in-phase
phase opposition
voltage maxima and minima
current maxima and minima
voltage standing-wave ratio (VSWR or SWR) S
slotted line
input impedance Z_{in}
short-circuited line
open-circuited line
quarter-wave transformer
Smith chart
unit circle
normalized impedance
normalized load resistance r_L
normalized load reactance x_L
WTG and WTL
SWR circle
admittance Y
conductance G and susceptance B
impedance matching
matching network
single-stub matching
transient response
bounce diagram

PROBLEMS

Sections 2-1 to 2-4: Transmission-Line Model

2.1* A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming that the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz

(b) $l = 50$ km, $f = 60$ Hz

(c) $l = 20$ cm, $f = 600$ MHz

(d) $l = 1$ mm, $f = 100$ GHz

2.2 Calculate the line parameters R', L', G', and C' for a coaxial line with an inner conductor diameter of 0.5 cm and an outer conductor diameter of 1 cm, filled with an insulating material where $\mu = \mu_0$, $\varepsilon_r = 4.5$, and $\sigma = 10^{-3}$ S/m. The conductors are made of copper with $\mu_c = \mu_0$ and $\sigma_c = 5.8 \times 10^7$ S/m. The operating frequency is 1 GHz.

2.3* A 1-GHz parallel-plate transmission line consists of 1.2-cm-wide copper strips separated by a 0.15-cm-thick layer of polystyrene. Appendix B gives $\mu_c = \mu_0 = 4\pi \times 10^{-7}$ (H/m) and $\sigma_c = 5.8 \times 10^7$ (S/m) for copper, and $\varepsilon_r = 2.6$ for polystyrene. Use Table 2-1 to determine the line parameters of the transmission line. Assume that $\mu = \mu_0$ and $\sigma \simeq 0$ for polystyrene.

2.4 Show that the transmission-line model shown in Fig. 2-37 yields the same telegrapher's equations given by Eqs. (2.14) and (2.16).

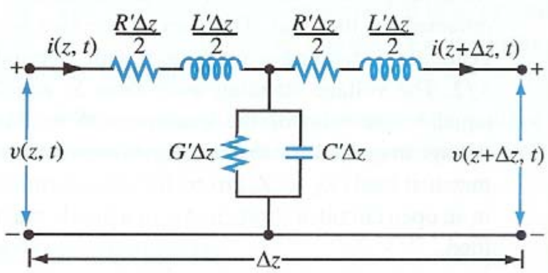

Figure 2-37: Transmission-line model for Problem 2.4.

2.5* Find α, β, u_p, and Z_0 for the coaxial line of Problem 2.2.

*Answer(s) available in Appendix D.
Solution available in CD-ROM.

Section2-5: The Lossless Line

2.6 In addition to not dissipating power, a lossless line has two important features: (1) it is dispersionless (u_p is independent of frequency); and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \qquad \text{(distortionless line)}$$

Such a line is called a ***distortionless*** line, because despite the fact that it is not lossless, it nonetheless possesses the previously mentioned features of the lossless line. Show that for a distortionless line,

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}$$

$$\beta = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

2.7* For a distortionless line [see Problem 2.6] with $Z_0 = 50\ \Omega$, $\alpha = 20$ (mNp/m), and $u_p = 2.5 \times 10^8$ (m/s), find the line parameters and λ at 100 MHz.

2.8 Find α and Z_0 of a distortionless line whose $R' = 2\ \Omega$/m and $G' = 2 \times 10^{-4}$ S/m.

2.9* A transmission line operating at 125 MHz has $Z_0 = 40\ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters R', L', G', and C'.

2.10 Using a slotted line, the voltage on a lossless [redacted]ansmission line was found to have a maximum magnitude of 1.5 V and a minimum magnitude of 0.6 V. Find the magnitude of the load's reflection coefficient.

2.11* Polyethylene with $\varepsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with a characteristic impedance of 50 Ω. The radius of the inner conductor is 1.2 mm.

(a) What is the radius of the outer conductor?

(b) What is the phase velocity of the line?

