Problem 2.31  A voltage generator with

\[ v_g(t) = 5 \cos(2\pi \times 10^9 t) \text{ V} \]

and internal impedance \( Z_g = 50 \text{ Ω} \) is connected to a 50-Ω lossless air-spaced transmission line. The line length is 5 cm and the line is terminated in a load with impedance \( Z_L = (100 - j100) \text{ Ω} \). Determine:

(a) \( \Gamma \) at the load.
(b) \( Z_{\text{in}} \) at the input to the transmission line.
(c) The input voltage \( \tilde{V}_i \) and input current \( \tilde{I}_i \).
(d) The quantities in (a)–(c) using CD Modules 2.4 or 2.5.

Solution:

(a) From Eq. (2.59),

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62e^{-j29.7^\circ}. \]

(b) All formulae for \( Z_{\text{in}} \) require knowledge of \( \beta = \omega / u_p \). Since the line is an air line, \( u_p = c \), and from the expression for \( v_g(t) \) we conclude \( \omega = 2\pi \times 10^9 \text{ rad/s} \). Therefore

\[ \beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m}. \]

Then, using Eq. (2.79),

\[ Z_{\text{in}} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \]

\[ = 50 \left[ \frac{(100 - j100) + j50\tan \left( \frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm} \right)}{50 + j(100 - j100)\tan \left( \frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm} \right)} \right] \]

\[ = 50 \left[ \frac{(100 - j100) + j50\tan \left( \frac{\pi}{3} \text{ rad} \right)}{50 + j(100 - j100)\tan \left( \frac{\pi}{3} \text{ rad} \right)} \right] = (12.5 - j12.7) \text{ Ω}. \]

(c) In phasor domain, \( \tilde{V}_g = 5 \text{ V}e^{j0^\circ} \). From Eq. (2.80),

\[ \tilde{V}_i = \frac{\tilde{V}_g Z_{\text{in}}}{Z_g + Z_{\text{in}}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40e^{-j34.0^\circ} \text{ (V)}, \]

and also from Eq. (2.80),

\[ \tilde{I}_i = \frac{\tilde{V}_i}{Z_{\text{in}}} = \frac{1.4e^{-j34.0^\circ}}{(12.5 - j12.7)} = 78.4e^{j115^\circ} \text{ (mA)}. \]
## Module 2.4: Transmission Line Simulator

### Set Line
- **Characteristic Impedance** $Z_0 = 50.0 + j 0.0 \ \Omega$
- **Frequency** $f = 169 \ \text{Hz}$
- **Relative Permittivity** $\varepsilon_r = 1.0$
- **Line Length** $l = 0.05 \ \text{m}$

### Low Loss Approximation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_0$</td>
<td>50.0 + j 0.0 \ \Omega</td>
</tr>
<tr>
<td>$f$</td>
<td>169 \ \text{Hz}</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>1.0</td>
</tr>
<tr>
<td>$l$</td>
<td>0.05 \ \text{m}</td>
</tr>
</tbody>
</table>

### Impedance
- **Impedance** $Z_L = 100 + j (-100) \ \Omega$
- **Admittance** $Y_L = 100 - j (100) \ \text{S}$

### Reflection Coefficient
- $\Gamma_d = -0.53543815 - j 0.3192389 \ \text{rad}$

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage $V(d)$</td>
<td>$1.161077 - j 0.782794 \ \text{V}$</td>
</tr>
<tr>
<td>Current $I(d)$</td>
<td>$0.076778 + j 0.015611 \ \text{A}$</td>
</tr>
<tr>
<td>Power Flow $P_{AV}$</td>
<td>$38.461538 \ \text{mW}$</td>
</tr>
</tbody>
</table>
**Problem 2.33** Two half-wave dipole antennas, each with an impedance of 75 Ω, are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. P2.33.

All lines are 50 Ω and lossless.

(a) Calculate $Z_{in1}$, the input impedance of the antenna-terminated line, at the parallel juncture.

(b) Combine $Z_{in1}$ and $Z_{in2}$ in parallel to obtain $Z'_L$, the effective load impedance of the feedline.

(c) Calculate $Z_{in}$ of the feedline.

**Solution:**

(a) 

$$Z_{in1} = Z_0 \left[ \frac{Z_{L1} + jZ_0 \tan \beta l_1}{Z_0 + jZ_{L1} \tan \beta l_1} \right]$$

$$= 50 \left\{ \frac{75 + j50 \tan \left( \frac{2\pi}{\lambda} \cdot 0.2\lambda \right)}{50 + j75 \tan \left( \frac{2\pi}{\lambda} \cdot 0.2\lambda \right)} \right\} = (35.20 - j8.62) \Omega.$$ 

(b) 

$$Z'_L = \frac{Z_{in1}Z_{in2}}{Z_{in1} + Z_{in2}} = \frac{(35.20 - j8.62)^2}{2(35.20 - j8.62)} = (17.60 - j4.31) \Omega.$$ 

(c)
$Z_{in} = \left\{ \frac{(17.60 - j4.31) + j50 \tan[(2\pi/\lambda)(0.3\lambda)]}{50 + j(17.60 - j4.31) \tan[(2\pi/\lambda)(0.3\lambda)]} \right\} = (107.57 - j56.7) \, \Omega.$