Chapter 18

Electric Potential

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Electric Potential

• Electric forces can do work on a charged object
• Electrical work is related to electric potential energy
  • Analogous in many ways to gravitational potential energy
• Electric potential is closely related to electric potential energy
• Conservation of energy will be revisited
• The ideas of forces, work, and energy will be extended to electric forces and systems
Electric Potential Energy

• A point charge in an electric field experiences a force: $\vec{F} = q\vec{E}$

• Assume the charge moves a distance $\Delta x$

• The work done by the electric force on the charge is $W = F \Delta x$

• The electric force is conservative, so the work done is independent of the path

• If the electric force does an amount of work $W$ on a charged particle, there is an accompanying change in electric potential energy

Section 18.1
Electric Potential Energy, cont.

- The electric potential energy is denoted at $\text{PE}_{\text{elec}}$
- The change in electric potential energy is $\Delta \text{PE}_{\text{elec}} = -W = -F \Delta x = -q F \Delta x$
- The change in potential energy depends on the endpoints of the motion, but not on the path taken

When $q$ is positive, the electric force $\vec{F}$ is parallel to $\vec{E}$.

Section 18.1
Potential Energy – Stored Energy

• A positive amount of energy can be stored in a system that is composed of the charge and the electric field
• Stored energy can be taken out of the system
  • This energy may show up as an increase in the kinetic energy of the particle

To move $q$ from $B$ to $A$, an external agent must exert a force $\vec{F}_{\text{ext}}$ to overcome the electric force.

If the charge moves back from $A$ to $B$, the system gives up potential energy.

Section 18.1
Potential Energy – Two Point Charges

- From Coulomb’s Law:
  \[ F = \frac{kq_1q_2}{r^2} \]
- If they are like charges, they will repel
  - Solid line on the graph
- If they are unlike charges, they will attract
  - Dotted line on the curve

If \( q_1 \) and \( q_2 \) are both positive, the electric force is repulsive and the work \( W_E < 0 \).
Two Point Charges, cont.

- The electric potential energy is given by
  \[ PE_{\text{elec}} = \frac{kq_1q_2}{r} \text{ or } \frac{q_1q_2}{4\pi\varepsilon_0 r} \]

- Note that \( PE_{\text{elec}} \) varies as \( 1/r \) (C) while the force varies as \( 1/r^2 \) (B)
Two Point Charges, final

- $P_{E_{elec}}$ approaches zero when the two charges are very far apart
  - $r$ becomes infinitely large
- The electric force also approaches zero in this limit
- The changes in potential energy are important

\[
\Delta P_{E_{elec}} = P_{E_{elec,f}} - P_{E_{elec,i}} = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}
\]
Electric and Gravitational PEs

- Both vary as $1/r$
- The electric potential energy falls to zero when the separation between two charges is infinite
- The gravitational potential energy falls to zero when the separation between two masses is infinite
PE_{elec} and Superposition

• The results for two point charges can be extended by using the superposition principle.

• If there is a collection of point charges, the total potential energy is the sum of the potential energies of each pair of charges.

• Complicated charge distributions can always be treated as a collection of point charges arranged in some particular manner.

• The electric forces between a collection of charges will always be conservative.
Problem Solving Strategy

- **Recognize the principle**
  - Determine the system whose energy is conserved
  - It could be two point charges, a collection of point charges, or a single charge in an electric field

- **Sketch the problem**
  - Draw a figure showing the initial and final states of all the charges in the system

- **Identify the relationships**
  - Identify the change in electric potential energy
Problem Solving Strategy, cont.

- **Solve**
  - The approach to solve the problem will depend on the particular problem
    - If there are no external forces, energy is conserved
    - If there is a non-zero external force, the work done by that force will equal the change in potential plus kinetic energy
    - The electric force is conservative, so the work done by the electric force is independent of the path and equals $-\Delta PE_{elec}$

- **Check**
  - Consider what your answer means
  - Make sure your answer makes sense
Electric Potential: Voltage

- Electric potential energy can be treated in terms of a test charge
  - Similar to the treatment of the electric field produced by a charge
- A test charge is placed at a given location and its potential energy is measured
Electric Potential Defined

• The electric potential is defined as

\[ V = \frac{PE_{elec}}{q} \]

• Often referred as “the potential”

• SI unit is the Volt, V
  • Named in honor of Alessandro Volta
  • 1 V = 1 J/C = 1 N \cdot m / C

• The units of the electric field can also be given in terms of the Volt: 1 V / m = 1 N / C
The diagrams show the electric potential in a television picture tube (a CRT).

Two parallel plates are charged and form an “accelerator”.

