middle| V_0 \middle| \psi_n^0 \right\rangle$$

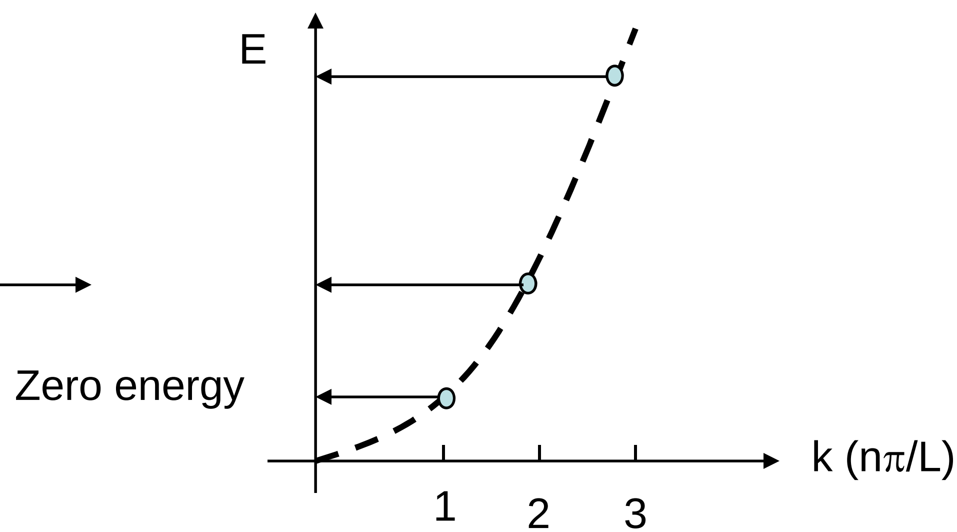
$$= \int_0^L V_0 \sin^2 \frac{n\pi x}{L} dx$$

$$= \int_0^L V_0 \frac{1 - \cos \frac{2n\pi x}{L}}{2} dx$$

$$= V_0 \frac{L}{2}$$

Wrong, why?





Time-dependent perturbation theory

$$i\hbar\frac{\partial}{\partial t}\psi = H\psi \qquad i\hbar\frac{\partial}{\partial t}\psi_n^0 = H^0\psi \qquad i\hbar\frac{\partial}{\partial t}\psi_n^0 = E_n^0\psi_n^0 \qquad \psi_n^0 = C_n^0 e^{-\frac{iEt}{\hbar}}$$

Initial state

$$\psi_n^0 = C_n^0 e^{-\frac{iEt}{\hbar}} \qquad \left|\psi_n^0\right\rangle$$

$$i\hbar\frac{\partial}{\partial t}\psi = (H^0 + H^1)\psi$$

Final state

$$\psi^1 = \sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}}$$

$$i\hbar\frac{\partial}{\partial t}\sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}}\psi_k^0 = (H^0 + H^1)\sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}}\psi_k^0$$

$$i\hbar \frac{\partial}{\partial t} \sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}} \psi_k^0 = (H^0 + H^1) \sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}} \psi_k^0$$

$$i\hbar \sum_k C_k^1 \frac{\partial}{\partial t} e^{-\frac{iE_k t}{\hbar}} \psi_k^0 + i\hbar \sum_k \dot{C}_k^1 e^{-\frac{iE_k t}{\hbar}} \psi_k^0 = H^0 \sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}} \psi_k^0 + H^1 \sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}} \psi_k^0$$

$$i\hbar \sum_k \dot{C}_k^1 e^{-\frac{iE_k t}{\hbar}} \psi_k^0 = H^1 \sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}} \psi_k^0$$

$$i\hbar \sum_k \dot{C}_k^1 e^{-\frac{iE_k t}{\hbar}} \left|\psi_k^0\right\rangle = H' \sum_k C_k^1 e^{-\frac{iE_k t}{\hbar}} \left|\psi_k^0\right\rangle$$

$$i\hbar \dot{C}_m^1 = \sum_k C_k^1 e^{-\frac{i(E_k - E_m)t}{\hbar}} \left\langle \psi_m^0 \middle| H' \middle| \psi_k^0 \right\rangle$$

$$\dot{C}_m^1 = -\frac{i}{\hbar} \sum_k C_k^1 e^{-\frac{i(E_k - E_m)t}{\hbar}} \left\langle \psi_m^0 \middle| H' \middle| \psi_k^0 \right\rangle$$

$$\dot{C}_m^1 = -\frac{i}{\hbar}\sum_k C_k^1 e^{-\frac{i(E_k - E_m)t}{\hbar}} \left\langle \psi_m^0 \middle| H' \middle| \psi_k^0 \right\rangle$$

Initial state $\left| \psi_n^0 \right\rangle$

$$\dot{C}_m^1 = \frac{1}{i\hbar} e^{-\frac{i(E_k - E_m)t}{\hbar}} \left\langle \psi_m^0 \middle| H' \middle| \psi_n^0 \right\rangle$$

