EMag I. Prof. Xingwei Wang

Homework #1

Due day: Sept. 17(Monday) before class.

Problem 1.1 A 2-kHz sound wave traveling in the x-direction in air was observed to have a differential pressure $p(x,t) = 10$ N/m^2 at $x = 0$ and $t = 50\ \mu$s. If the reference phase of $p(x,t)$ is 36°, find a complete expression for $p(x,t)$. The velocity of sound in air is 330 m/s.

Solution: The general form is given by Eq. (1.17),

$$p(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right),$$

where it is given that $\phi_0 = 36°$. From Eq. (1.26), $T = 1/f = 1/(2\times 10^3) = 0.5$ ms. From Eq. (1.27),

$$\lambda = \frac{u_p}{f} = \frac{330}{2\times 10^3} = 0.165 \text{ m}.$$

Also, since

$$\begin{aligned} p(x=0,\ t=50\ \mu\text{s}) = 10\ (\text{N/m}^2) &= A\cos\left(\frac{2\pi\times 50\times 10^{-6}}{5\times 10^{-4}} + 36°\frac{\pi \text{ rad}}{180°}\right) \\ &= A\cos(1.26 \text{ rad}) = 0.31A, \end{aligned}$$

it follows that $A = 10/0.31 = 32.36$ N/m^2. So, with t in (s) and x in (m),

$$\begin{aligned} p(x,t) &= 32.36\cos\left(2\pi\times 10^6\frac{t}{500} - 2\pi\times 10^3\frac{x}{165} + 36°\right) \quad (\text{N/m}^2) \\ &= 32.36\cos(4\pi\times 10^3 t - 12.12\pi x + 36°) \quad (\text{N/m}^2). \end{aligned}$$

Problem 1.3 A harmonic wave traveling along a string is generated by an oscillator that completes 180 vibrations per minute. If it is observed that a given crest, or maximum, travels 300 cm in 10 s, what is the wavelength?

Solution:

$$f = \frac{180}{60} = 3 \text{ Hz.}$$

$$u_\text{p} = \frac{300 \text{ cm}}{10 \text{ s}} = 0.3 \text{ m/s.}$$

$$\lambda = \frac{u_\text{p}}{f} = \frac{0.3}{3} = 0.1 \text{ m} = 10 \text{ cm.}$$

Problem 1.4 A wave traveling along a string is given by

$$y(x,t) = 2\sin(4\pi t + 10\pi x) \quad \text{(cm)},$$

where x is the distance along the string in meters and y is the vertical displacement. Determine: (a) the direction of wave travel, (b) the reference phase ϕ_0, (c) the frequency, (d) the wavelength, and (e) the phase velocity.

Solution:

(a) We start by converting the given expression into a cosine function of the form given by (1.17):

$$y(x,t) = 2\cos\left(4\pi t + 10\pi x - \frac{\pi}{2}\right) \quad \text{(cm)}.$$

Since the coefficients of t and x both have the same sign, the wave is traveling in the negative x-direction.

(b) From the cosine expression, $\phi_0 = -\pi/2$.

(c) $\omega = 2\pi f = 4\pi$,

$$f = 4\pi/2\pi = 2 \text{ Hz.}$$

(d) $2\pi/\lambda = 10\pi$,

$$\lambda = 2\pi/10\pi = 0.2 \text{ m.}$$

(e) $u_\text{p} = f\lambda = 2 \times 0.2 = 0.4$ (m/s).