**Problem 2.38** The input impedance of a 31-cm–long lossless transmission line of unknown characteristic impedance was measured at 1 MHz. With the line terminated in a short circuit, the measurement yielded an input impedance equivalent to an inductor with inductance of 0.064 $\mu$H, and when the line was open-circuited, the measurement yielded an input impedance equivalent to a capacitor with capacitance of 40 pF. Find $Z_0$ of the line, the phase velocity, and the relative permittivity of the insulating material.

**Solution:** Now $\omega = 2\pi f = 6.28 \times 10^6$ rad/s, so

$$Z_{sc}^{in} = j\omega L = j2\pi \times 10^6 \times 0.064 \times 10^{-6} = 0.40 \Omega$$

and $Z_{oc}^{in} = 1/j\omega C = 1/(j2\pi \times 10^6 \times 40 \times 10^{-12}) = -j4000 \Omega$.

From Eq. (2.94), $Z_0 = \sqrt{Z_{sc}^{in}Z_{oc}^{in}} = \sqrt{(0.4 \Omega)(-j4000 \Omega)} = 40 \Omega$. Using Eq. (2.48),

$$u_p = \frac{\omega}{\beta} = \frac{\omega l}{\tan^{-1}\sqrt{-Z_{sc}^{in}/Z_{oc}^{in}}} = \frac{6.28 \times 10^6 \times 0.31}{\tan^{-1}(\pm\sqrt{-0.4/(-j4000)})} = 1.95 \times 10^6 \times (\pm 0.01 + n\pi) \text{ m/s},$$

where $n \geq 0$ for the plus sign and $n \geq 1$ for the minus sign. For $n = 0$, $u_p = 1.94 \times 10^8$ m/s = 0.65$c$ and $\epsilon_r = (c/u_p)^2 = 1/0.65^2 = 2.4$. For other values of $n$, $u_p$ is very slow and $\epsilon_r$ is unreasonably high.
Problem 2.40  A 100-MHz FM broadcast station uses a 300-Ω transmission line between the transmitter and a tower-mounted half-wave dipole antenna. The antenna impedance is 73 Ω. You are asked to design a quarter-wave transformer to match the antenna to the line.

(a) Determine the electrical length and characteristic impedance of the quarter-wave section.

(b) If the quarter-wave section is a two-wire line with $D = 2.5$ cm, and the wires are embedded in polystyrene with $\varepsilon_r = 2.6$, determine the physical length of the quarter-wave section and the radius of the two wire conductors.

Solution:
(a) For a match condition, the input impedance of a load must match that of the transmission line attached to the generator. A line of electrical length $\lambda/4$ can be used. From Eq. (2.97), the impedance of such a line should be

$$Z_0 = \sqrt{Z_{in}Z_L} = \sqrt{300 \times 73} = 148 \Omega.$$ 

(b) $$\frac{\lambda}{4} = \frac{u_0}{4f} = \frac{c}{4\sqrt{\varepsilon_r}f} = \frac{3 \times 10^8}{4\sqrt{2.6} \times 100 \times 10^6} = 0.465 \text{ m},$$

and, from Table 2-2,

$$Z_0 = \frac{120}{\sqrt{\varepsilon}} \ln \left[ \frac{D}{d} + \sqrt{\left( \frac{D}{d} \right)^2 - 1} \right] \Omega.$$ 

Hence,

$$\ln \left[ \frac{D}{d} + \sqrt{\left( \frac{D}{d} \right)^2 - 1} \right] = \frac{148 \sqrt{2.6}}{120} = 1.99,$$

which leads to

$$\frac{D}{d} + \sqrt{\left( \frac{D}{d} \right)^2 - 1} = 7.31,$$

and whose solution is $D/d = 3.73$. Hence, $d = D/3.73 = 2.5 \text{ cm}/3.73 = 0.67 \text{ cm}$. 


Problem 2.41  A 50-Ω lossless line of length $l = 0.375\lambda$ connects a 300-MHz generator with $\tilde{V}_g = 300$ V and $Z_g = 50$ Ω to a load $Z_L$. Determine the time-domain current through the load for:

(a) $Z_L = (50 - j50)$ Ω   
(b) $Z_L = 50$ Ω   
(c) $Z_L = 0$ (short circuit)

For (a), verify your results by deducing the information you need from the output products generated by CD Module 2.4.