[redacted] A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)$ Ω. The wavelength is 8 cm. Find the following:

(a) The reflection coefficient at the load.

(b) The standing-wave ratio on the line.

(c) The position of the voltage maximum nearest the load.

(d) The position of the current maximum nearest the load.

[redacted]* On a 150-Ω lossless transmission line, the following observations were noted: distance of first voltage minimum from the load = 3 cm; distance of first voltage maximum from the load = 9 cm; $S = 3$. Find Z_L.

[redacted] Using a slotted line, the following results were obtained: distance of first minimum from the load = 4 cm; distance of second minimum from the load = 14 cm; voltage standing-wave ratio = 1.5. If the line is lossless and $Z_0 = 50\ \Omega$, find the load impedance.

2.15* A load with impedance $Z_L = (25 - j50)$ Ω is to be connected to a lossless transmission line with characteristic impedance Z_0, with Z_0 chosen such that the standing-wave ratio is the smallest possible. What should Z_0 be?

2.16 A 50-Ω lossless line terminated in a purely resistive load has a voltage standing-wave ratio of 3. Find all possible values of Z_L.

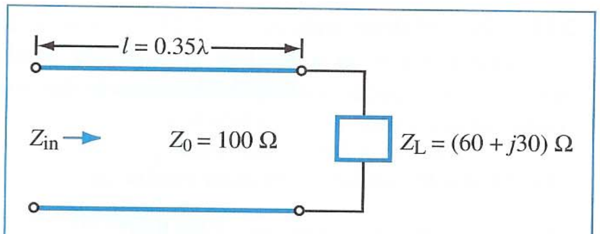

Figure 2-38: Circuit for Problem 2.18.

Section 2-6: Input Impedance

2.17* At an operating frequency of 300 MHz, a lossless 50-Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)$ Ω. Find the input impedance.

2.18 A lossless transmission line of electrical length $l = 0.35\lambda$ is terminated in a load impedance as shown in Fig. 2-38. Find Γ, S, and Z_{in}.

2.19 Show that the input impedance of a quarter-wavelength–long lossless line terminated in a short circuit appears as an open circuit.

2.20 Show that at the position where the magnitude of the voltage on the line is a maximum, the input impedance is purely real.

2.21* A voltage generator with

$$v_g(t) = 5\cos(2\pi \times 10^9 t) \text{ V}$$

and internal impedance $Z_g = 50$ Ω is connected to a 50-Ω lossless air-spaced transmission line. The line length is 5 cm and it is terminated in a load with impedance $Z_L = (100 - j100)$ Ω. Find the following:

(a) Γ at the load.

(b) Z_{in} at the input to the transmission line.

(c) The input voltage $\widetilde{V}_i$ and input current $\widetilde{I}_i$.

2.22 A 6-m section of 150-Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \quad \text{(V)}$$

and $Z_g = 150$ Ω. If the line, which has a relative permittivity $\epsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)$ Ω, find the following:

(a) λ on the line.

(b) The reflection coefficient at the load.

(c) The input impedance.

(d) The input voltage $\widetilde{V}_i$.

(e) The time-domain input voltage $v_i(t)$.

2.23* Two half-wave dipole antennas, each with an impedance of 75 Ω, are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. 2-39. All lines are 50 Ω and lossless.

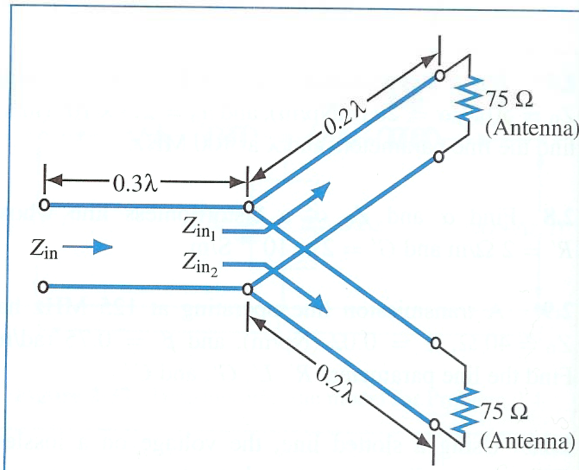

Figure 2-39: Circuit for Problem 2.23.