Section 18.2
Accelerating Charged Particles, Analysis

- Assumptions and set up
  - A positive test charge
  - The electric field has a magnitude of $E$
  - The plates are separated by a distance $L$
- The electric force on the test charge is $F = q\, E$ and the charge moves a distance $L$
- The work done on the test charge by the electric field is $W = -q\, E\, L$
- There is a potential difference between the plates of

\[ \Delta V = \frac{\Delta PE_{\text{elec}}}{q} = -E\, L \]
Accelerating an Electron

- Reverse the direction of the electric field
  - This compensates for the electron’s negative charge and allows it to still accelerate in the same direction as the original test charge
- Assume the electron’s initial velocity is zero and apply conservation of energy
  - $KE_i + PE_{elec, i} = KE_f + PE_{elec, f}$ and $KE_f = -\Delta PE_{elec}$
  - In terms of potential, $\Delta PE_{elec} = q \Delta V = -e \Delta V$
  - Solving for the final speed of the electron gives
    $$v_f = \sqrt{\frac{2e\Delta V}{m}}$$
Electric Potential and Field

- The electric field may vary with position.
- The magnitude and direction of the electric field are related to how the electric potential changes with position.
- $\Delta V = -E \Delta x$ or $E = -\Delta V / \Delta x$

The electric field is proportional to the rate of change of $V$ with position.

$$|\vec{E}| = -\frac{\Delta V}{\Delta x}$$

Section 18.2
Electron-Volt

• Energy units can be derived from the product of the electric potential and the charge
• Often we are concerned with the energy gained or lost as an electron or ion moves through a potential difference
• It is convenient to define a unit of energy called the **electron-volt** (eV)
  • One electron volt is defined as the amount of energy gained or lost when an electron travels though a potential difference of 1 V
  • $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$
Electric Potential Due to Point Charge

- The electric potential at a distance $r$ away from a single point charge $q$ is given by
  \[ V = \frac{kq}{r} \] or \[ V = \frac{q}{4\pi\varepsilon_0 r} \]

- The solid curve shows the result for a positive charge, the dotted line is for a negative charge.
Changes in Potential

• Since changes in potential (and potential energy) are important, a “reference point” must be defined
• The standard convention is to choose $V = 0$ at $r = \infty$
• In many problems, the Earth may be taken as $V = 0$
  • This is the origin of the term electric ground
  • The convention is that ground is where $V = 0$
Electric Field Near a Metal

- A solid metal sphere carries an excess positive charge, \( q \)
- The excess charge resides on the surface
- The field inside the metal is zero
The field outside any spherical ball of charge is given by

\[ E = \frac{kq}{r^2} = \frac{q}{4\pi \varepsilon_0 r^2} \]

- \( r \) is the distance from the center of the ball.
- This holds outside the sphere only, the field is still zero inside the sphere.
Potential Near a Metal

- Since the field outside the sphere is the same as that of a point charge, the potential is also the same

\[ V = \frac{kq}{r} \text{ for } r > r_{\text{sphere}} \]

- The potential is constant inside the metal

Section 18.2
Potential, cont.

• The potential at all locations in the metal sphere must equal the potential at the outer edge of the sphere

• Everywhere inside a metal sphere

\[ V = \frac{kq}{r_{\text{sphere}}} \]
Summary, Field & Potential

• The excess charge on a metal in equilibrium always resides at the surface.
• Because the electric field inside a metal is zero, the potential is constant throughout a piece of metal.
• The electric field lines approach perpendicular to the surface of the metal sphere.
  • This is true for any shape, spherical or not.
Lightning Rod

- A sketch of the electric field near a lightning rod
- The field lines are perpendicular to the surface of the metal rod
- The field lines are largest near the sharp tip of the rod and smaller near the flat side
Lightning Rod Model

- A lightning rod can be modeled as two metal spheres connected by a metal wire.
- The smaller sphere represents the tip and the larger sphere represents the flatter body.

\[ V_1 \approx \frac{kQ_1}{r_1} \]
\[ V_2 \approx \frac{kQ_2}{r_2} \]

\[ V_1 = V_2 \] because the spheres are connected by a wire.

Section 18.2
Lightning Rod Analysis

- Assume a net positive charge on the system.
- Because the spheres are connected by a wire, they are a single piece of metal.
- They must have the same potential.

\[ V_{\text{sphere } 1} = \frac{kQ_1}{r_1} = V_{\text{sphere } 2} = \frac{kQ_2}{r_2} \]

- The charges are related by:

\[ \frac{Q_1}{r_1} = \frac{Q_2}{r_2} \]
Lightning Rod Analysis, cont.

