2.47 Use the Smith chart to find the reflection coefficient corresponding to a load impedance of

(a) \( Z_L = 3Z_0 \)
(b) \( Z_L = (2 - j2)Z_0 \)
(c) \( Z_L = -j2Z_0 \)
(d) \( Z_L = 0 \) (short circuit)

Solution: Refer to Fig. P2.47.
(a) Point A is \( z_L = 3 + j0 \). \( \Gamma = 0.5 \exp 0^\circ \)
(b) Point B is \( z_L = 2 - j2 \). \( \Gamma = 0.62 \exp -29.7^\circ \)
(c) Point C is \( z_L = 0 - j2 \). \( \Gamma = 1.0 \exp -53.1^\circ \)
(d) Point D is \( z_L = 0 + j0 \). \( \Gamma = 1.0 \exp 180.0^\circ \)
Figure P2.47: Solution of Problem 2.47.
Use the Smith chart to find the normalized load impedance corresponding to a reflection coefficient of

(a) \( \Gamma = 0.5 \)
(b) \( \Gamma = 0.5 \angle 60^\circ \)
(c) \( \Gamma = -1 \)
(d) \( \Gamma = 0.3 \angle -30^\circ \)
(e) \( \Gamma = 0 \)
(f) \( \Gamma = j \)

\[ Z_L = \frac{Z_0}{1 + \Gamma} \]

Figure P2.49: Solution of Problem 2.49.

Solution: Refer to Fig. P2.49.
(a) Point $A'$ is $\Gamma = 0.5$ at $z_L = 3 + j0$.
(b) Point $B'$ is $\Gamma = 0.5 \exp j60^\circ$ at $z_L = 1 + j1.15$.
(c) Point $C'$ is $\Gamma = -1$ at $z_L = 0 + j0$.
(d) Point $D'$ is $\Gamma = 0.3 \exp -j30^\circ$ at $z_L = 1.60 - j0.53$.
(e) Point $E'$ is $\Gamma = 0$ at $z_L = 1 + j0$.
(f) Point $F'$ is $\Gamma = j$ at $z_L = 0 + j1$. 
2.50 Use the Smith chart to determine the input impedance $Z_{in}$ of the two-line configuration shown in Fig. P2.50.

$Z_{01} = 100 \Omega \quad Z_{02} = 50 \Omega \quad Z_L = (75 - j50) \Omega$

Solution:
Starting at point $A$, namely at the load, we normalize $Z_L$ with respect to $Z_{02}$:

$$z_L = \frac{Z_L}{Z_{02}} = \frac{75 - j50}{50} = 1.5 - j1.$$  \hspace{1cm} \text{(point $A$ on Smith chart 1)}

From point $A$ on the Smith chart, we move on the SWR circle a distance of $5\lambda/8$ to point $B_r$, which is just to the right of point $B$ (see figure). At $B_r$, the normalized input impedance of line 2 is:

$$z_{in2} = 0.48 - j0.36 \quad \text{(point $B_r$ on Smith chart)}$$

Next, we unnormalize $z_{in2}$:

$$Z_{in2} = Z_{02}z_{in2} = 50 \times (0.48 - j0.36) = (24 - j18) \ \Omega.$$
To move along line 1, we need to normalize with respect to $Z_{01}$. We shall call this $z_{L1}$:

$$z_{L1} = \frac{Z_{in2}}{Z_{01}} = \frac{24 - j18}{100} = 0.24 - j0.18 \quad \text{(point } B\ell \text{ on Smith chart 2)}$$

After drawing the SWR circle through point $B\ell$, we move $3\lambda/8$ towards the generator, ending up at point $C$ on Smith chart 2. The normalized input impedance of line 1 is:

$$z_{in} = 0.66 - j1.25$$

which upon unnormalizing becomes:

$$Z_{in} = (66 - j125) \, \Omega.$$
Using a slotted line on a 50-Ω air-spaced lossless line, the following measurements were obtained: \( S = 1.6 \) and \( |\bar{V}|_{\text{max}} \) occurred only at 10 cm and 24 cm from the load. Use the Smith chart to find \( Z_L \).

\[ Z_L = 0.82 - j0.39. \]
Therefore \( Z_{L} = z_{L}Z_{0} = (0.82 - j0.39) \times 50 \ \Omega = (41.0 - j19.5) \ \Omega. \)
A 200-Ω transmission line is to be matched to a computer terminal with \( Z_L = (50 - j25) \Omega \) by inserting an appropriate reactance in parallel with the line. If \( f = 800 \) MHz and \( \varepsilon_r = 4 \), determine the location nearest to the load at which inserting:

(a) A capacitor can achieve the required matching, and the value of the capacitor.

(b) An inductor can achieve the required matching, and the value of the inductor.

Solution:

(a) After entering the specified values for \( Z_L \) and \( Z_0 \) into Module 2.6, we have \( z_L \) represented by the red dot in Fig. P2.66(a), and \( y_L \) represented by the blue dot. By moving the cursor a distance \( d = 0.093\lambda \), the blue dot arrives at the intersection point between the SWR circle and the \( S = 1 \) circle. At that point

\[
y(d) = 1.026126 - j1.5402026.
\]

To cancel the imaginary part, we need to add a reactive element whose admittance is positive, such as a capacitor. That is:

\[
\omega C = (1.54206) \times Y_0
\]

\[
= \frac{1.54206}{Z_0} = \frac{1.54206}{200} = 7.71 \times 10^{-3},
\]

which leads to

\[
C = \frac{7.71 \times 10^{-3}}{2\pi \times 8 \times 10^8} = 1.53 \times 10^{-12} \text{ F}.
\]
(b) Repeating the procedure for the second intersection point [Fig. P2.66(b)] leads to

\[ y(d) = 1.000001 + j1.520691, \]

at \( d_2 = 0.447806\lambda \).

To cancel the imaginary part, we add an inductor in parallel such that

\[ \frac{1}{\omega L} = \frac{1.520691}{200}, \]

from which we obtain

\[ L = \frac{200}{1.52 \times 2\pi \times 8 \times 10^8} = 2.618 \times 10^{-8} \text{ H}. \]
Figure P2.66(b)