(a) Calculate Z_{in_1}, the input impedance of the antenna-terminated line, at the parallel juncture.

(b) Combine Z_{in_1} and Z_{in_2} in parallel to obtain Z'_{L}, the effective load impedance of the feedline.

(c) Calculate Z_{in} of the feedline.

Section2-7: Special Cases

2.24 At an operating frequency of 300 MHz, it is desired to use a section of a lossless 50-Ω transmission line terminated in a short circuit to construct an equivalent load with reactance $X = 40$ Ω. If the phase velocity of the line is $0.75c$, what is the shortest possible line length that would exhibit the desired reactance at its input?

2.25* A lossless transmission line is terminated in a short circuit. How long (in wavelengths) should the line be for it to appear as an open circuit at its input terminals?

2.26 The input impedance of a 31-cm-long lossless transmission line of unknown characteristic impedance was measured at 1 MHz. With the line terminated in a short circuit, the measurement yielded an input impedance equivalent to an inductor with inductance of 0.064 μH, and when the line was open-circuited, the measurement yielded an input impedance equivalent to a capacitor with capacitance of 40 pF. Find Z_0 of the line, the phase velocity, and the relative permittivity of the insulating material.

2.27* A 75-Ω resistive load is preceded by a $\lambda/4$ section of a 50-Ω lossless line, which itself is preceded by another $\lambda/4$ section of a 100-Ω line. What is the input impedance?

2.28 A 100-MHz FM broadcast station uses a 300-Ω transmission line between the transmitter and a tower-mounted half-wave dipole antenna. The antenna impedance is 73 Ω. You are asked to design a quarter-wave transformer to match the antenna to the line.

(a) Determine the electrical length and characteristic impedance of the quarter-wave section.

(b) If the quarter-wave section is a two-wire line with $d = 2.5$ cm, and the spacing between the wires is made of polystyrene with $\varepsilon_r = 2.6$, determine the physical length of the quarter-wave section and the radius of the two wire conductors.

2.29* A 50-MHz generator with $Z_g = 50$ Ω is connected to a load $Z_L = (50 - j25)$ Ω. The time-average power transferred from the generator into the load is maximum when $Z_g = Z_L^*$, where Z_L^* is the complex conjugate of Z_L. To achieve this condition without changing Z_g, the effective load impedance can be modified by adding an open-circuited line in series with Z_L, as shown in Fig. 2-40. If the line's $Z_0 = 100$ Ω, determine the shortest length of line (in wavelengths) necessary for satisfying the maximum-power-transfer condition.

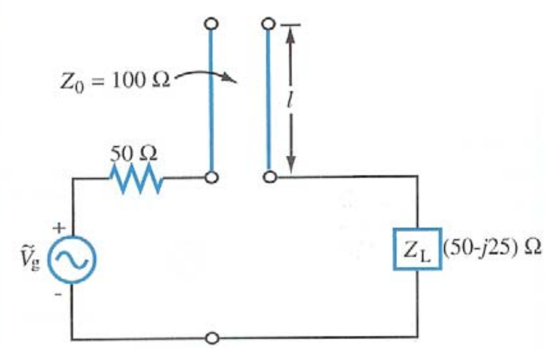

Figure 2-40: Transmission-line arrangement for Problem 2.29.

2.30 A 50-Ω lossless line of length $l = 0.375\lambda$ connects a 300-MHz generator with $\tilde{V}_g = 300$ V and $Z_g = 50$ Ω to a load Z_L. Determine the time-domain current through the load for:

