PROBLEM 8.31

A 6-kip force is applied to the machine element AB as shown. Knowing that the uniform thickness of the element is 0.8 in., determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.

SOLUTION

Thickness = 0.8 in.

At the section containing points a, b, and c,

\[ P = 6 \cos 35^\circ = 4.9149 \text{ kips} \quad V = 6 \sin 35^\circ = 3.4415 \text{ kips} \]

\[ M = (6 \sin 35^\circ)(16) - (6 \cos 35^\circ)(8) = 15.744 \text{ kip} \cdot \text{in} \]

\[ A = (0.8)(3.0) = 2.4 \text{ in}^2 \]

\[ I = \frac{1}{12} (0.8)(3.0)^3 = 1.80 \text{ in}^4 \]

(a) At point a:

\[ \sigma_x = \frac{P}{A} \frac{Mc}{I} = \frac{4.9149}{2.4} \frac{(15.744)(1.5)}{1.80} \sigma_x = -11.07 \text{ ksi} \]

\[ \tau_{xy} = 0 \]

(b) At point b:

\[ \sigma_x = \frac{P}{A} \frac{Mc}{I} = \frac{4.9149}{2.4} \]

\[ \tau_{xy} = \frac{3V}{2A} = \frac{3}{2} \frac{3.4415}{2.4} \]

\[ \tau_{xy} = 2.15 \text{ ksi} \]

(c) At point c:

\[ \sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(15.744)(1.5)}{1.80} \]

\[ \sigma_x = 15.17 \text{ ksi} \]

\[ \tau_{xy} = 0 \]
PROBLEM 8.44

Forces are applied at points A and B of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress at (a) point H, (b) point K.

SOLUTION

At the section containing points H and K,

\[ P = 2500 \text{ lb} \quad \text{(compression)} \]
\[ V_y = -600 \text{ lb} \]
\[ V_x = 0 \]
\[ M_x = (3.5 - 1)(600) = 1500 \text{ lb} \cdot \text{in} \]
\[ M_y = 0 \]
\[ M_z = -(2.5)(600) = -1500 \text{ lb} \cdot \text{in} \]

For semicircle,

\[ Q = \frac{2}{3} c^3 \]
\[ = 42.667 \times 10^{-3} \text{in}^3 \]
PROBLEM 8.44  (Continued)

(a) At point $H$:

\[
\sigma_H = \frac{P}{A} + \frac{Mc}{I} = \frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106 \times 10^{-3}} = 24.87 \times 10^3 \text{ psi}
\]

\[
\tau_H = \frac{T_c}{J} = \frac{(1500)(0.4)}{40.212 \times 10^{-3}} = 14.92 \times 10^3 \text{ psi}
\]

\[
\sigma_{ave} = \frac{24.87}{2} = 12.435 \text{ ksi}
\]

\[
R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423 \text{ ksi}
\]

\[
\sigma_{max} = \sigma_{ave} + R = 31.9 \text{ ksi}
\]

\[
\sigma_{min} = \sigma_{ave} - R = -6.99 \text{ ksi}
\]

(b) At point $K$:

\[
\sigma_K = \frac{P}{A} = \frac{2500}{0.50265} = -4.974 \times 10^3 \text{ psi}
\]

\[
\tau_K = \frac{T_c + VQ}{J} = \frac{(1500)(0.4) + (600)(42.667 \times 10^{-3})}{40.212 \times 10^{-3} + (20.106 \times 10^{-3})(0.8)} = 16.512 \times 10^3 \text{ psi}
\]

\[
\sigma_{ave} = \frac{-4.974}{2} = -2.487 \text{ ksi}
\]

\[
R = \sqrt{\left(\frac{-4.974}{2}\right)^2 + (16.512)^2} = 16.698 \text{ ksi}
\]

\[
\sigma_{max} = \sigma_{ave} + R = 14.21 \text{ ksi}
\]

\[
\sigma_{min} = \sigma_{ave} - R = -19.18 \text{ ksi}
\]

\[
\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 16.70 \text{ ksi}
\]
PROBLEM 8.45

Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.

SOLUTION

Calculate forces and couples at section containing points a, b, and c.

\[ h = 10.5 \text{ in.} \]
\[ P = 50 \text{ kips} \quad V_x = 6 \text{ kips} \quad V_z = 2 \text{ kips} \]
\[ M_{x} = (10.5 - 2)(6) = 51 \text{ kip} \cdot \text{in} \]
\[ M_{z} = (10.5)(2) = 21 \text{ kip} \cdot \text{in} \]

Section properties.
\[ A = (1.8)(4.8) = 8.64 \text{ in}^2 \]
\[ I_{x} = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4 \]
\[ I_{z} = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4 \]

Stresses.
\[ \sigma = -\frac{P}{A} + \frac{M_{x}x}{I_{z}} + \frac{M_{z}z}{I_{x}} \]
\[ \tau = \frac{V_{x}Q}{I_{z}t} \]
PROBLEM 8.45  (Continued)

(a) **Point a:**  \( x = 0, \ z = 0.9 \text{ in.}, \ Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3 \)

\[
\sigma = -\frac{50}{8.64} + 0 + \frac{(21)(0.9)}{2.3328} = 2.31 \text{ ksi} \uparrow
\]
\[
\tau = \frac{6(5.184)}{(16.5888)(1.8)} = 1.042 \text{ ksi} \uparrow
\]

(b) **Point b:**  \( x = 1.2 \text{ in.}, \ z = 0.9 \text{ in.}, \ Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3 \)

\[
\sigma = -\frac{50}{8.64} + \frac{(51)(1.2)}{16.5888} + \frac{(21)(0.9)}{2.3328} = 6.00 \text{ ksi} \uparrow
\]
\[
\tau = \frac{6(3.888)}{(16.5888)(1.8)} = 0.781 \text{ ksi} \uparrow
\]

(c) **Point c:**  \( x = 2.4 \text{ in.}, \ z = 0.9 \text{ in.}, \ Q = 0 \)

\[
\sigma = -\frac{50}{8.64} + \frac{(51)(2.4)}{16.5888} + \frac{(21)(0.9)}{2.3328} = 9.69 \text{ ksi} \uparrow
\]
\[
\tau = 0 \uparrow
\]