14.204 (22.212) Strength of Materials
Test #3: Torsion

You must show all work and units and clearly identify your answers for full credit. You may NOT use the back of the page. Additional paper is available at the front desk upon request. Turn in any additional paper used.

GIVEN:
A torque of $T = 3\, \text{kN\cdot m}$ is applied to the solid brass cylinder shown. Bronze has a shear modulus of 40 GPa, an elastic modulus of 105 GPa, and a Poisson’s ratio of 0.33. The Polar Moment of Inertia for a solid cylinder is $(0.5\pi)(\text{radius})^4$.

REQUIRED:
1. Write the equations and detail the variables for:
   a. The Angle of Twist with respect to material properties and geometry (5 points).
   b. Maximum Shear Stress from torsion loading (5 points).
   c. Shear Stress from torsion loading relative to Maximum Shear Stress (5 points).
   d. Maximum Shear Strain from torsion loading (5 points).
   e. Shear Strain from torsion loading relative to Maximum Shear Strain (5 points).
2. Determine the angle of twist of the cylinder from the provided loading (15 points).
3. Determine the maximum shear strain from the provided loading (15 points).
4. Determine the maximum shear stress from the provided loading (15 points).
5. Determine the shearing stress and strain at point D, which lies on a 15 mm radius circle drawn at the end of the cylinder (30 points).
6. Extra Credit: Determine the torque carried by the portion of the cylinder within the 15 mm radius circle on the end of the cylinder as shown in the provided figure (5 points).

SOLUTION:
#2
\[ \phi = \frac{T_c}{J_G} = \frac{(3 \text{ kN-m})(0.03 \text{ m})}{(1.2724 \times 10^{-6} \text{ m}^4)(40 \times 10^6 \text{ kN/m}^2)} \]

\[ J = \frac{1}{2} \pi R^4 = \frac{1}{2} (\pi)(0.03 \text{ m})^4 \]

\[ J = 1.2724 \times 10^{-6} \text{ m}^4 \]

\[ \phi = 0.01179 \text{ Radians} = 0.676 \text{ Degrees} \quad \text{Answer #2} \]

#3
\[ \gamma_{\text{max}} = \frac{C \phi}{\ell} = \frac{(0.03 \text{ m})(0.01179 \text{ Radians})}{(0.1 \text{ m})} = 1.77 \times 10^{-3} \]

\[ \gamma_{\text{max}} = 1.77 \times 10^{-3} = 0.0177 \quad \text{Answer #3} \]

#4
\[ T_{\text{max}} = \frac{T_c}{J} = \frac{(3 \text{ kN-m})(0.03 \text{ m})}{1.2724 \times 10^{-6} \text{ m}^4} = 70732 \text{ kN} = 70.7 \text{ MPa} \]

\[ T_{\text{max}} = 70.7 \text{ MPa} \]

\[ T = \ell T_{\text{max}} \]

\[ T = 0.03 \text{ m}(70.7 \text{ MPa}) \]

\[ T = 2.121 \text{ MPa} \]

\[ T_{\text{max}} = 35.35 \text{ MPa} \quad \text{Answer #4} \]

#5
\[ \gamma = \frac{\ell}{\ell} \gamma_{\text{max}} \]

\[ \gamma = \frac{0.0015 \text{ m}}{0.03 \text{ m}}(0.0177) \]

\[ \gamma = 8.85 \times 10^{-3} \quad \text{Answer #5} \]

#6
\[ T_0 = \frac{T_0}{\ell_0} \]

\[ T_0 = \frac{J_0 T_0}{\ell_0} \]

\[ J_0 = \frac{\ell}{2} (0.015 \text{ m})^4 \]

\[ T_0 = \frac{(1/2)(0.015 \text{ m})^4(35.35 \text{ MPa})}{(0.015 \text{ m})} = 0.187 \text{ kN-m} = 187 \text{ N-m} \]

\[ T_0 = 0.187 \text{ kN-m} \quad \text{Answer #6} \]