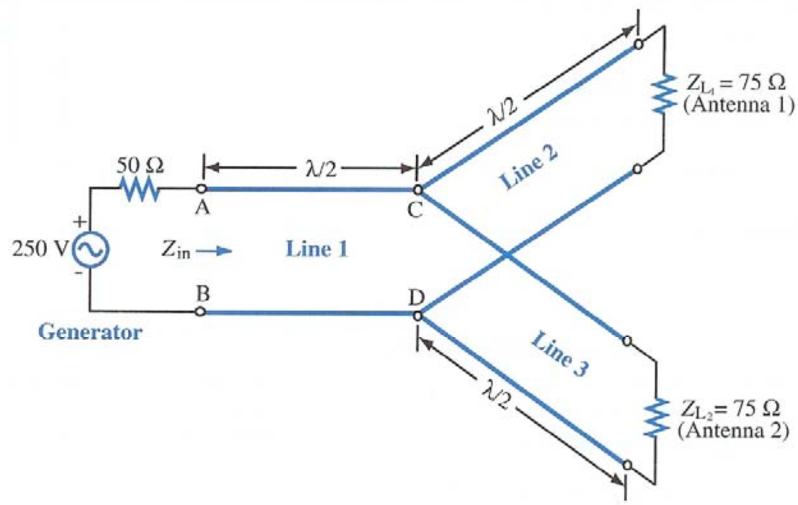

Figure 2-41: Antenna configuration for Problem 2.32.

(a) $Z_L = (50 - j50)$ Ω

(b) $Z_L = 50$ Ω

(c) $Z_L = 0$ (short circuit)

Section 2-8: Power Flow on Lossless Line

2.31* A generator with $\widetilde{V}_g = 300$ V and $Z_g = 50$ Ω is connected to a load $Z_L = 75$ Ω through a 50-Ω lossless line of length $l = 0.15\lambda$.

(a) Compute Z_{in}, the input impedance of the line at the generator end.

(b) Compute $\widetilde{I}_i$ and $\widetilde{V}_i$.

(c) Compute the time-average power delivered to the line, $P_{in} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_i\widetilde{I}_i^*]$.

(d) Compute $\widetilde{V}_L$, $\widetilde{I}_L$, and the time-average power delivered to the load, $P_L = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_L\widetilde{I}_L^*]$. How does P_{in} compare to P_L? Explain.

(e) Compute the time-average power delivered by the generator, P_g, and the time-average power dissipated in Z_g. Is conservation of power satisfied?

2.32 If the two-antenna configuration shown in Fig. 2-41 is connected to a generator with $\widetilde{V}_g = 250$ V and $Z_g = 50$ Ω, how much average power is delivered to each antenna?

2.33* For the circuit shown in Fig. 2-42, calculate the average incident power, the average reflected power, and the average power transmitted into the infinite 100-Ω line. The $\lambda/2$ line is lossless and the infinitely long line is slightly lossy. (Hint: The input impedance of an infinitely long line is equal to its characteristic impedance so long as $\alpha \neq 0$.)

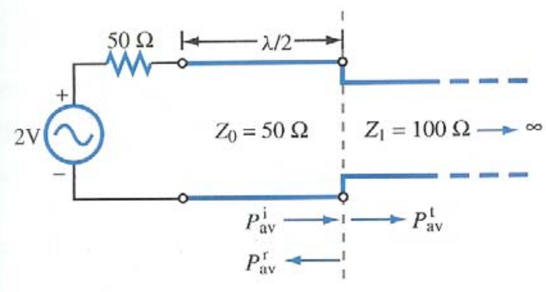

Figure 2-42: Circuit for Problem 2.33.

2.34 An antenna with a load impedance

$$Z_L = (75 + j25)\ \Omega$$

is connected to a transmitter through a 50-Ω lossless transmission line. If under matched conditions (50-Ω load) the transmitter can deliver 20 W to the load, how much power can it deliver to the antenna? Assume that $Z_g = Z_0$.

Section 2-9: Smith Chart

2.35* Use the Smith chart to find the reflection coefficient corresponding to a load impedance of

(a) $Z_L = 3Z_0$

(b) $Z_L = (2 - 2j)Z_0$

(c) $Z_L = -2jZ_0$

(d) $Z_L = 0$ (short circuit)

2.36 Use the Smith chart to find the normalized load impedance corresponding to a reflection coefficient of

(a) $\Gamma = 0.5$

(b) $\Gamma = 0.5\angle 60^\circ$

(c) $\Gamma = -1$

(d) $\Gamma = 0.3\angle -30^\circ$

(e) $\Gamma = 0$

(f) $\Gamma = j$

2.37* On a lossless transmission line terminated in a load $Z_L = 100\ \Omega$, the standing-wave ratio was measured to be 2.5. Use the Smith chart to find the two possible values of Z_0.

