Chapter 7
Momentum, Impulse, and Collisions
Introduction

- Objects have been treated as “point particles”
  - Mass is located at a single point in space
  - This assumption is very useful
  - This is the correct way to deal with many situations
- Not all types of motion can be dealt with using this approach
- May have to consider the object as an extended object
  - Can imagine the object as a collection of small pieces
  - The pieces can be treated as point particles
  - Have to include forces that exist within the system of interacting particles
Momentum

• The momentum of a particle depends on its mass and velocity

• Momentum is defined as $\mathbf{p} = m \mathbf{v}$
  • The direction of the momentum is the same as the velocity
  • SI unit is kg · m / s

• A particular value of the momentum can be achieved in different ways
  • A small mass moving at a high velocity
  • A large mass moving with a low velocity
Momentum of a System

- To find the total momentum of a system of particles, you need to add the momenta of all the individual particles in the system.

\[ \mathbf{p}_{\text{total}} = \sum_i \mathbf{p}_i = \sum_i m_i \mathbf{v}_i \]

- The particles may be pieces of a solid object or individual particles associated with each other.
Force and Momentum

- Assume the force and acceleration are constant

\[ \vec{F} = m\vec{a} = m \frac{\Delta \vec{V}}{\Delta t} = m \frac{\vec{V}_f - \vec{V}_i}{\Delta t} \]

- Since momentum is mass times velocity, the force can be related to the momentum

\[ \vec{F} \Delta t = m(\vec{V}_f - \vec{V}_i) = \vec{p}_f - \vec{p}_i \]

- *Impulse* = \( \vec{F} \Delta t = \Delta \vec{p} \)

- This is the *impulse theorem*
More About Impulse

• Impulse is a vector quantity
• Its direction is parallel to the total force
Impulse and Variable Forces

- The force does not need to be constant
- The magnitude of the force grows rapidly from zero to a maximum value
- The force then decreases to zero
- Impulse = area under the force-time curve
- Still impulse = $\Delta \vec{p}$
Impulse and Average Force

- It may be difficult to calculate the form of the force-time curve
- Often the time interval is very small
  - Example bat hitting ball
- The average force can be used to find the impulse
  - $\text{impulse} = \bar{F}_{avg} \Delta t = \Delta \vec{p}$
Impulse Revisited

• Impulse is equal to the area under the curve
• The same value of the impulse can be obtained in different ways
  • A large force with a short time
  • A small force with a long time
• Applications include air bags

Section 7.2
Airbag Example

• An example of extending the time is an airbag
  • Your collision with the airbag involves a much longer interaction time than if you were to collide with the steering column
  • This leads to a smaller force
Conservation of Momentum

- Impulse and momentum concepts can be applied to collisions
- The total momentum just before the collision is equal to the total momentum just after the collision
- The total momentum of the system is conserved

Section 7.3
Conservation of Momentum, System

- Conservation of momentum can be applied to systems of many particles
  - The particles may undergo many collisions with each other
  - The system is assumed to be closed
  - The total momentum of the entire system is conserved
  - Remember the total energy is also conserved
- Also applied to solid objects
  - The solid object can be thought of as a collection of many point particles subject to forces (action-reaction pairs) with momentum still conserved
Momentum and External Forces

• The interaction forces between particles in a system do not change the momentum of the system.
• External forces may act from outside the system.
• The external forces may cause the particles to accelerate and therefore the momentum is not conserved.
  • In many cases, the external force is very small when compared to the collision forces.
  • Then assuming momentum is conserved is still a useful way to analyze collisions.
Collisions

• A collision changes the particles’ velocities
• The kinetic energies of the individual particles will also change
• Collisions fall into two categories
  • Elastic collisions
    • The system’s kinetic energy is conserved
  • Inelastic collisions
    • Some kinetic energy is lost during the collision
• Momentum is conserved in both types of collisions
More About Energy in Collisions

- Elastic collisions
  - Kinetic energy is converted into potential energy and then back into kinetic energy
  - So kinetic energy is conserved
- Inelastic collisions
  - If the object does not return the kinetic energy to the system after the collision, the collision is inelastic
  - The kinetic energy after the collision is less than the kinetic energy before the collision
Problem Solving

- **Recognize the principle**
  - The momentum of the system is conserved when the external forces are zero
  - Conservation of Momentum can be applied when the collision force between the particles is much larger than the external forces

- **Sketch the problem**
  - Make a sketch of the system
  - Show the coordinate axes
  - Show the initial and final velocities of the particles in the system
    - When given
Problem Solving, cont.

