EMag I. Prof. Xingwei Wang

Homework #3 solution

Due day: Oct. 01(Monday) before class.

Problem 2.6 A coaxial line with inner and outer conductor diameters of 0.5 cm and 1 cm, respectively, is filled with an insulating material with $\varepsilon_r = 4.5$ and $\sigma = 10^{-3}$ S/m. The conductors are made of copper.

(a) Calculate the line parameters at 1 GHz.

Solution: **(a)** Given

$$a = (0.5/2) \text{ cm} = 0.25 \times 10^{-2} \text{ m},$$
$$b = (1.0/2) \text{ cm} = 0.50 \times 10^{-2} \text{ m},$$

combining Eqs. (2.5) and (2.6) gives

$$\begin{aligned} R' &= \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right) \\ &= \frac{1}{2\pi}\sqrt{\frac{\pi(10^9 \text{ Hz})(4\pi\times10^{-7} \text{ H/m})}{5.8\times10^7 \text{ S/m}}}\left(\frac{1}{0.25\times10^{-2}\text{ m}}+\frac{1}{0.50\times10^{-2}\text{ m}}\right) \\ &= 0.788\ \Omega\text{/m.} \end{aligned}$$

From Eq. (2.7),

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times10^{-7}\text{ H/m}}{2\pi}\ln 2 = 139 \text{ nH/m.}$$

From Eq. (2.8),

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi\times10^{-3}\text{ S/m}}{\ln 2} = 9.1 \text{ mS/m.}$$

From Eq. (2.9),

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\varepsilon_r\varepsilon_0}{\ln(b/a)} = \frac{2\pi\times4.5\times(8.854\times10^{-12}\text{ F/m})}{\ln 2} = 362 \text{ pF/m.}$$

Problem 2.8 Find α, β, u_p, and Z_0 for the coaxial line of Problem 2.6.

Solution: From Eq. (2.22),

$$\begin{aligned}
\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{(0.788\ \Omega/\mathrm{m}) + j(2\pi \times 10^9\ \mathrm{s}^{-1})(139 \times 10^{-9}\ \mathrm{H/m})} \\
&\quad \times \sqrt{(9.1 \times 10^{-3}\ \mathrm{S/m}) + j(2\pi \times 10^9\ \mathrm{s}^{-1})(362 \times 10^{-12}\ \mathrm{F/m})} \\
&= (109 \times 10^{-3} + j44.5)\ \mathrm{m}^{-1}.
\end{aligned}$$

Thus, from Eqs. (2.25a) and (2.25b), $\alpha = 0.109$ Np/m and $\beta = 44.5$ rad/m.

From Eq. (2.29),

$$\begin{aligned}
Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} &= \sqrt{\frac{(0.788\ \Omega/\mathrm{m}) + j(2\pi \times 10^9\ \mathrm{s}^{-1})(139 \times 10^{-9}\ \mathrm{H/m})}{(9.1 \times 10^{-3}\ \mathrm{S/m}) + j(2\pi \times 10^9\ \mathrm{s}^{-1})(362 \times 10^{-12}\ \mathrm{F/m})}} \\
&= (19.6 + j0.030)\ \Omega.
\end{aligned}$$

From Eq. (2.33),

$$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^9}{44.5} = 1.41 \times 10^8\ \mathrm{m/s}.$$

Problem 2.19 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)$ Ω. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,

(b) the standing-wave ratio on the line,

(c) the position of the voltage maximum nearest the load,

(d) the position of the current maximum nearest the load.

Solution:

(a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57e^{-j79.8^\circ}.$$

(b) From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

(c) From Eq. (2.70)

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8 \text{ cm}}{4\pi} \frac{\pi \text{ rad}}{180^\circ} + \frac{n \times 8 \text{ cm}}{2}$$
$$= -0.89 \text{ cm} + 4.0 \text{ cm} = 3.11 \text{ cm}.$$

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.72),

$$d_{min} = d_{max} - \lambda/4 = 3.11 \text{ cm} - 8 \text{ cm}/4 = 1.11 \text{ cm}.$$