Problem 6.3 A coil consists of 100 turns of wire wrapped around a square frame of sides 0.25 m. The coil is centered at the origin with each of its sides parallel to the x- or y-axis. Find the induced emf across the open-circuited ends of the coil if the magnetic field is given by

(a) $\mathbf{B} = \hat{\mathbf{z}}\, 20e^{-3t}$ (T)

(b) $\mathbf{B} = \hat{\mathbf{z}}\, 20 \cos x \, \cos 10^3 t$ (T)

(c) $\mathbf{B} = \hat{\mathbf{z}}\, 20 \cos x \, \sin 2y \, \cos 10^3 t$ (T)

Solution: Since the coil is not moving or changing shape, $V^{\mathrm{m}}_{\mathrm{emf}} = 0$ V and $V_{\mathrm{emf}} = V^{\mathrm{tr}}_{\mathrm{emf}}$. From Eq. (6.6),

$$V_{\mathrm{emf}} = -N \frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} = -N \frac{d}{dt} \int_{-0.125}^{0.125} \int_{-0.125}^{0.125} \mathbf{B} \cdot (\hat{\mathbf{z}}\, dx\, dy),$$

where $N = 100$ and the surface normal was chosen to be in the $+\hat{\mathbf{z}}$ direction.

(a) For $\mathbf{B} = \hat{\mathbf{z}} 20e^{-3t}$ (T),

$$V_{\mathrm{emf}} = -100 \frac{d}{dt} (20e^{-3t} (0.25)^2) = 375e^{-3t} \quad \text{(V)}.$$

(b) For $\mathbf{B} = \hat{\mathbf{z}} 20 \cos x \cos 10^3 t$ (T),

$$V_{\mathrm{emf}} = -100 \frac{d}{dt} \left(20 \cos 10^3 t \int_{x=-0.125}^{0.125} \int_{y=-0.125}^{0.125} \cos x \, dx\, dy \right) = 124.6 \sin 10^3 t \quad \text{(kV)}.$$

(c) For $\mathbf{B} = \hat{\mathbf{z}} 20 \cos x \sin 2y \cos 10^3 t$ (T),

$$V_{\mathrm{emf}} = -100 \frac{d}{dt} \left(20 \cos 10^3 t \int_{x=-0.125}^{0.125} \int_{y=-0.125}^{0.125} \cos x \sin 2y \, dx\, dy \right) = 0.$$

Problem 6.6 The square loop shown in Fig. P6.6 is coplanar with a long, straight wire carrying a current

$$I(t) = 5\cos(2\pi \times 10^4 t) \quad \text{(A)}.$$

(a) Determine the emf induced across a small gap created in the loop.

(b) Determine the direction and magnitude of the current that would flow through a 4-Ω resistor connected across the gap. The loop has an internal resistance of 1 Ω.

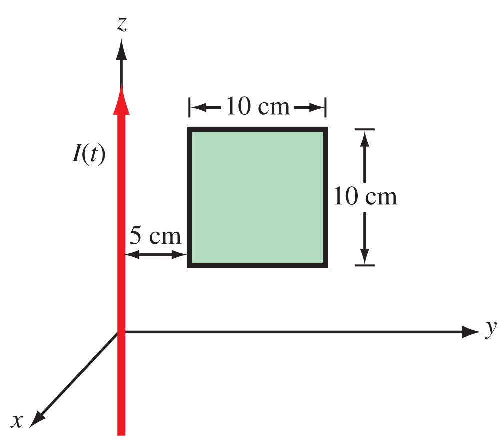

Figure P6.6: Loop coplanar with long wire (Problem 6.6).

Solution:

(a) The magnetic field due to the wire is

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}$ and $r = y$. The flux passing through the loop is

$$\begin{aligned}
\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} &= \int_{5\text{ cm}}^{15\text{ cm}} \left(-\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y}\right) \cdot [-\hat{\mathbf{x}}\, 10 \text{ (cm)}]\; dy \\
&= \frac{\mu_0 I \times 10^{-1}}{2\pi} \ln \frac{15}{5} \\
&= \frac{4\pi \times 10^{-7} \times 5\cos(2\pi \times 10^4 t) \times 10^{-1}}{2\pi} \times 1.1 \\
&= 1.1 \times 10^{-7} \cos(2\pi \times 10^4 t) \quad \text{(Wb)}.
\end{aligned}$$

$$
\begin{aligned}
V_{\text{emf}} = -\frac{d\Phi}{dt} &= 1.1 \times 2\pi \times 10^4 \sin(2\pi \times 10^4 t) \times 10^{-7} \\
&= 6.9 \times 10^{-3} \sin(2\pi \times 10^4 t) \quad \text{(V)}.
\end{aligned}
$$

(b)

$$
I_{\text{ind}} = \frac{V_{\text{emf}}}{4+1} = \frac{6.9 \times 10^{-3}}{5} \sin(2\pi \times 10^4 t) = 1.38 \sin(2\pi \times 10^4 t) \quad \text{(mA)}.
$$

At $t = 0$, $\mathbf{B}$ is a maximum, it points in $-\hat{\mathbf{x}}$-direction, and since it varies as $\cos(2\pi \times 10^4 t)$, it is decreasing. Hence, the induced current has to be CCW when looking down on the loop, as shown in the figure.

Problem 6.7 The rectangular conducting loop shown in Fig. P6.7 rotates at 6,000 revolutions per minute in a uniform magnetic flux density given by

$$\mathbf{B} = \hat{\mathbf{y}}\,50 \quad \text{(mT)}.$$

Determine the current induced in the loop if its internal resistance is 0.5 Ω.

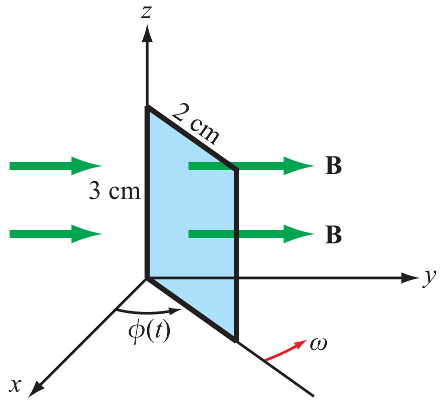

Figure P6.7: Rotating loop in a magnetic field (Problem 6.7).

Solution:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S} = \hat{\mathbf{y}}\,50 \times 10^{-3} \cdot \hat{\mathbf{y}}(2 \times 3 \times 10^{-4}) \cos\phi(t) = 3 \times 10^{-5} \cos\phi(t),$$

$$\phi(t) = \omega t = \frac{2\pi \times 6 \times 10^3}{60}\, t = 200\pi t \quad \text{(rad/s)},$$

$$\Phi = 3 \times 10^{-5} \cos(200\pi t) \quad \text{(Wb)},$$

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = 3 \times 10^{-5} \times 200\pi \sin(200\pi t) = 18.85 \times 10^{-3} \sin(200\pi t) \quad \text{(V)},$$

$$I_{\text{ind}} = \frac{V_{\text{emf}}}{0.5} = 37.7 \sin(200\pi t) \quad \text{(mA)}.$$

The direction of the current is CW (if looking at it along $-\hat{\mathbf{x}}$-direction) when the loop is in the first quadrant ($0 \leq \phi \leq \pi/2$). The current reverses direction in the second quadrant, and reverses again every quadrant.