Problem 2.66  A 200-Ω transmission line is to be matched to a computer terminal with \( Z_L = (50 - j25) \Omega \) by inserting an appropriate reactance in parallel with the line. If \( f = 800 \text{ MHz} \) and \( \varepsilon_r = 4 \), determine the location nearest to the load at which inserting:

(a) A capacitor can achieve the required matching, and the value of the capacitor.

(b) An inductor can achieve the required matching, and the value of the inductor.

Solution:

(a) After entering the specified values for \( Z_L \) and \( Z_0 \) into Module 2.6, we have \( z_L \) represented by the red dot in Fig. P2.66(a), and \( y_L \) represented by the blue dot. By moving the cursor a distance \( d = 0.093 \lambda \), the blue dot arrives at the intersection point between the SWR circle and the \( S = 1 \) circle. At that point

\[
y(d) = 1.026126 - j1.5402026.
\]
To cancel the imaginary part, we need to add a reactive element whose admittance is positive, such as a capacitor. That is:

\[
\omega C = \left(1.54206\right) \times Y_0
= \frac{1.54206}{Z_0} = \frac{1.54206}{200} = 7.71 \times 10^{-3},
\]
which leads to

\[
C = \frac{7.71 \times 10^{-3}}{2 \pi \times 8 \times 10^8} = 1.53 \times 10^{-12} \text{ F}.
\]
(b) Repeating the procedure for the second intersection point [Fig. P2.66(b)] leads to

\[ y(d) = 1.000001 + j1.520691, \]

at \( d_2 = 0.447806\lambda \).

To cancel the imaginary part, we add an inductor in parallel such that

\[ \frac{1}{\omega L} = \frac{1.520691}{200}, \]

from which we obtain

\[ L = \frac{200}{1.52 \times 2\pi \times 8 \times 10^8} = 2.618 \times 10^{-8} \text{ H.} \]
Figure P2.66(b)
Problem 2.68 A 50-Ω lossless line is to be matched to an antenna with \( Z_L = (75 - j20) \) Ω using a shorted stub. Use the Smith chart to determine the stub length and distance between the antenna and stub.

Solution: Refer to Fig. P2.68(a) and Fig. P2.68(b), which represent two different solutions.  
\[
z_L = \frac{Z_L}{Z_0} = \frac{(75 - j20) \Omega}{50 \Omega} = 1.5 - j0.4
\]
and is located at point Z-LOAD in both figures. Since it is advantageous to work in admittance coordinates, \( y_L \) is plotted as point Y-LOAD in both figures. Y-LOAD is at 0.041λ on the WTG scale.
For the first solution in Fig. P2.68(a), point \( Y-LOAD-IN-1 \) represents the point at which \( g = 1 \) on the SWR circle of the load. \( Y-LOAD-IN-1 \) is at \( 0.145\lambda \) on the WTG scale, so the stub should be located at \( 0.145\lambda - 0.041\lambda = 0.104\lambda \) from the load (or some multiple of a half wavelength further). At \( Y-LOAD-IN-1 \), \( b = 0.52 \), so a stub with an input admittance of \( \gamma_{stub} = 0 - j0.52 \) is required. This point is \( Y-STUB-IN-1 \) and is at \( 0.423\lambda \) on the WTG scale. The short circuit admittance is denoted by point \( Y-SHT \), located at \( 0.250\lambda \). Therefore, the short stub must be \( 0.423\lambda - 0.250\lambda = 0.173\lambda \) long (or some multiple of a half wavelength longer).

Figure P2.68: (b) Second solution to Problem 2.68.

For the second solution in Fig. P2.68(b), point \( Y-LOAD-IN-2 \) represents the point at which \( g = 1 \) on the SWR circle of the load. \( Y-LOAD-IN-2 \) is at \( 0.355\lambda \) on the WTG scale, so the stub should be located at \( 0.355\lambda - 0.041\lambda = 0.314\lambda \) from the
load (or some multiple of a half wavelength further). At \( Y\text{-LOAD-IN-2} \), \( b = -0.52 \), so a stub with an input admittance of \( y_{\text{stub}} = 0 + j0.52 \) is required. This point is \( Y\text{-STUB-IN-2} \) and is at 0.077\( \lambda \) on the WTG scale. The short circuit admittance is denoted by point \( Y\text{-SHT} \), located at 0.250\( \lambda \). Therefore, the short stub must be 0.077\( \lambda \) – 0.250\( \lambda \) + 0.500\( \lambda \) = 0.327\( \lambda \) long (or some multiple of a half wavelength longer).
Problem 3.12  Two lines in the $x$–$y$ plane are described by the expressions:

\begin{align*}
\text{Line 1} & \quad x + 2y = -6, \\
\text{Line 2} & \quad 3x + 4y = 8.
\end{align*}

Use vector algebra to find the smaller angle between the lines at their intersection point.

![Figure P3.12: Lines 1 and 2.](image)

**Solution:** Intersection point is found by solving the two equations simultaneously:

\begin{align*}
-2x - 4y &= 12, \\
3x + 4y &= 8.
\end{align*}

The sum gives $x = 20$, which, when used in the first equation, gives $y = -13$.

Hence, intersection point is $(20, -13)$.

Another point on line 1 is $x = 0$, $y = -3$. Vector $\mathbf{A}$ from $(0, -3)$ to $(20, -13)$ is

\[
\mathbf{A} = \hat{x}(20) + \hat{y}(-13 + 3) = 20\hat{x} - 10\hat{y},
\]

\[
|\mathbf{A}| = \sqrt{20^2 + 10^2} = \sqrt{500}.
\]

A point on line 2 is $x = 0$, $y = 2$. Vector $\mathbf{B}$ from $(0, 2)$ to $(20, -13)$ is

\[
\mathbf{B} = \hat{x}(20) + \hat{y}(-13 - 2) = 20\hat{x} - 15\hat{y},
\]

\[
\theta_{AB} = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}| \cdot |\mathbf{B}|}\right).
\]
\[ |\mathbf{B}| = \sqrt{20^2 + 15^2} = \sqrt{625}. \]

Angle between \( \mathbf{A} \) and \( \mathbf{B} \) is
\[
\theta_{AB} = \cos^{-1}\left( \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}| |\mathbf{B}|} \right) = \cos^{-1}\left( \frac{400 + 150}{\sqrt{500} \cdot \sqrt{625}} \right) = 10.3^\circ.
\]