- **Identify the relationships**
  - Write the conservation of momentum equation for the system
  - Is the kinetic energy conserved?
    - If KE is conserved, then the collision is elastic
      - Write the kinetic energy equation for both particles
      - Use the system of equations to solve for unknown quantities
    - If KE is not conserved, then the collision is inelastic
      - Use the conservation of momentum equation

- **Solve** for the unknown(s)

- **Check**
  - Consider what your answer means
  - Check that the answer makes sense
Identify the System

• When applying the principle of conservation of momentum, it is important to first *identify the system*
• It is usually best to choose the system so that all the important forces act between different parts of the system
• Choose the system so that the external forces are equal to zero
  • Or at least very small
• If the external forces are exactly zero, the total momentum of the system will be conserved exactly
Elastic Collision Example

- Recognize the Principle
  - External forces are zero
  - Total momentum is conserved
- Sketch the problem
  - Shown to the right
  - Everything is along the x-axis

Before collision

After collision

\[ m_1 \quad \vec{v}_{1i} \quad \vec{v}_{2i} \quad m_2 \]

\[ \vec{v}_{1f} \quad \vec{v}_{2f} \quad x \]
Elastic Collision Example, cont.

• Identify the relationships
  • Elastic collision, so kinetic energy is conserved
  • Equations:

    Conservation of momentum

    \[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]

    Conservation of kinetic energy

    \[ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \]

• Solve for the unknowns
Power of Conservation Principles

• The two conservation principles were all that were needed to solve the one-dimensional collision problem
• The collision can be completely solved
• This means we don’t need to know anything about the forces acting during the collision
  • The nature of the interaction forces, the time, etc., have no effect on the outcome of the collision
• The conservation principles completely describe the results
Inelastic Collisions in One Dimension

- In many collisions, kinetic energy is not conserved
  - The KE after the collision is smaller than the KE before the collision
- These collisions are called *inelastic*
- The total energy of the universe is still conserved
  - The “lost” kinetic energy goes into other forms of energy
- Momentum is conserved
- Momentum gives the following equation:
  \[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]
  - Leaves two unknowns

Section 7.4
Completely Inelastic Collisions

- In a completely inelastic collision, the objects stick together.
- They will have the same velocity after the collision.
- Therefore, there is only one unknown and the equation can be solved.

Section 7.4
Kinetic Energy in Inelastic Collisions

• Although kinetic energy is not conserved, total energy is conserved
• The kinetic energy is converted into other forms of energy
  • These could include
    • Heat
    • Sound
    • Elastic potential energy
Collisions in Two Dimensions

- The components of the velocity must be taken into account
- Conservation of momentum includes both components of the velocity
- Follow the general problem solving strategy
  - Include any additional information given
  - Is the collision elastic?
    - If yes, kinetic energy is conserved
    - If not, is there any information about one of the final velocities?
Collision in Two Dimensions Example: Earth-Asteroid

- We want to use a rocket to deflect an incoming asteroid
- The system is two colliding particles
  - Rocket and asteroid
  - External forces are gravity from the Sun and Earth
    - Small compared to the forces involved in the collision, so it is correct to assume momentum is conserved
Collision Example: Earth-Asteroid, cont.

- Choose the initial velocity of the asteroid as the $+y$ direction
- Choose the initial velocity of the rocket as the $+x$ direction
- The rocket and the asteroid stick together, so it is a completely inelastic collision
- Write the conservation of momentum equations for each direction
- Solve for the final velocity
Conservation of Momentum and Analysis of Inelastic Events

• In all the previous examples, mass has been constant
• The principle of conservation of momentum can be applied in situations where the mass changes
Example: Changing Mass

- Treat the car as an object whose mass changes.
- Can be treated as a one-dimensional problem.
- The car initially moves in the x-direction.
- The gravel has no initial velocity component in the x-direction.
- The gravel remains in the car, the total mass of the object is the mass of the car plus the mass of the gravel.

Section 7.5
Changing Mass Example, cont.

- Solving for the final velocity gives

\[ v_f = \frac{m_c v_o}{m_c + m_g} \]

- Momentum is *not* conserved in the y-direction
  - There are external forces acting on the car and gravel
Problem Solving Strategy – Inelastic Events

- **Recognize the Principle**
  - The momentum of a system in a given direction is conserved only when the net external force in that direction is zero or negligible

- **Sketch the Problem**
  - Include a coordinate system
  - Use the given information to determine the initial and final velocity components
    - When possible
Problem Solving Strategy – Inelastic Events, cont.

