Problem 2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(20 \times 10^{-2} \text{ m}) \times (20 \times 10^{3} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 1.33 \times 10^{-5} \text{ (negligible).}$$

(b) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(50 \times 10^{3} \text{ m}) \times (60 \times 10^{0} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.01 \text{ (borderline).}$$

(c) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(20 \times 10^{-2} \text{ m}) \times (600 \times 10^{6} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.40 \text{ (nonnegligible).}$$

(d) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(1 \times 10^{-3} \text{ m}) \times (100 \times 10^{9} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.33 \text{ (nonnegligible).}$$

Problem 2.2 A two-wire copper transmission line is embedded in a dielectric material with $\varepsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}$ S/m. Its wires are separated by 3 cm and their radii are 1 mm each.

(a) Calculate the line parameters R', L', G', and C' at 2 GHz.

(b) Compare your results with those based on CD Module 2.1. Include a printout of the screen display.

Solution:

(a) Given:

$$
\begin{aligned}
f &= 2 \times 10^9 \text{ Hz}, \\
d &= 2 \times 10^{-3} \text{ m}, \\
D &= 3 \times 10^{-2} \text{ m}, \\
\sigma_c &= 5.8 \times 10^7 \text{ S/m (copper)}, \\
\varepsilon_r &= 2.6, \\
\sigma &= 2 \times 10^{-6} \text{ S/m}, \\
\mu &= \mu_c = \mu_0.
\end{aligned}
$$

From Table 2-1:

$$
\begin{aligned}
R_s &= \sqrt{\pi f \mu_c / \sigma_c} \\
&= [\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7} / 5.8 \times 10^7]^{1/2} \\
&= 1.17 \times 10^{-2}\ \Omega, \\
R' &= \frac{2R_s}{\pi d} = \frac{2 \times 1.17 \times 10^{-2}}{2\pi \times 10^{-3}} = 3.71\ \Omega/\text{m}, \\
L' &= \frac{\mu}{\pi} \ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right] \\
&= 1.36 \times 10^{-6} \text{ H/m}, \\
G' &= \frac{\pi \sigma}{\ln[(D/d) + \sqrt{(D/d)^2 - 1}]} \\
&= 1.85 \times 10^{-6} \text{ S/m}, \\
C' &= \frac{G' \varepsilon}{\sigma} \\
&= \frac{1.85 \times 10^{-6} \times 8.85 \times 10^{-12} \times 2.6}{2 \times 10^{-6}} \\
&= 2.13 \times 10^{-11} \text{ F/m}.
\end{aligned}
$$

#2.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{r_L + jX_L - Z_0}{r_L + jX_L + Z_0}$$

$$|\Gamma| = \left|\frac{r_L + jX_L - Z_0}{r_L + jX_L + Z_0}\right| = \left|\frac{(r_L - Z_0) + jX_L}{(r_L + Z_0) + jX_L}\right| = \sqrt{\frac{(r_L - Z_0)^2 + X_L^2}{(r_L + Z_0)^2 + X_L^2}} \le 1$$

When $r_L = 0$. $|\Gamma| = 1$,

Problem 1.26 Find the phasors of the following time functions:

(a) $\upsilon(t) = 9\cos(\omega t - \pi/3)$ (V)

(b) $\upsilon(t) = 12\sin(\omega t + \pi/4)$ (V)

(c) $i(x,t) = 5e^{-3x}\sin(\omega t + \pi/6)$ (A)

(d) $i(t) = -2\cos(\omega t + 3\pi/4)$ (A)

(e) $i(t) = 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$ (A)

Solution:

(a) $\widetilde{V} = 9e^{-j\pi/3}$ V.

(b) $v(t) = 12\sin(\omega t + \pi/4) = 12\cos(\pi/2 - (\omega t + \pi/4)) = 12\cos(\omega t - \pi/4)$ V, $\widetilde{V} = 12e^{-j\pi/4}$ V.

(c)

$$\begin{aligned}
i(t) &= 5e^{-3x}\sin(\omega t + \pi/6) \text{ A} = 5e^{-3x}\cos[\pi/2 - (\omega t + \pi/6)] \text{ A} \\
&= 5e^{-3x}\cos(\omega t - \pi/3) \text{ A}, \\
\widetilde{I} &= 5e^{-3x}e^{-j\pi/3} \text{ A}.
\end{aligned}$$

(d)

$$\begin{aligned}
i(t) &= -2\cos(\omega t + 3\pi/4), \\
\widetilde{I} &= -2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A}.
\end{aligned}$$

(e)

$$\begin{aligned}
i(t) &= 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6) \\
&= 4\cos[\pi/2 - (\omega t + \pi/3)] + 3\cos(\omega t - \pi/6) \\
&= 4\cos(-\omega t + \pi/6) + 3\cos(\omega t - \pi/6) \\
&= 4\cos(\omega t - \pi/6) + 3\cos(\omega t - \pi/6) = 7\cos(\omega t - \pi/6), \\
\widetilde{I} &= 7e^{-j\pi/6} \text{ A}.
\end{aligned}$$