Problem 2.1  A transmission line of length $l$ connects a load to a sinusoidal voltage source with an oscillation frequency $f$. Assuming the velocity of wave propagation on the line is $c$, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) \[ \frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(20 \times 10^{-2} \text{ m}) \times (20 \times 10^3 \text{ Hz})}{3 \times 10^8 \text{ m/s}} = 1.33 \times 10^{-5} \text{ (negligible)}. \]

(b) \[ \frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(50 \times 10^3 \text{ m}) \times (60 \times 10^0 \text{ Hz})}{3 \times 10^8 \text{ m/s}} = 0.01 \text{ (borderline)}. \]

(c) \[ \frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(20 \times 10^{-2} \text{ m}) \times (600 \times 10^6 \text{ Hz})}{3 \times 10^8 \text{ m/s}} = 0.40 \text{ (nonnegligible)}. \]

(d) \[ \frac{l}{\lambda} = \frac{lf}{u_p} = \frac{(1 \times 10^{-3} \text{ m}) \times (100 \times 10^9 \text{ Hz})}{3 \times 10^8 \text{ m/s}} = 0.33 \text{ (nonnegligible)}. \]
Problem 2.2  A two-wire copper transmission line is embedded in a dielectric material with \( \varepsilon_r = 2.6 \) and \( \sigma = 2 \times 10^{-6} \) S/m. Its wires are separated by 3 cm and their radii are 1 mm each.

(a) Calculate the line parameters \( R', L', G', \) and \( C' \) at 2 GHz.

(b) Compare your results with those based on CD Module 2.1. Include a printout of the screen display.

Solution:

(a) Given:

\[
\begin{align*}
f & = 2 \times 10^9 \text{ Hz}, \\
d & = 2 \times 10^{-3} \text{ m}, \\
D & = 3 \times 10^{-2} \text{ m}, \\
\sigma_c & = 5.8 \times 10^7 \text{ S/m (copper)}, \\
\varepsilon_r & = 2.6, \\
\sigma & = 2 \times 10^{-6} \text{ S/m}, \\
\mu & = \mu_c = \mu_0.
\end{align*}
\]

From Table 2-1:

\[
R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \left[ \frac{\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7} \right]^{1/2} = 1.17 \times 10^{-2} \text{ } \Omega,
\]

\[
R' = \frac{2R_s}{\pi d} = \frac{2 \times 1.17 \times 10^{-2}}{2\pi \times 10^{-3}} = 3.71 \text{ } \Omega/\text{m},
\]

\[
L' = \frac{\mu}{\pi} \ln \left( \frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1} \right) = 1.36 \times 10^{-6} \text{ } \text{H/m},
\]

\[
G' = \frac{\pi \sigma}{\ln[(D/d) + \sqrt{(D/d)^2 - 1}]} = 1.85 \times 10^{-6} \text{ } \text{S/m},
\]

\[
C' = \frac{G' \varepsilon}{\sigma} = \frac{1.85 \times 10^{-6} \times 8.85 \times 10^{-12} \times 2.6}{2 \times 10^{-6}} = 2.13 \times 10^{-11} \text{ } \text{F/m}.
\]

(b) Solution via Module 2.1:
Module 2.1 Two-Wire Line

Select:
- Impedance vs. Distance D

Real Part of Characteristic Impedance

Input
- Wire Diameter: \( d = 2.0 \) [mm]
- Centers distance: \( D = 30.0 \) [mm]
- Frequency: \( f = 2.0 \times 10^9 \) [Hz]
- \( \varepsilon_r = 2.6 \)
- \( \sigma = 2.0 \times 10^{-6} \) [S/m]
- \( \sigma_c = 5.8 \times 10^7 \) [S/m]

Output
- Structure Data:
  - \( Z_0 = 253.037142 - 0.026617 \) [Ω]
  - \( C' = 21.241303 \) [pF/m]
  - \( L' = 1.360034 \) [μH/m]
  - \( R' = 3.713907 \) [Ω/m]
  - \( G' = 2.0 \times 10^{-6} \) [S/m]
  - \( \lambda_0 = 0.15 \) [m] in vacuum
  - \( \lambda = 9.3026 \) [cm] in guide
  - \( \alpha = 0.007572 \) [Np/m]
  - \( \beta = 67.542213 \) [rad/m]

Update
Problem 2.5  For a parallel-plate transmission line, the line parameters are given by:

\[ R' = 1 \text{ (Ω/m)}, \]
\[ L' = 167 \text{ (nH/m)}, \]
\[ G' = 0, \]
\[ C' = 172 \text{ (pF/m)}. \]

Find \( \alpha, \beta, u_p, \) and \( Z_0 \) at 1 GHz.

