1. What is the probability that the first raindrop lands in the shaded region?

*Hint:* View a raindrop as a single point in the triangle. What is the sample space? What is the event? How are events to be measured?

**Solution:** The sample space consists of the three subtriangles in which the drop can land. The event is the shaded subtriangle. The subtriangle have equal area, so they are all equally likely, and we measure by counting subtriangles. Since two of the three subtriangles is shaded, the probability is $\frac{2}{3} \approx 0.667$.

2. What is the probability that both raindrops land in the shaded region?

**Solution:** There are 3 possible outcomes (subsquares) for the first drop. For each of those outcomes, there are 3 possible outcomes for the second drop. This gives a sample space of size $3 \times 3 = 9$. Once again the outcomes are equally likely, so counting is our measure. The event (two drops in the shaded region) can happen in 4 of these ways (two choices of subtriangle for each drop), so the probability is $\frac{4}{9} \approx 0.444$.

3. What is the probability that no raindrops land in the shaded region?

**Solution:** As in the previous problem, we have a sample space of size 9. The event (two drops in the unshaded white region) can happen in only 1 of these ways, so the probability is $\frac{1}{9} = 0.111$.

4. What is the probability that at least one raindrop lands in the shaded region?

**Solution #1:** The event that at least one raindrop lands in the shaded region is the complement of the event that no raindrops land in the shaded region. So this problem asks for the complement of the event in the previous problem, and the probability is $1 - \frac{1}{9} = \frac{8}{9} \approx 0.889$.

**Solution #2:** Let $D_1$ be the event that the first drop lands in the shaded region. Let $D_2$ be the event that the second drop lands in the shaded region.

We need to find $P(D_1 \cup D_2)$, the probability of the union of these events ($D_1$ or $D_2$ or both).

From Problem 1 above we know that $P(D_1) = P(D_2) = \frac{2}{3}$, and from Problem 2 we know that $P(D_1 \cap D_2) = \frac{4}{9}$ (the probability of $D_1$ and $D_2$). From the inclusion-exclusion formula we have

$$P(D_1 \cup D_2) = P(D_1) + P(D_2) - P(D_1 \cap D_2) = \frac{2}{3} + \frac{2}{3} - \frac{4}{9} = \frac{6 + 6 - 4}{9} = \frac{8}{9}$$

once again.

5. What is the probability that exactly one raindrop lands in the shaded region?

**Solution #1:** If the first raindrop falls in the shaded region, there are 2 choices of location for the this first drop, and then one choice for the second drop (the white subtriangle).

If the second raindrop falls in the shaded region, there are 2 choices of location for the second drop, and then one choice for the first drop (the white subtriangle).

This gives a total of $2 + 2 = 4$ ways for exactly one drop to land in the shaded region, and the probability is $\frac{4}{9} \approx 0.444$.

**Solution #2:** This event is the complement of the union of two events in parts 2 & 3 above (both in the shaded region or none). So the probability is the complement of the sum of the probabilities we computed above; that is, $1 - \left(\frac{2}{9} + \frac{2}{9}\right) = \frac{5}{9} \approx 0.444$. 

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Read the problems carefully and show your work.

Two raindrops randomly fall on a triangular region. The region is divided into 3 congruent subtriangles as shown.