2.38 A lossless 50-Ω transmission line is terminated in a load with $Z_L = (50 + j25)\ \Omega$. Use the Smith chart to find the following:

(a) The reflection coefficient Γ.

(b) The standing-wave ratio.

(c) The input impedance at 0.35λ from the load.

(d) The input admittance at 0.35λ from the load.

(e) The shortest line length for which the input impedance is purely resistive.

(f) The position of the first voltage maximum from the load.

2.39* A lossless 50-Ω transmission line is terminated in a short circuit. Use the Smith chart to find the following:

(a) The input impedance at a distance 2.3λ from the load.

(b) The distance from the load at which the input admittance is $Y_{in} = -j0.04$ S.

2.40 Use the Smith chart to find y_L if $z_L = 1.5 - j0.7$.

2.41* A lossless 100-Ω transmission line 3λ/8 in length is terminated in an unknown impedance. If the input impedance is $Z_{in} = -j2.5\ \Omega$,

(a) Use the Smith chart to find Z_L.

(b) What length of open-circuit line could be used to replace Z_L?

2.42 A 75-Ω lossless line is 0.6λ long. If $S = 1.8$ and $\theta_r = -60°$, use the Smith chart to find $|\Gamma|$, Z_L, and Z_{in}.

2.43* Using a slotted line on a 50-Ω air-spaced lossless line, the following measurements were obtained: $S = 1.6$ and $|\widetilde{V}|_{max}$ occurred only at 10 cm and 24 cm from the load. Use the Smith chart to find Z_L.

2.44 At an operating frequency of 5 GHz, a 50-Ω lossless coaxial line with insulating material having a relative permittivity $\varepsilon_r = 2.25$ is terminated in an antenna with an impedance $Z_L = 150$ Ω. Use the Smith chart to find Z_{in}. The line length is 30 cm.

Section 2-10: Impedance Matching

2.45* A 50-Ω lossless line 0.6λ long is terminated in a load with $Z_L = (50 + j25)$ Ω. At 0.3λ from the load, a resistor with resistance $R = 30$ Ω is connected as shown in Fig. 2-43. Use the Smith chart to find Z_{in}.

2.46 A 50-Ω lossless line is to be matched to an antenna with $Z_L = (75 - j20)$ Ω using a shorted stub. Use the Smith chart to determine the stub length and distance between the antenna and stub.

2.47* Repeat Problem 2.46 for a load with $Z_L = (100 + j50)$ Ω.

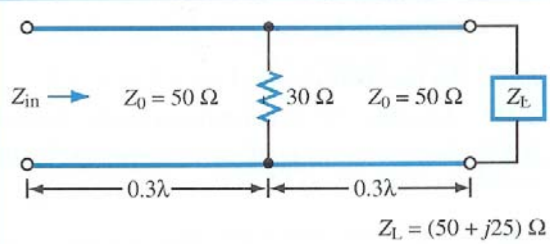

Figure 2-43: Circuit for Problem 2.45.

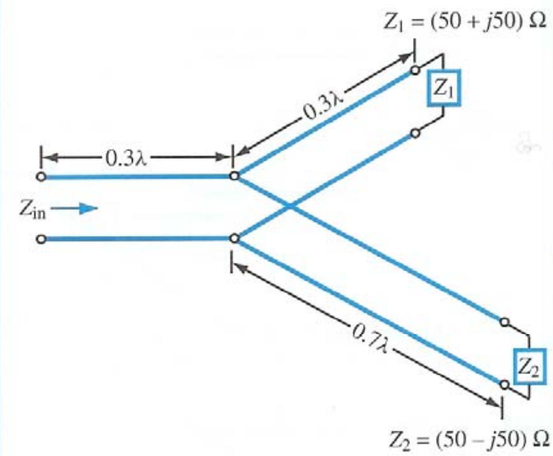

Figure 2-44: Network for Problem 2.48.

