Chapter 10

Fluids
Fluids

- A fluid may be either a liquid or a gas
- Some characteristics of a fluid
  - Flows from one place to another
  - Shape varies according to its container
- To describe the behavior of fluids, new quantities are needed, but the analysis is still based on Newton’s Laws of Motion
Pressure

• The pressure in a fluid is connected with the force that the fluid exerts on a particular surface
• The force is related to pressure by
  \[ P = \frac{F}{A} \]
  • F is the magnitude of the force
  • A is the area
• The force is perpendicular to the area
• Units: The SI unit of pressure is a Pa (Pascal)
  • 1 Pa = 1 N / m²
  • In the US Customary system, the unit is psi (lb/in²)

Section 10.1
Pressure, cont.

• The pressure in the water exerts a force on the submarine
• The force is everywhere perpendicular to the surface of the submarine
• The force on the window is $F = PA_{\text{window}}$
Pressure, final

- Pressure is a scalar quantity
- The box is filled with a fluid
- The fluid exerts an outward pressure on all the sides of the box
- The force associated with the pressure in a fluid is always perpendicular to the surface under consideration

Pressure forces are always perpendicular to a surface.
Atmospheric Pressure

• The pressure of the Earth’s atmosphere near sea level is $P_{atm} = 1.01 \times 10^5 \text{ Pa}$

  • The pressure of the atmosphere actually varies in response to changes in the weather and other factors

• In many cases, the atmosphere exerts a force on both sides of an object, so the net force is zero

\[ F = P_{atm}A \]
Vacuum

• A region where the atmospheric pressure is zero is a vacuum
  • At least approximately zero
• A vacuum was first demonstrated by Otto von Guericke about 400 years ago
• He used a device called a Magdeburg sphere
Magdeburg Sphere

- Constructed of two metal hemispheres
- One hemisphere contains a small tube connected to a valve through which air was removed by a pump
- Inside, a vacuum was created
Guericke’s Experiment

- A simpler case is shown to aid in the analysis
- Consider one plate, the outside air exerts a force it
  - \( F = P_{\text{atm}} A \)
  - \( A \) is the area of the plate
- Since the inside is a vacuum, \( F \approx 0 \)
- The forces act to hold the plates together

There is no force on the inside surfaces because \( P \approx 0 \) inside.
The pressure discussed so far is absolute pressure.

To measure pressure, gauge pressure is often used.

This device determines the pressure by measuring the force on a moveable plate.
Gauge Pressure, cont.

- The total force on the plate inside the gauge is due to the pressures on both sides plus the force exerted by the spring.
- If the gauge plate is in equilibrium, the $\Sigma F = 0$.
- The deflection of the gauge is proportional to the difference between the pressure one wants to measure ($P_{\text{abs}}$) and atmospheric pressure.
- $P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}}$.
Density

• Density is another quantity needed to describe fluids
• Density is defined as mass per unit volume
• The symbol for density is $\rho$

\[ \rho = \frac{M}{V} \]

• This assumes the density is constant throughout the volume
• The density of a substance can vary with temperature and may also depend on pressure
Density, cont.

- For most liquids the density is approximately independent of the pressure
  - These liquids are called *incompressible*
- The density of a gas depends strongly on the pressure
  - Gases are *compressible*
- See table 10.1 for densities of various materials
  - Note the temperature and pressure for the gases
- Units: SI unit is kg/m³
Specific Gravity

• Specific gravity is the ratio of the density of a substance to the density of water
• Specific gravity is important for determining buoyancy of an object
  • It is used to determine whether or not an object will float in water
Fluids and Gravity

- Pressure does vary from place to place in a fluid
- For a cubical box, each wall has an area $h^2$ and a height $h$
- The liquid is at rest, so equilibrium applies
  - $\Sigma F = 0$
Fluids and Gravity, 2

• The forces along the horizontal directions are equal in magnitude and opposite in direction, so they cancel.

• The forces on the top and the bottom do not cancel.
  • The forces are in opposite directions, but the pressure at the bottom must support the pressure at the top plus the weight of the fluid in the box.