Solution: At 1 GHz, \( \omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s}. \) Application of (2.22) gives:

\[
\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}
\]
\[
= \left[ (1 + j2\pi \times 10^9 \times 167 \times 10^{-9})(0 + j2\pi \times 10^9 \times 172 \times 10^{-12}) \right]^{1/2}
\]
\[
= \left[ (1 + j1049)(j1.1) \right]^{1/2}
\]
\[
= \left[ \sqrt{1 + (1049)^2} e^{j\tan^{-1}1049} \times 1.1 e^{j90^\circ} \right]^{1/2}
\]
\[
= \left[ 1049 e^{j90^\circ} \times 1.1 e^{j90^\circ} \right]^{1/2}
\]
\[
= \left[ 1154 e^{j90^\circ} \right]^{1/2}
\]
\[
= 34 e^{j90^\circ} = 34 \cos 90^\circ + j34 \sin 90^\circ = 34 + j0.
\]

Hence,

\[
\alpha = 0.016 \text{ Np/m},
\]
\[
\beta = 34 \text{ rad/m}.
\]

\[
u_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10^9}{34} = 1.85 \times 10^8 \text{ m/s}.
\]

\[
Z_0 = \left[ \frac{R' + j\omega L'}{G' + j\omega C'} \right]^{1/2}
\]
\[
= \left[ 1049 e^{j90^\circ} \times 1.1 e^{j90^\circ} \right]^{1/2}
\]
\[
= \left[ 954 e^{-j0.05^\circ} \right]^{1/2}
\]
\[
= 31 e^{-j0.025^\circ} \simeq (31 - j0.01) \Omega.
\]
Problem 2.18  Polyethylene with $\varepsilon_r = 2.25$ is used as the insulating material in a lossless coaxial line with characteristic impedance of 50 $\Omega$. The radius of the inner conductor is 1.2 mm.

(a) What is the radius of the outer conductor?
(b) What is the phase velocity of the line?

Solution: Given a lossless coaxial line, $Z_0 = 50$ $\Omega$, $\varepsilon_r = 2.25$, $a = 1.2$ mm:

(a) From Table 2-2, $Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln \left( \frac{b}{a} \right)$ which can be rearranged to give

$$b = ae^{Z_0\sqrt{\varepsilon_r}/60} = (1.2 \text{ mm})e^{50\sqrt{2.25}/60} = 4.2 \text{ mm}.$$ 

(b) Also from Table 2-2,

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.25}} = 2.0 \times 10^8 \text{ m/s}.$$ 

Problem 2.19 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50) \Omega$. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,
(b) the standing-wave ratio on the line,
(c) the position of the voltage maximum nearest the load,
(d) the position of the current maximum nearest the load.
(e) Verify quantities in parts (a)–(d) using CD Module 2.4. Include a printout of the screen display.

Solution:

(a) From Eq. (2.59),
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57 e^{-j79.8^\circ}.$$

(b) From Eq. (2.73),
$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

(c) From Eq. (2.70)
$$d_{\text{max}} = \frac{\theta \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8 \text{ cm} \pi \text{ rad}}{4\pi} + \frac{n \times 8 \text{ cm}}{2} = -0.89 \text{ cm} + 4.0 \text{ cm} = 3.11 \text{ cm}.$$

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.72),
$$d_{\text{min}} = d_{\text{max}} - \frac{\lambda}{4} = 3.11 \text{ cm} - 8 \text{ cm}/4 = 1.11 \text{ cm}.$$

(e) The problem statement does not specify the frequency, so in Module 2.4 we need to select the combination of $f$ and $\varepsilon_r$ such that $\lambda = 5$ cm. With $\varepsilon_r$ chosen as 1,
$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-2}} = 3.75 \text{ GHz}.$$

The generator parameters are irrelevant to the problem.

The results listed in the output screens are very close to those given in parts (a) through (d).
Figure P2.19(a)
Figure P2.19(b)