2.48 Use the Smith chart to find Z_{in} of the feed line shown in Fig. 2-44. All lines are lossless with $Z_0 = 50$ Ω.

2.49* Repeat Problem 2.48 for the case where all three transmission lines are $\lambda/4$ in length.

Section 2-11: Transients on Transmission Lines

2.50 Generate a bounce diagram for the voltage $V(z, t)$ for a 1-m-long lossless line characterized by $Z_0 = 50$ Ω and $u_p = 2c/3$ (where c is the velocity of light) if the line is fed by a step voltage applied at $t = 0$ by a generator circuit with $V_g = 60$ V and $R_g = 100$ Ω. The line is terminated in a load $Z_L = 25$ Ω. Use the bounce diagram to plot $V(t)$ at a point midway along the length of the line from $t = 0$ to $t = 25$ ns.

2.51 Repeat Problem 2.50 for the current I on the line.

2.52 In response to a step voltage, the voltage waveform shown in Fig. 2-45 was observed at the sending end of a lossless transmission line with $R_g = 50\ \Omega$, $Z_0 = 50\ \Omega$, and $\varepsilon_r = 2.25$. Determine the following:

(a) The generator voltage.

(b) The length of the line.

(c) The load impedance.

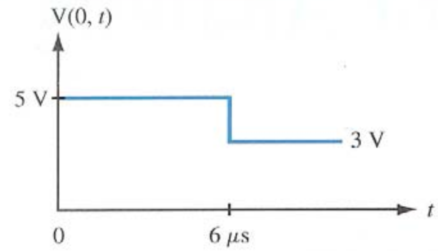

Figure 2-45: Voltage waveform for Problems 2.52 and 2.54.

2.53* In response to a step voltage, the voltage waveform shown in Fig. 2-46 was observed at the sending end of a shorted line with $Z_0 = 50\ \Omega$ and $\varepsilon_r = 4$. Determine V_g, R_g, and the line length.

2.54 Suppose the voltage waveform shown in Fig. 2-45 was observed at the sending end of a 50-Ω transmission line in response to a step voltage introduced by a generator with $V_g = 15$ V and an unknown series resistance R_g. The line is 1 km in length, its velocity of propagation is 1×10^8 m/s, and it is terminated in a load $Z_L = 100\ \Omega$.

(a) Determine R_g.

(b) Explain why the drop in level of $V(0, t)$ at $t = 6\ \mu$s cannot be due to reflection from the load.

(c) Determine the shunt resistance R_f and location of the fault responsible for the observed waveform.

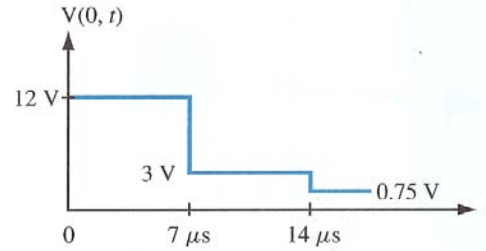

Figure 2-46: Voltage waveform of Problem 2.53.

2.55 A generator circuit with $V_g = 200$ V and $R_g = 25\ \Omega$ was used to excite a 75-Ω lossless line with a rectangular pulse of duration $\tau = 0.4\ \mu$s. The line is 200 m long, its $u_p = 2 \times 10^8$ m/s, and it is terminated in a load $Z_L = 125\ \Omega$.

(a) Synthesize the voltage pulse exciting the line as the sum of two step functions, $V_{g_1}(t)$ and $V_{g_2}(t)$.

(b) For each voltage step function, generate a bounce diagram for the voltage on the line.

(c) Use the bounce diagrams to plot the total voltage at the sending end of the line.

2.56 For the circuit of Problem 2.55, generate a bounce diagram for the current and plot its time history at the middle of the line.

2.57–2.65 Additional Solved Problems — complete solutions on .