- **Identify the Relationships**
  - Express the conservation of momentum condition for the direction(s) identified
  - Use the given information to determine the increase or decrease of the kinetic energy

- **Solve**
  - Solve for the unknown quantities
    - Generally the final velocity

- **Check**
  - Consider what the answer means
  - Does the answer make sense
Inelastic Processes and Collisions

- Most inelastic processes are similar to collisions
- Total momentum is conserved
- The separation is just like a collision in reverse
Example: Asteroid Splitting

- Instead of using a rocket to collide with an asteroid, we could try to break it apart
- A bomb is used to separate the asteroid into parts
- Assuming the masses of the pieces are equal, the parts of the asteroid will move apart with velocities that are equal in magnitude and opposite in direction
It is important to distinguish between internal and external forces:

- Internal forces act between the particles of the system.
- External forces come from outside the system.
- The total force is the sum of the internal and external forces in the system.

The force exerted by $m_1$ on $m_2$ and the force exerted by $m_2$ on $m_1$ are internal forces.

Section 7.6
Forces, cont.

- The internal forces come in action-reaction pairs
- For the entire system, \( \sum \vec{F}_{\text{int}} = 0 \)
- For the entire system, \( \sum \vec{F} = M_{\text{tot}} \vec{a}_{CM} \)
  - The “cm” stands for center of mass
  - This is the same form as Newton’s Second Law for a point particle
What Is Center of Mass?

• The center of mass can be thought of as the balance point of the system.

\[ x_{CM} \] is the “balance point” of the system.

Section 7.6
Calculating Center of Mass

• The x- and y-coordinates of the center of mass can be found by

\[
x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\sum_i m_i x_i}{M_{tot}}
\]

\[
y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{\sum_i m_i y_i}{M_{tot}}
\]

• In three dimensions, there would be a similar expression for \(z_{CM}\)

• To apply the equations, you must first choose a coordinate system with an origin

• The values of \(x_{CM}\) and \(y_{CM}\) refer to that coordinate system
Example: Center of Mass

- All the point particles must be included in the center of mass calculation
  - This can become complicated
- For a symmetric object, the center of mass is the center of symmetry of the object
- The center of mass need not be located inside the object

![Diagram showing center of mass calculation](image)
Motion of the Center of Mass

- The two skaters push off from each other.
- No friction, so momentum is conserved.
- The center of mass does not move although the skaters separate.
- Center of mass motion is caused only by the external forces acting on the system.

Section 7.6
The complicated motion of an object can be viewed as a combination of translational motion and rotational motion.

- Translational motion is often referred to as linear motion.
Translational Motion, cont.

- The translational motion of any system of particles is described by Newton’s Second Law as applied to an equivalent particle of mass $M_{\text{tot}}$
  - The equivalent particle is located at the center of mass
  - We can treat the motion as if all the mass were located at the center of mass
  - The center of mass motion will be precisely the same as that of a point particle
Example: Bouncing Ball and Momentum Conservation

- Consider the example of the motion of a pool ball when colliding with the edge of the table
  - Interested in determining the ball’s velocity after the collision
  - There is no force acting on the ball in the x-direction
  - The normal force of the edge of the table exerts an impulse on the ball in the y-direction
Example: Pool Ball, cont.

- Apply conservation of momentum to the x-direction
- Solving the resulting equations for the final velocity gives \( v_{fy} = \pm v_{iy} \)
  - Choose the negative
- The final velocity is directed opposite to the initial velocity
  - The outgoing angle is equal to incoming angle

Top view of a cue ball bouncing from the edge of a pool table.

Section 7.7
Importance of Conservation Principles

- Two conservation principles so far
  - Conservation of Energy
  - Conservation of Momentum
- Allow us to analyze problems in a very general and powerful way
  - For example, collisions can be analyzed in terms of conservation principles that completely determine the outcome
  - Analysis of the interaction forces was not necessary
Importance of Conservation Principles, cont.

• Conservation principles are extremely general statements about the physical world
  • Conservation principles can be used where Newton’s Laws cannot be used
• Careful tests of conservation principles can sometimes lead to new discoveries
  • Example is the discovery of the neutrino