EMag I. Prof. Xingwei Wang

Homework #2_solution

Due day: Sept. 24(Monday) before class.

Problem 2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(20 \times 10^{-2} \text{ m}) \times (20 \times 10^{3} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 1.33 \times 10^{-5} \text{ (negligible)}.$$

(b) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(50 \times 10^{3} \text{ m}) \times (60 \times 10^{0} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.01 \text{ (borderline)}.$$

(c) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(20 \times 10^{-2} \text{ m}) \times (600 \times 10^{6} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.40 \text{ (nonnegligible)}.$$

(d) $$\frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(1 \times 10^{-3} \text{ m}) \times (100 \times 10^{9} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.33 \text{ (nonnegligible)}.$$

Problem 2.2 A two-wire copper transmission line is embedded in a dielectric material with $\varepsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}$ S/m. Its wires are separated by 3 cm and their radii are 1 mm each.

(a) Calculate the line parameters R', L', G', and C' at 2 GHz.

Solution:

(a) Given:

$$
\begin{aligned}
f &= 2 \times 10^{9} \text{ Hz},\\
d &= 2 \times 10^{-3} \text{ m},\\
D &= 3 \times 10^{-2} \text{ m},\\
\sigma_c &= 5.8 \times 10^{7} \text{ S/m (copper)},\\
\varepsilon_r &= 2.6,\\
\sigma &= 2 \times 10^{-6} \text{ S/m},\\
\mu &= \mu_c = \mu_0.
\end{aligned}
$$

Problem 2.4 A 1-GHz parallel-plate transmission line consists of 1.2-cm-wide copper strips separated by a 0.15-cm-thick layer of polystyrene. Appendix B gives $\mu_c = \mu_0 = 4\pi \times 10^{-7}$ (H/m) and $\sigma_c = 5.8 \times 10^{7}$ (S/m) for copper, and $\varepsilon_r = 2.6$ for polystyrene. Use Table 2-1 to determine the line parameters of the transmission line. Assume $\mu = \mu_0$ and $\sigma \simeq 0$ for polystyrene.

Solution:

$$
\begin{aligned}
R' &= \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{1.2 \times 10^{-2}}\left(\frac{\pi \times 10^{9} \times 4\pi \times 10^{-7}}{5.8 \times 10^{7}}\right)^{1/2} = 1.38 \quad (\Omega/\text{m}),\\
L' &= \frac{\mu d}{w} = \frac{4\pi \times 10^{-7} \times 1.5 \times 10^{-3}}{1.2 \times 10^{-2}} = 1.57 \times 10^{-7} \quad (\text{H/m}),\\
G' &= 0 \qquad \text{because } \sigma = 0,\\
C' &= \frac{\varepsilon w}{d} = \varepsilon_0 \varepsilon_r \frac{w}{d} = \frac{10^{-9}}{36\pi} \times 2.6 \times \frac{1.2 \times 10^{-2}}{1.5 \times 10^{-3}} = 1.84 \times 10^{-10} \quad (\text{F/m}).
\end{aligned}
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