• In general, the pressure always increases as one goes deeper into a fluid.
• Since the fluid is in equilibrium:
  \[ \Sigma F_y = P_{\text{bot}} A - P_{\text{top}} A - m g = 0 \]
• The density gives an expression for the mass in terms of the volume: \( m = \rho V = \rho h^3 \)
• Substituting and rearranging,
  \[ P_{\text{bot}} - P_{\text{top}} = \rho g h \]
• This is often written as \( P = P_o + \rho g h \)
  • \( P_o \) is the pressure at the “reference point” and \( P \) is the pressure at a height \( h \) relative to the reference point
• Note that the final result is not dependent on the shape of the “box”
• This is a general relation that tells how the pressure varies with depth in an incompressible fluid
  • Cannot be applied to gases
Often we consider fluids with a surface open to the atmosphere.
This is a convenient reference level.
- \( P_0 = P_{atm} \)

The general equation applies only to incompressible liquids since we assumed a uniform density.

\[
P_{A} = P_0 + \rho gh
\]

\[
P_{B} = P_{atm} + \rho gh
\]
Pressure in a U-Tube

- A U-tube contains a liquid in a U-shaped tube
- The pressure in the liquid depends only on the depth $h$ below the reference level
- The U-tube can contain different liquids

$A$ and $C$ are at same depth so $P_A = P_C$.

$B$ and $D$ are at same depth so $P_B = P_D$. 

Surfaces ($P_{atm}$)

Reference level

$h_A = h_C$

$h_B = h_D$

$P_A < P_B$
Uses of U-Tubes

- U-tubes and other types of containers can be used in many different ways
- These include
  - Measuring the density of a liquid
  - Measuring pressure differences
Barometers

- A barometer measures the absolute pressure of the atmosphere.
- One type consists of a straight tube, partially filled with liquid and closed at the top, with an open bottom inverted into an open dish.
- \( P_{\text{atm}} = \rho g h \).
Units of Pressure

- Many different units may be used for measuring pressures.
- The units of inches or mm of mercury come from measuring the height in a barometer.
- The values of atmospheric pressure can be used as conversion factors.

**TABLE 10.2 Units Used for Measuring Pressure**

<table>
<thead>
<tr>
<th>Pressure Unit</th>
<th>Value of Atmospheric Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pascal</td>
<td>$1.01 \times 10^5$ Pa</td>
</tr>
<tr>
<td>Pounds per square inch</td>
<td>$14.7$ lb/in.$^2$</td>
</tr>
<tr>
<td>Inches</td>
<td>$29.9$ in.</td>
</tr>
<tr>
<td>Millimeters of mercury</td>
<td>$760$ mm</td>
</tr>
<tr>
<td>Atmosphere</td>
<td>$1$ atm</td>
</tr>
</tbody>
</table>
Pumping, Surface Pump

- The soil is porous enough to assume the pressure at the underground surface is $P_{\text{atm}}$
- A vacuum pump could be located at the top of the well
  - Easier to fix if it breaks
  - $h = \frac{P_{\text{atm}}}{\rho g}$
  - Limited to $\approx 10$ m

Section 10.2
Pumping, Bottom of Well

- A high pressure pump could be placed at the bottom of the well
- There is no theoretical limit on the magnitude of $P_{\text{bot}}$
Pressure in a Compressible Fluid

- Gases are compressible fluids and so the previous discussion and equations cannot be applied.
- The density of the gas depends on pressure.
- The pressure at a particular point in a fluid in a gravitational field must support the weight of the fluid above that point.

\[ F = P_{\text{atm}} \cdot A \]
Pressure in a Compressible Fluid, cont.

- Let \( y \) be the height above the surface in the fluid.
- For a gas, as \( y \) increases and the pressure drops, the density becomes less.
- For a gas,
  \[ P = P_o e^{-y/y_o} \]
- For air at sea level, \( P_o = P_{atm} \) and \( y_o = 1.0 \times 10^4 \) m
  - This equation assumes the temperature stays constant.
• An incompressible fluid follows the dotted line
• A compressible fluid follows the curved line
  • The atmosphere would be compressible
Pressure in a Fluid, Summary

- The pressure in a fluid must vary with depth
- $P$ must increase as one goes deeper into the fluid
- For a liquid, $P = P_0 - \rho g y$
  - This is an incompressible fluid
- For a gas, $P = P_0 \, e^{y/y_o}$
  - This is a compressible fluid
- In both cases, the pressure difference between the top and bottom surfaces of an arbitrary volume of fluid must support the weight of the fluid within the volume
- The pressure increases as the depth increases
Pascal’s Principle