Solution:

\[ Z_L - Z_0 \]
\[ Z_L + Z_0 \]

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j50 - 50}{50 - j50 + 50} = -\frac{j50}{100 - j50} = 0.45e^{-j63.43^\circ}. \]
Application of Eq. (2.79) gives:

\[ Z_{\text{in}} = Z_0 \left[ \frac{Z_L + jZ_0 \tan \beta l}{Z_L + jZ_0 \tan \beta l} \right] = 50 \left[ \frac{(50 - j50) + j50 \tan 135^\circ}{50 + j(50 - j50) \tan 135^\circ} \right] = (100 + j50) \, \Omega. \]

Using Eq. (2.82) gives

\[ V_0^+ = \left( \frac{\bar{V}_g Z_{\text{in}}}{Z_L + Z_{\text{in}}} \right) \left( \frac{1}{e^{j\beta l} + e^{-j\beta l}} \right) \]

\[ = \frac{300(100 + j50)}{50 + (100 + j50)} \left( \frac{1}{e^{j135^\circ} + 0.45 e^{-j63.43^\circ} e^{-j135^\circ}} \right) \]

\[ = 150 e^{-j135^\circ} \, (V), \]

\[ \bar{I}_L = \frac{V_0^+}{Z_0} (1 - \Gamma) = \frac{150 e^{-j135^\circ}}{50} (1 - 0.45 e^{-j63.43^\circ}) = 2.68 e^{-j108.44^\circ} \, (A), \]

\[ i_L(t) = \Re \{ \bar{I}_L e^{j\omega t} \} \]

\[ = \Re \{2.68 e^{-j108.44^\circ} e^{j6\pi \times 10^8 t}\} \]

\[ = 2.68 \cos(6\pi \times 10^8 t - 108.44^\circ) \, (A). \]

(b)

\[ Z_L = 50 \, \Omega, \]

\[ \Gamma = 0, \]

\[ Z_{\text{in}} = Z_0 = 50 \, \Omega, \]

\[ V_0^+ = \frac{300\times50}{50 + 50} \left( \frac{1}{e^{j135^\circ} + 0} \right) = 150 e^{-j135^\circ} \, (V), \]

\[ \bar{I}_L = \frac{V_0^+}{Z_0} \frac{150 e^{-j135^\circ}}{50} e^{-j135^\circ} = 3 e^{-j135^\circ} \, (A), \]

\[ i_L(t) = \Re \{3 e^{-j135^\circ} e^{j6\pi \times 10^8 t}\} = 3 \cos(6\pi \times 10^8 t - 135^\circ) \, (A). \]

(c)

\[ Z_L = 0, \]

\[ \Gamma = -1, \]

\[ Z_{\text{in}} = Z_0 \left( \frac{0 + jZ_0 \tan 135^\circ}{Z_0 + 0} \right) = jZ_0 \tan 135^\circ = -j50 \, (\Omega), \]

\[ V_0^+ = \frac{300(-j50)}{50 - j50} \left( \frac{1}{e^{j135^\circ} - e^{-j135^\circ}} \right) = 150 e^{-j135^\circ} \, (V), \]
\[
I_L = \frac{V_0^+}{Z_0} [1 - \Gamma] = \frac{150 e^{-j135^\circ}}{50} [1 + 1] = 6 e^{-j135^\circ} \quad (A),
\]
\[
i_L(t) = 6 \cos(6\pi \times 10^8 t - 135^\circ) \quad (A).
\]
From output of Module 2.4, at \( d = 0 \) (load)
\[
\tilde{I}(d) = 2.68 \angle -1.89 \text{ rad},
\]
which corresponds to
\[
\tilde{I}(d) = 2.68 \angle -108.29^\circ.
\]
The equivalent time-domain current at \( f = 300 \text{ MHz} \) is
\[
i_L(t) = 2.68 \cos(6\pi \times 10^8 t - 108.29^\circ) \quad (A).
\]