- The relative magnitudes of the electrical fields can also be found

\[ \frac{E_1}{E_2} = \frac{r_2}{r_1} \]

- The field is larger near the surface of the smaller sphere

- This means the field is largest near the sharp edges of the lightning rod

- The large electric field causes the nearby air molecules to be ionized

- The electrons and ions near the tip are able to carry charge from the air (lightning) to the rod

Section 18.2
Discontinuous Jumps

- When moving a particle between two points A and B that are a distance $\Delta x$ apart, $F \Delta x = W = -\Delta PE$
- This gives
  \[ F = -\frac{\Delta PE}{\Delta x} \]
- If $\Delta x$ is very small, the $\Delta PE$ must also be very small or else $F$ would be very large
- As the distance becomes infinitesimal this means that the electric potential energy cannot jump discontinuously when a particle moves between two points
- The electric potential also cannot jump discontinuously
Shielding the Electric Field

• A region can be designed where the electric field is zero
  • Go inside a metal cavity
• Example: Your car acts like the piece of metal to shield you from a lightning strike
Equipotential Surfaces

A useful way to visualize electric fields is through plots of *equipotential surfaces*:
- Contours where the electric potential is constant
- Equipotential lines are in two-dimensions
- In B, several surfaces are shown at constant potentials
Equipotential Surfaces, cont.

- Equipotential surfaces are always perpendicular to the direction of the electric field.
  - Due to the relationship between $E$ and $V$
- For motion parallel to an equipotential surface, $V$ is constant and $\Delta V = 0$.
  - The electric field component parallel to the surface is zero.

$\Delta V = 0$, so $E_x = 0$
Equipotential Surface – Point Charge

- The electric field lines emanate radially outward from the charge
  - The equipotential surfaces are perpendicular to the field
- The equipotential surfaces are a series of concentric spheres
  - Different spheres correspond to different values of $V$

Section 18.3
Capacitors

- A capacitor can be used to store charge and energy
- This example is a parallel-plate capacitor
- Connect the two metal plates to wires that can carry charge on or off the plates
Capacitors, cont.

- Each plate produces a field $E = \frac{Q}{2 \varepsilon_0 A}$
- In the region between the plates, the fields from the two plates add, giving
  \[ E = \frac{Q}{\varepsilon_0 A} \]
  - This is the electric field between the plates of a parallel-plate capacitor
- There is a potential difference across the plates
  \[ \Delta V = E d \] where $d$ is the distance between the plates
Capacitance Defined

- From the equations for electric field and potential,

\[ \Delta V = Ed = \frac{Qd}{\varepsilon_0 A} \]

- Capacitance, \( C \), is defined as

\[ C = \frac{Q}{\Delta V} \text{ therefore } \Delta V = \frac{Q}{C} \]

- In terms of \( C \),

\[ C = \frac{\varepsilon_0 A}{d} \text{ (parallel-plate capacitor)} \]

- \( A \) is the area of a single plate and \( d \) is the plate separation

Section 18.4
Capacitance, Notes

- Other configurations will have other specific equations
- All will employ two plates of some sort
- In all cases, the charge on the capacitor plates is proportional to the potential difference across the plates
- SI unit of capacitance is Coulombs / Volt and is called a Farad
  - 1 F = 1 C/V
  - The Farad is named in honor of Michael Faraday
Storing Energy in a Capacitor

- Applications using capacitors depend on the capacitor’s ability to store energy and the relationship between charge and potential difference (voltage).
- When there is a nonzero potential difference between the two plates, energy is stored in the device.
- The energy depends on the charge, voltage, and capacitance of the capacitor.
Energy in a Capacitor, cont.

- To move a charge $\Delta Q$ through a potential difference $\Delta V$ requires energy.
- The energy corresponds to the shaded area in the graph.
- The total energy stored is equal to the energy required to move all the packets of charge from one plate to the other.

\[ \Delta V = \text{voltage across capacitor} \]

\[ \Delta Q \]

\[ \text{Total area} = P E_{\text{elec}} = P E_{\text{cap}} = \frac{1}{2} Q \Delta V \]

\[ \text{Area} = (\Delta Q)(\Delta V) = \text{energy to move } \Delta Q \text{ across plates} \]

Section 18.4
Energy in a Capacitor, Final

- The total energy corresponds to the area under the $\Delta V$ – $Q$ graph
- Energy = Area = $\frac{1}{2} Q \Delta V = P E_{cap}$
  - $Q$ is the final charge
  - $\Delta V$ is the final potential difference
- From the definition of capacitance, the energy can be expressed in different forms

$$PE_{cap} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \frac{Q^2}{C}$$

- These expressions are valid for all types of capacitors

Section 18.4
Capacitors in Series

- When dealing with multiple capacitors, the equivalent capacitance is useful.
- In series:
  - \( \Delta V_{\text{total}} = \Delta V_{\text{top}} + \Delta V_{\text{bottom}} \)
Capacitors in Series, cont.