• If you change the pressure at a particular point in the fluid, the pressure everywhere in the fluid must change by the same amount

• This is *Pascal’s Principle*
  
  • If the pressure at one location in a closed container of fluid changes, this change is transmitted equally to all locations in the fluid
    • Can apply to liquids or gases
    • The gas must be held in a closed container and be in static equilibrium
Hydraulics

• Pascal’s Principle is the basis of hydraulics
• A simple hydraulic lift is shown
• It is typically filled with oil and has pistons at both ends
  • The pistons are moveable
  • They are tight enough to avoid leaks
Hydraulic Lift

• Assume the car is in static equilibrium
  • It could also be moving very slowly
• The car’s weight is the force acting on the right-hand piston
  • $F_R = m \, g$
• There is an increase in pressure on the right of $\Delta P = \frac{F_R}{A_R}$
• According to Pascal’s Principle, this same pressure will also act on the left-hand piston
Hydraulic Lift, cont.

- From Pascal’s Principle:
  \[ F_L = F_R \frac{A_L}{A_R} \]

- Since the piston on the right is so much larger than the one on the left, a relatively small force on the left can lift a large object on the right.

- The hydraulic lift is a device that amplifies forces.
Suppose the piston on the right moves upward through a small distance $y_R$.

The work done by the right-hand piston is $W_R = F_R y_R$.

The piston on the left moves downward a distance of $y_L$ and the force does work which is $W_L = F_L y_L$. 
• We can expect the hydraulic lift to conserve mechanical energy
• The work done on the left-hand piston will equal the work done by the right-hand piston
• Since the fluid is incompressible, the volume of oil on the left equals the volume of oil on the right
• This leads to confirmation that $W_R = W_L$ or $F_R y_R = F_L y_L$
• A hydraulic device can amplify forces, but it must “de-amplify” the displacement

Section 10.3
Archimedes’ Principle

- Archimedes’ Principle applies to objects immersed in fluids
  - Fully immersed or floating
- Archimedes’ Principle
  - When an object is immersed in a fluid, the fluid exerts an upward force on the object that is equal to the weight of the fluid displaced by the object
- This upward force is called the **buoyant force**
Buoyant Force

- The buoyant force is due to the increase of pressure with depth in a fluid.
- The pressure at the bottom is greater than the pressure at the top.
- This leads to a greater upward force on the bottom surface.
- The buoyant force is the difference between the force on the top and the force on the bottom of the object.

Section 10.4
Example: Archimedes’ Principle, Immersed Object

- The box is fully immersed in the fluid
- The buoyant force equals the weight of the displaced fluid
- Since the box is completely immersed, the volume of the displaced fluid equals the volume of the box
- \( F_B = \rho_L \ g \ V \)
Archimedes’ Principle, Review

- If the weight of the box is exactly equal to $F_B$, the box will remain where it is.
- If the box is heavier than $F_B$, it will sink to the bottom.
- If the weight of the box is smaller than the weight of the fluid it displaces, it will rise to the surface and float.
  - For example, if the box was empty.
- The weight of the displaced fluid equals the weight of the box.
  - This would be the amount below the “waterline”.

Section 10.4
Example: Archimedes’ Principle, Apparent Weight

- The minimum force required to lift a submerged object is called the *apparent weight*.
- The apparent weight of the box is smaller than its true weight by an amount equal to the buoyant force acting on it.
Archimedes’ Principle: Final Notes

• Archimedes’ Principle
  • Applies to all shapes
    • Not just the boxes used in the examples
  • Applies to object immersed in gases as well as liquids
    • Compressible or incompressible fluids
Fluids in Motion

- The general theory of fluids in motion is very complex since the velocity of a fluid usually varies from place to place within the fluid.
- Assumptions made to simplify:
  - The fluid density is constant
    - The fluid is incompressible
  - The flow is steady
    - The velocity is independent of time
    - It can still vary from point to point in space
  - There is no friction
    - No viscosity
  - There are no complex flow patterns
    - Such as turbulence

Section 10.5
Ideal Fluids

- A fluid that satisfies the four assumptions is called an *ideal fluid*
- An ideal fluid can be used to illustrate some key aspects of fluid dynamics
Principle of Continuity

- Consider the velocity through the pipe
  - In the x-direction
  - The fluid enters the pipe on the left with speed $v_L$
  - It exits on the right with speed $v_R$
- The amount of fluid that flows through the pipe must be conserved

Section 10.5
Principle of Continuity, cont.

