Problem 6.12 The electromagnetic generator shown in Fig. 6-12 is connected to an electric bulb with a resistance of 150 Ω. If the loop area is 0.1 m^2 and it rotates at 3,600 revolutions per minute in a uniform magnetic flux density $B_0 = 0.4$ T, determine the amplitude of the current generated in the light bulb.

Solution: From Eq. (6.38), the sinusoidal voltage generated by the a-c generator is $V_{emf} = A\omega B_0 \sin(\omega t + C_0) = V_0 \sin(\omega t + C_0)$. Hence,

$$V_0 = A\omega B_0 = 0.1 \times \frac{2\pi \times 3,600}{60} \times 0.4 = 15.08 \quad \text{(V)},$$

$$I = \frac{V_0}{R} = \frac{15.08}{150} = 0.1 \quad \text{(A)}.$$

Problem 6.18 An electromagnetic wave propagating in seawater has an electric field with a time variation given by $\mathbf{E} = \hat{\mathbf{z}}E_0 \cos \omega t$. If the permittivity of water is $81\varepsilon_0$ and its conductivity is 4 (S/m), find the ratio of the magnitudes of the conduction current density to displacement current density at each of the following frequencies:

(a) 1 kHz

(b) 1 MHz

(c) 1 GHz

(d) 100 GHz

Solution: From Eq. (6.44), the displacement current density is given by

$$\mathbf{J}_{\rm d} = \frac{\partial}{\partial t}\mathbf{D} = \varepsilon \frac{\partial}{\partial t}\mathbf{E}$$

and, from Eq. (4.67), the conduction current is $\mathbf{J} = \sigma \mathbf{E}$. Converting to phasors and taking the ratio of the magnitudes,

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_{\rm d}}\right| = \left|\frac{\sigma \widetilde{\mathbf{E}}}{j\omega\varepsilon_r\varepsilon_0\widetilde{\mathbf{E}}}\right| = \frac{\sigma}{\omega\varepsilon_r\varepsilon_0} .$$

(a) At $f = 1$ kHz, $\omega = 2\pi \times 10^3$ rad/s, and

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_{\rm d}}\right| = \frac{4}{2\pi \times 10^3 \times 81 \times 8.854 \times 10^{-12}} = 888 \times 10^3.$$

The displacement current is negligible.

(b) At $f = 1$ MHz, $\omega = 2\pi \times 10^6$ rad/s, and

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_{\rm d}}\right| = \frac{4}{2\pi \times 10^6 \times 81 \times 8.854 \times 10^{-12}} = 888.$$

The displacement current is practically negligible.

(c) At $f = 1$ GHz, $\omega = 2\pi \times 10^9$ rad/s, and

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_{\rm d}}\right| = \frac{4}{2\pi \times 10^9 \times 81 \times 8.854 \times 10^{-12}} = 0.888.$$

Neither the displacement current nor the conduction current are negligible.

(d) At $f = 100$ GHz, $\omega = 2\pi \times 10^{11}$ rad/s, and

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_{\rm d}}\right| = \frac{4}{2\pi \times 10^{11} \times 81 \times 8.854 \times 10^{-12}} = 8.88 \times 10^{-3}.$$

The conduction current is practically negligible.

Problem 6.25 Given an electric field

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \sin ay \cos(\omega t - kz),$$

where E_0, a, ω, and k are constants, find $\mathbf{H}$.

Solution:

$$\begin{aligned}
\mathbf{E} &= \hat{\mathbf{x}} E_0 \sin ay \cos(\omega t - kz), \\
\widetilde{\mathbf{E}} &= \hat{\mathbf{x}} E_0 \sin ay \, e^{-jkz}, \\
\widetilde{\mathbf{H}} &= -\frac{1}{j\omega\mu} \nabla \times \widetilde{\mathbf{E}} \\
&= -\frac{1}{j\omega\mu} \left[\hat{\mathbf{y}} \frac{\partial}{\partial z} (E_0 \sin ay \, e^{-jkz}) - \hat{\mathbf{z}} \frac{\partial}{\partial y} (E_0 \sin ay \, e^{-jkz}) \right] \\
&= \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay - \hat{\mathbf{z}} \, ja \cos ay] e^{-jkz}, \\
\mathbf{H} &= \mathfrak{Re}[\widetilde{\mathbf{H}} e^{j\omega t}] \\
&= \mathfrak{Re} \left\{ \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay + \hat{\mathbf{z}} a \cos ay \, e^{-j\pi/2}] e^{-jkz} e^{j\omega t} \right\} \\
&= \frac{E_0}{\omega\mu} \left[\hat{\mathbf{y}} k \sin ay \cos(\omega t - kz) + \hat{\mathbf{z}} a \cos ay \cos\left(\omega t - kz - \frac{\pi}{2} \right) \right] \\
&= \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay \cos(\omega t - kz) + \hat{\mathbf{z}} a \cos ay \sin(\omega t - kz)] \, .
\end{aligned}$$

Problem 6.29 The magnetic field in a given dielectric medium is given by

$$\mathbf{H} = \hat{\mathbf{y}}\, 6\cos 2z \sin(2\times 10^7 t - 0.1x) \quad \text{(A/m)},$$

where x and z are in meters. Determine:

(a) $\mathbf{E}$,

(b) the displacement current density $\mathbf{J}_\text{d}$, and

(c) the charge density ρ_v.