- Find an expression for $C_{total}$

\[ \Delta V_{total} = \frac{Q}{C_{top}} + \frac{Q}{C_{bottom}} \]

and $C_{total} = \frac{Q}{V_{total}}$

Rearranging,

\[ \frac{1}{C_{total}} = \frac{1}{C_{top}} + \frac{1}{C_{bottom}} \]
Capacitors in Series, final

- The two capacitors are *equivalent* to a single capacitor, $C_{\text{equiv}}$
- In general, this equivalent capacitance can be written as

\[
\frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2}
\]
Capacitors in Parallel

- Capacitors can also be connected in parallel.
- In parallel:
  - $Q_{\text{total}} = Q_1 + Q_2$; $V_1 = V_2$ and $C_{\text{equiv}} = C_1 + C_2$
Combinations of Three or More Capacitors

- For capacitors in parallel: \( C_{\text{equiv}} = C_1 + C_2 + C_3 + \ldots \)

- For capacitors in series: \( \frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \)

- These results apply to all types of capacitors

- When a circuit contains capacitors in both series and parallel, the above rules apply to the appropriate combinations

- A single equivalent capacitance can be found
Dielectrics

- Most real capacitors contain two metal "plates" separated by a thin insulating region
  - Many times these plates are rolled into cylinders
- The region between the plates typically contains a material called a **dielectric**
Dielectrics, cont.

- Inserting the dielectric material between the plates changes the value of the capacitance.
- The change is proportional to the **dielectric constant,** $\kappa$.
- If $C_{\text{vac}}$ is the capacitance without the dielectric and $C_d$ is with the dielectric, then $C_d = \kappa C_{\text{vac}}$.
- Generally, $\kappa > 1$, so inserting a dielectric increases the capacitance.
- $\kappa$ is a dimensionless factor.
Dielectrics, final

- When the plates of a capacitor are charged, the electric field established extends into the dielectric material.
- Most good dielectrics are highly ionic and lead to a slight change in the charge in the dielectric.
- Since the field decreases, the potential difference decreases and the capacitance increases.

Section 18.5
Dielectric Summary

• The results of adding a dielectric to a capacitor apply to any type capacitor
• Adding a dielectric increases the capacitance by a factor $\kappa$
• Adding a dielectric reduces the electric field inside the capacitor by a factor $\kappa$
  • The actual value of the dielectric constant depends on the material
    • See table 18.1 for the value of $\kappa$ for some materials
Dielectric Breakdown

• As more and more charge is added to a capacitor, the electric field increases
• For a capacitor containing a dielectric, the field can become so large that it rips the ions in the dielectric apart
  • This effect is called **dielectric breakdown**
• The free ions are able to move through the material
  • They move rapidly toward the oppositely charged plate and destroy the capacitor
• The value of the field at which this occurs depends on the material
  • See table 18.1 for the values for various materials

Section 18.5
Lightning

- During a lightning strike, large amounts of electric charge move between a cloud and the surface of the Earth, or between clouds.
- There is a dielectric breakdown of the air.
Lightning, cont.

- Most charge motion involves electrons
  - They are easier to move than protons
- The electric field of a lightning strike is directed from the Earth to the cloud
- After the dielectric breakdown, electrons travel from the cloud to the Earth
Lightning, final

- In a thunderstorm, the water droplets and ice crystals gain a negative charge as they move in the cloud.
- They carry the negative charge to the bottom of the cloud.
  - This leaves the top of the cloud positively charged.
- The negative charges in the bottom of the cloud repel electrons from the Earth’s surface.
- This causes the Earth’s surface to be positively charged and establishes an electric field similar to the field between the plates of a capacitor.
- Eventually the field increases enough to cause dielectric breakdown, which is the lightning bolt.
Biological Applications

- In the 1790’s Louis Galvani showed that nerves and muscles used electrical potential.
- A defibrillator uses an externally applied potential to shock the heart into normal beating.
- An electrocardiogram (ECG or EKG) monitors potential at various points in the chest to show heart movements.
- An electroencephalogram (EEG) detects potentials in the brain.
Electric Potential Energy Revisited

• One way to view electric potential energy is that the potential energy is stored in the electric field itself
  • Whenever an electric field is present in a region of space, potential energy is located in that region
• The potential energy between the plates of a parallel plate capacitor can be determined in terms of the field between the plates:

\[ PE_{elec} = \frac{1}{2} Q\Delta V = \frac{1}{2} QEd = \frac{1}{2} \varepsilon_o E^2 (Ad) \]

where Ad is the volume of the region
Electric Potential Energy, final

- The **energy density** of the electric field can be defined as the energy / volume:

\[ u_{\text{elec}} = \frac{1}{2} \varepsilon_0 E^2 \]

- These results give the energy density for any arrangement of charges.
- Potential energy is present whenever an electric field is present.
  - Even in a region of space where no charges are present.