- The mass of fluid flowing in from the left during a time interval $\Delta t$ must equal the mass flowing out of the right during that time.

- The Principle of Continuity
  - The rate at which the fluid flows into the pipe must equal the rate at which the fluid flows out of the pipe.

- This means that a small-diameter flow stream with a large speed can carry the same amount of fluid per unit time as a large diameter flow stream with small speed.

Section 10.5
Principle of Continuity, Equation

- For an incompressible fluid, the volume that flows into the pipe must equal the volume that flows out
  - Since mass = density x volume and the density is constant
- To analyze: $V_{in} = (v_L \Delta t) A_L$ and $V_{out} = (v_R \Delta t) A_R$
  - Don’t confuse V for volume with v for velocity
- According to the principle of continuity, $V_{in} = V_{out}$
- Simplifying: $v_L A_L = v_R A_R$
  - This is called the equation of continuity
Flow Rate

- The flow rate is the product of the area and speed
- Flow rate = \( Q = vA \)
- The equation of continuity says the incoming flow rate equals the outgoing flow rate
- Units: SI unit for flow rate is \( m^3 / s \)
  - Liters/second are also widely used
  - USC generally uses gallons/minute
Bernoulli’s Equation

• The flow of an ideal fluid through a pipe can be analyzed in terms of work and energy
• We will look at the pressure, speed, and height of the fluid at two different points
• The points are $\Delta t$ apart, with the first point being at $t = 0$
• We will apply the work-energy theorem to get a relationship among the pressure, speed, and height at the two times

Section 10.5
Bernoulli’s Equation, set up

Initial situation, $t = 0$

- Mass = $m$
- Volume $A_1 \Delta x_1$
- $P_1 A_1$
- $\vec{v}_1$
- $h_1$
- $\Delta x_1$

The volume $1$ will move to point $2$

Final situation, time $\Delta t$

- Volume $A_2 \Delta x_2$
- Mass = $m$
- $\vec{v}_2$
- $h_2$
- $\Delta x_2$

Section 10.5
Bernoulli’s Equation, Derived

- Looking at the work:
  - $W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1$
  - $W_2 = -F_2 \Delta x_2 = -P_2 A_2 \Delta x_2$
  - $W_{total} = W_1 + W_2 = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \Delta KE + \Delta PE$

- $\Delta KE + \Delta PE = \frac{1}{2} m v_2^2 + m g h_2 - \left( \frac{1}{2} m v_1^2 + m g h_1 \right)$

- The volume is $A \Delta x$ and the mass can be expressed in terms of the density

- Bernoulli’s Equation is
  \[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \]
Bernoulli’s Equation, Interpreted

- The pressure terms represent energy associated with pressure in the fluid
  - A higher value of $P$ corresponds to higher energy because the fluid can exert a higher force
- $P$ – energy due to pressure
- $\frac{1}{2} \rho v^2$ – kinetic energy
- $\rho g h$ – gravitational potential energy
- Bernoulli’s equation says that the total mechanical energy of the fluid is conserved as it travels from place to place, but some of this energy can be converted from kinetic energy to potential energy or potential to kinetic

Section 10.5
Bernoulli’s Equation – Application (Airplane Wing)

- There is a bulge at the top of the wing
- This causes the speed of the air to be slightly higher over the top of the wing compared to the bottom of the wing
- An increase in speed causes a decrease in pressure
- $F_{\text{total}} = (P_{\text{bot}} - P_{\text{top}}) A$
  - This produces the “lift” on the airplane wing

Section 10.5
Airplane Wings, Details

• There are two pressure terms
  • One is due to difference in air speeds
  • The other is due to the difference in heights between the top and bottom of the wind
    • The high air speeds and small difference in height indicates the pressure difference is due almost entirely to the speed terms in Bernoulli’s equation