Example 1: $H' = V_0$

$$C_m^1 = -\frac{i}{\hbar}\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle \int_0^t e^{-\frac{i(E_k - E_m)t}{\hbar}} dt$$

$$= \frac{i}{\hbar}\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle \frac{\hbar}{i(E_k - E_m)} e^{-\frac{i(E_k - E_m)t}{\hbar}} \Big|_0^t$$

$$= \frac{\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle}{(E_k - E_m)} \left(1 - e^{-\frac{i(E_k - E_m)t}{\hbar}} \right)$$

$$= \frac{\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle}{(E_k - E_m)} e^{-\frac{i(E_k - E_m)t}{2\hbar}} \left(2i \sin \frac{(E_k - E_m)t}{2\hbar} \right)$$

$$C_m^1 = \frac{i}{\hbar}\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle e^{-\frac{i(E_k - E_m)t}{2\hbar}} \frac{\sin\frac{(E_k - E_m)t}{2\hbar}}{(E_k - E_m)/2\hbar}$$

$$C_m^1 = \frac{i\pi}{\hbar}\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle e^{-\frac{i(E_k - E_m)t}{2\hbar}} \frac{\sin\frac{(E_k - E_m)t}{2\hbar}}{\pi(E_k - E_m)/2\hbar}$$

$$\left|C_m^1\right|^2 = \frac{\pi^2}{\hbar^2}\left|\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle\right|^2 \delta\left(E_k - E_m\right)$$

Scattering

$$\delta(x) = \lim_{L\to\infty} \frac{\sin(Lx)}{\pi x}$$

$$\dot{C}_m^1 = \frac{1}{i\hbar} e^{-\frac{i(E_k - E_m)t}{\hbar}} \left\langle \psi_m^0 \middle| H' \middle| \psi_n^0 \right\rangle$$

Example 2: $H' = V_0 \cos \omega_{op} t$

$$C_m^1 = -\frac{i}{\hbar} \left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle \int_0^t e^{-\frac{i(E_k - E_m)t}{\hbar}} \cos \omega t dt$$

$$= -\frac{i}{\hbar} \left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle \int_0^t \left[e^{-\frac{i(E_k - E_m - \hbar\omega)t}{\hbar}} + e^{-\frac{i(E_k - E_m + \hbar\omega)t}{\hbar}} \right] dt$$

$$= \frac{i}{\hbar} \left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle \frac{\hbar}{i(E_k - E_m)} e^{-\frac{i(E_k - E_m - \hbar\omega)t}{\hbar}} \Big|_0^t$$

$$= \frac{\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle}{(E_k - E_m)} \left(1 - e^{-\frac{i(E_k - E_m)t}{\hbar}} \right)$$

$$= \frac{\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle}{(E_k - E_m - \hbar\omega)} e^{-\frac{i(E_k - E_m - \hbar\omega)t}{2\hbar}} \left(2i \sin \frac{(E_k - E_m - \hbar\omega)t}{2\hbar} \right)$$

$$C_m^1 = \frac{it}{\hbar}\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle e^{-\frac{i(E_k - E_m - \hbar\omega)t}{2\hbar}} \left(\frac{\sin\frac{(E_k - E_m - \hbar\omega)t}{2\hbar}}{(E_k - E_m - \hbar\omega)t/2\hbar} \right)$$

$$\left|C_m^1\right|^2 = \frac{t^2}{\hbar^2}\left|\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle\right|^2 \left(\frac{\sin\frac{(E_k - E_m - \hbar\omega)t}{2\hbar}}{(E_k - E_m - \hbar\omega)t/2\hbar} \right)^2$$

$$\left|C_{total}^1\right|^2 = \frac{t^2}{\hbar^2}\left|\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle\right|^2 \int \left(\frac{\sin\frac{(\omega_k - \omega_m - \omega)t}{2}}{\frac{(\omega_k - \omega_m - \omega)t}{2}} \right)^2 \rho(\hbar\omega)d\hbar\omega$$

$$\left|C_{total}^1\right|^2 = \frac{t^2}{\hbar^2}\left|\left\langle \psi_m^0 \middle| V_0 \middle| \psi_n^0 \right\rangle\right|^2 \frac{2\hbar}{t} \int \left(\frac{\sin\frac{(\omega_k - \omega_m - \omega)t}{2}}{\frac{(\omega_k - \omega_m - \omega)t}{2}} \right)^2 \rho(\hbar\omega)d(\omega_k - \omega_m - \omega)t/2$$

$$\left|C_{total}^{1}\right|^{2} = \frac{t^{2}}{\hbar^{2}} \left|\left\langle \psi_{m}^{0} \middle| V_{0} \middle| \psi_{n}^{0} \right\rangle\right|^{2} \frac{2\hbar\pi}{t}$$

$$\left|C_{total}^{1}\right|^{2} = \frac{2\pi t}{\hbar} \left|\left\langle \psi_{m}^{0} \middle| V_{0} \middle| \psi_{n}^{0} \right\rangle\right|^{2}$$

$$W = \frac{d}{dt}\left|C_{total}^{1}\right|^{2} = \frac{2\pi}{\hbar} \left|\left\langle \psi_{m}^{0} \middle| V_{0} \middle| \psi_{n}^{0} \right\rangle\right|^{2} \rho(\hbar\omega)$$

$$W = \frac{2\pi}{\hbar} \left|\left\langle \psi_{m}^{0} \middle| V_{0} \middle| \psi_{n}^{0} \right\rangle\right|^{2} \rho(\hbar\omega)$$

Fermi-Golden rule

$$W_{nm} = \frac{2\pi}{\hbar} \left|\left\langle \psi_{m}^{0} \middle| V_{0} \middle| \psi_{n}^{0} \right\rangle\right|^{2} \delta(E_{m} - E_{k} - \hbar\omega)$$

Fermi-Golden rule