1. An urn holds 5 red, 6 blue, and 7 green marbles.
(a) Let $X$ be the number of blue marbles that result if 3 marbles are drawn one at a time at random without replacement from the urn. Find a formula for $f_X(k)$.
(b) Let $Y$ be the number of blue marbles that result if 3 marbles are drawn one at a time at random with replacement from the urn. Find a formula for $f_X(k)$.

2. Kitty generates random numbers between 1 and 6 by rolling a fair cubical die.
Caleb generates random numbers between 1 and 6 by tossing a fair coin five times, counting the heads, and adding 1 to the result.
Ursula generates random numbers between 1 and 6 by tossing a fair coin three times to generate a binary number with 3 bits ($H = 1$, $T = 0$). If she gets 000 or 111 she discards the result and flips 3 more times. Otherwise, she accepts the binary number resulting from the 3 tosses.

Let $K$, $C$, and $U$ be the random variables describing the outcomes of these 3 random number generators. Describe the three probability functions $f_K$, $f_C$, and $f_U$.

3. Let $X$ be a uniform continuous random variable on the interval $[2, 8]$.
(a) What is $P(X = 4)$?
(b) What is $P(X \leq 4)$?
(c) What is $P(4 \leq X \leq 7)$?
(d) Find a formula for $f_X(x)$.

4. A point $v = (x, y)$ is chosen uniformly at random from the triangular region with vertices at the points $(0, 0)$, $(2, 0)$, and $(2, 5)$. Let $X$ denote the $x$-coordinate of the point $v$. Let $Y$ denote the $y$-coordinate of the point $v$.
(a) Find a formula for $F_X(x)$.
(b) Find a formula for $F_Y(y)$.
(c) What is $P(X \geq 1)$?
(d) What is $P(X \geq Y)$?

Hint: Draw a picture of the triangle and the regions inside described by each of these events.

Answers:

\[
\begin{align*}
\frac{1}{2} = & \begin{cases} 
\frac{1}{2} & \text{if } x \in [2, 4] \\
0 & \text{otherwise}
\end{cases} \\
\frac{2}{5} = & \begin{cases} 
\frac{1}{5} & \text{if } y \in [0, 2] \\
\frac{4}{5} & \text{if } y \in [2, 5] \\
0 & \text{otherwise}
\end{cases} \\
0 = & \begin{cases} 
\frac{1}{6} & \text{if } \frac{1}{2} - \frac{2}{5}y \leq \frac{1}{2} \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\]

Caleb’s distribution is not uniform. Some outcomes are more likely than others. In fact, this is a binomial distribution.

Kitty’s distribution is uniform, so that $f_X(x) = \frac{1}{5}$ for $x \in [2, 4]$ and $f_X(x) = \frac{4}{5}$ for $x \in [2, 5]$. This is a uniform distribution.

Ursula’s distribution is not uniform. Some outcomes are more likely than others. In fact, this is a hypergeometric distribution.

The process is equivalent to tossing a biased coin three times, where $H = 1$ and $T = 0$. The probability of heads is $\frac{1}{2}$.

This is a binomial distribution.

\[
\begin{align*}
\frac{1}{2} = & \begin{cases} 
\frac{1}{2} & \text{if } \frac{1}{2} - \frac{2}{5}y \leq \frac{1}{2} \\
0 & \text{otherwise}
\end{cases} \\
\frac{2}{5} = & \begin{cases} 
\frac{1}{5} & \text{if } y \in [0, 2] \\
\frac{4}{5} & \text{if } y \in [2, 5] \\
0 & \text{otherwise}
\end{cases} \\
0 = & \begin{cases} 
\frac{1}{6} & \text{if } \frac{1}{2} - \frac{2}{5}y \leq \frac{1}{2} \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\]