• Bernoulli’s equation applied to the airplane wing assumed an incompressible fluid
  • Air is compressible
  • There is about a 15% difference due to differences in density
    • This means this result for lift is accurate to within about 15%
Real Fluids

- Real fluids have frictional forces internal to the fluid and between the fluid and the walls of the container.
- These are called *viscous* forces.
- These forces depend on the speed of the fluid:
  - They are often negligible for small speeds.
- In many cases, however, viscosity is important.
Viscosity

- Consider the fluid flowing through a tube.
- The fluid molecules near the wall of the tube experience strong forces from the molecules in the wall.
- The molecules close to the wall move relatively slowly.

Section 10.6
Viscosity, cont.

- The motion of each layer in the fluid is strongly influenced by the adjacent layers.
- Since the adjacent layers are moving at different speeds, each layer exerts a force on the layers next to it.
- "Thick" fluids experience large forces and have a large viscosity.
  - Honey is an example.
- "Thin" fluids have much smaller viscosities.
  - Water is an example.
Poiseuille’s Law

- For a viscous fluid to move through a tube, the pressure difference between the ends of the tube must overcome the viscous force.
- The average fluid speed is given by Poiseuille’s Law:

\[ v_{\text{ave}} = \frac{P_1 - P_2}{8\eta L} r^2 \]
Poiseuille’s Law, cont.

- The viscosity of the fluid is given the symbol $\eta$
- Values of viscosity are shown in the table
- This average velocity can be used to find the average flow rate of the fluid: $Q = v_{\text{avg}} A$

**TABLE 10.3 Viscosities of Some Common Fluids**

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Viscosity $(N \cdot s/m^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water (20°C)</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Blood (at body temperature, 37°C)</td>
<td>$2.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>Air (20°C)</td>
<td>$1.8 \times 10^{-5}$</td>
</tr>
<tr>
<td>Honey (20°C)</td>
<td>1000</td>
</tr>
<tr>
<td>Methanol (30°C)</td>
<td>$5.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>Ethanol (30°C)</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Stoke’s Law

• The drag force on a spherical object of radius $r$ that moves slowly through a fluid with a viscosity $\eta$ is

$$\vec{F}_{\text{drag}} = -6 \pi \eta r \vec{v}$$

• This is the same viscosity as in Poiseuille’s Law

• The negative sign indicates that the drag force is opposite to the velocity
Two things may happen at the free surface of a fluid:
- Which happens depends on the intermolecular forces.
- In most liquids, the molecules form the shape giving the smallest possible surface area:
  - That shape is a sphere.
- The liquid may also spread out in a flat layer.

Section 10.6
• The fluid has a *surface tension*, \( \gamma \), that is a measure of the energy required to increase the surface area.

• Surface tensions of some fluids are given in the table.

• Surface tension is responsible for effects such as a bug “walking” on the water.

**TABLE 10.4** Surface Tensions of Some Common Liquids at Room Temperature

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Surface Tension (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>( 7.3 \times 10^{-2} )</td>
</tr>
<tr>
<td>Ethanol</td>
<td>( 2.3 \times 10^{-2} )</td>
</tr>
<tr>
<td>Mercury</td>
<td>( 48 \times 10^{-2} )</td>
</tr>
</tbody>
</table>
Capillary Pressure

- The forces between the liquid’s molecules and a wall can also affect the shape of a free liquid surface.
- The surface of the liquid in a capillary tube is not flat.
- The water molecules are attracted to the walls of the tube and this pulls the water up a small amount at the walls.
- This is a concave meniscus.

Section 10.6
• The mercury molecules are attracted more strongly to each other than to the walls.
• This leads to a convex meniscus.
• The force that pulls the fluid along the walls is associated with a capillary pressure.
• The magnitude of the capillary pressure is proportional to the surface tension.
• The capillary pressure of a fluid in a tube of radius \( r \): 
  \[ P_{\text{cap}} = \frac{2\gamma}{r} \]
• This applies to water, other fluids such as mercury are more complicated.
Turbulence

- When fluid velocities are large, the flow patterns can be very complex and fluctuate with time.
- This behavior is called *turbulence*.
- The picture shows vortices produced in the baseball’s wake.

Section 10.7