Solution:

(a)

$$\begin{aligned}
\mathbf{H} &= \hat{\mathbf{y}}\, 6\cos 2z \sin(2\times 10^7 t - 0.1x) = \hat{\mathbf{y}}\, 6\cos 2z \cos(2\times 10^7 t - 0.1x - \pi/2),\\
\widetilde{\mathbf{H}} &= \hat{\mathbf{y}}\, 6\cos 2z\, e^{-j0.1x} e^{-j\pi/2} = -\hat{\mathbf{y}}\, j6\cos 2z\, e^{-j0.1x},\\
\widetilde{\mathbf{E}} &= \frac{1}{j\omega\varepsilon} \nabla \times \widetilde{\mathbf{H}}\\
&= \frac{1}{j\omega\varepsilon} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & -j6\cos 2z\, e^{-j0.1x} & 0 \end{vmatrix}\\
&= \frac{1}{j\omega\varepsilon} \left\{ \hat{\mathbf{x}} \left[-\frac{\partial}{\partial z} (-j6\cos 2z\, e^{-j0.1x}) \right] + \hat{\mathbf{z}} \left[\frac{\partial}{\partial x} (-j6\cos 2z\, e^{-j0.1x}) \right] \right\}\\
&= \hat{\mathbf{x}} \left(-\frac{12}{\omega\varepsilon} \sin 2z\, e^{-j0.1x} \right) + \hat{\mathbf{z}} \left(\frac{j0.6}{\omega\varepsilon} \cos 2z\, e^{-j0.1x} \right).
\end{aligned}$$

From the given expression for $\mathbf{H}$,

$$\begin{aligned}
\omega &= 2\times 10^7 \quad \text{(rad/s)},\\
\beta &= 0.1 \quad \text{(rad/m)}.
\end{aligned}$$

Hence,

$$u_\text{p} = \frac{\omega}{\beta} = 2\times 10^8 \quad \text{(m/s)},$$

and

$$\varepsilon_\text{r} = \left(\frac{c}{u_\text{p}}\right)^2 = \left(\frac{3\times 10^8}{2\times 10^8}\right)^2 = 2.25.$$

Using the values for ω and ε, we have

$$\begin{aligned}
\widetilde{\mathbf{E}} &= (-\hat{\mathbf{x}}\, 30\sin 2z + \hat{\mathbf{z}}\, j1.5\cos 2z) \times 10^3 e^{-j0.1x} \quad \text{(V/m)},\\
\mathbf{E} &= \left[-\hat{\mathbf{x}}\, 30\sin 2z \cos(2\times 10^7 t - 0.1x) - \hat{\mathbf{z}}\, 1.5\cos 2z \sin(2\times 10^7 t - 0.1x)\right] \quad \text{(kV/m)}.
\end{aligned}$$

(b)

$$\widetilde{\mathbf{D}} = \varepsilon\widetilde{\mathbf{E}} = \varepsilon_r\varepsilon_0\widetilde{\mathbf{E}} = (-\hat{\mathbf{x}}\,0.6\sin 2z + \hat{\mathbf{z}}\,j0.03\cos 2z) \times 10^{-6} e^{-j0.1x} \quad (\text{C/m}^2),$$

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t},$$

or

$$\widetilde{\mathbf{J}}_d = j\omega\widetilde{\mathbf{D}} = (-\hat{\mathbf{x}}\,j12\sin 2z - \hat{\mathbf{z}}\,0.6\cos 2z)e^{-j0.1x},$$

$$\mathbf{J}_d = \mathfrak{Re}[\widetilde{\mathbf{J}}_d e^{j\omega t}]$$
$$= \left[\hat{\mathbf{x}}\,12\sin 2z\sin(2\times 10^7 t - 0.1x) - \hat{\mathbf{z}}\,0.6\cos 2z\cos(2\times 10^7 t - 0.1x)\right] \quad (\text{A/m}^2).$$

(c) We can find ρ_v from

$$\nabla \cdot \mathbf{D} = \rho_v$$

or from

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}.$$

Applying Maxwell's equation,

$$\rho_v = \nabla \cdot \mathbf{D} = \varepsilon\nabla \cdot \mathbf{E} = \varepsilon_r\varepsilon_0\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z}\right)$$

yields

$$\rho_v = \varepsilon_r\varepsilon_0\left\{\frac{\partial}{\partial x}\left[-30\sin 2z\cos(2\times 10^7 t - 0.1x)\right]\right.$$
$$\left. + \frac{\partial}{\partial z}\left[-1.5\cos 2z\sin(2\times 10^7 t - 0.1x)\right]\right\}$$
$$= \varepsilon_r\varepsilon_0\left[-3\sin 2z\sin(2\times 10^7 t - 0.1x) + 3\sin 2z\sin(2\times 10^7 t - 0.1x)\right] = 0.$$