PROBLEM 2.61

A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a \( \frac{5}{8} \)-in.-diameter rod and it is subjected to an 800-lb tensile force. Knowing that an elongation of 0.45 in. and a decrease in diameter of 0.025 in. are observed in a 5-in. gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson’s ratio for the material.

SOLUTION

\[
A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left( \frac{5}{8} \right)^2 = 0.306796 \text{ in}^2
\]

\[P = 800 \text{ lb}\]

\[
\sigma_y = \frac{P}{A} = \frac{800}{0.306796} = 2.6076 \times 10^3 \text{ psi}
\]

\[
\varepsilon_y = \frac{\delta_y}{L} = \frac{0.45}{5.0} = 0.090
\]

\[
\varepsilon_x = \frac{\delta_x}{d} = \frac{-0.025}{0.625} = -0.040
\]

\[
E = \frac{\delta_y}{\varepsilon_y} = \frac{2.6076 \times 10^3}{0.090} = 28.973 \times 10^3 \text{ psi}
\]

\[E = 29.0 \times 10^3 \text{ psi} \uparrow\]

\[
\nu = \frac{\varepsilon_x}{\varepsilon_y} = \frac{-0.040}{0.090} = 0.44444
\]

\[\nu = 0.444 \uparrow\]

\[
\sigma = \frac{E}{2(1 + \nu)} = \frac{28.973 \times 10^3}{(2)(1 + 0.44444)} = 10.0291 \times 10^3 \text{ psi}
\]

\[\sigma = 10.03 \times 10^3 \text{ psi} \uparrow\]
PROBLEM 2.67

The brass rod $AD$ is fitted with a jacket that is used to apply a hydrostatic pressure of 48 MPa to the 240-mm portion $BC$ of the rod. Knowing that $E = 105$ GPa and $v = 0.33$, determine $(a)$ the change in the total length $AD$, $(b)$ the change in diameter at the middle of the rod.

SOLUTION

$$\sigma_x = \sigma_z = -p = -48 \times 10^6 \text{ Pa}, \quad \sigma_y = 0$$

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z)$$

$$= \frac{1}{105 \times 10^9} \left[ -48 \times 10^6 - (0.33)(0) - (0.33)(-48 \times 10^6) \right]$$

$$= 306.29 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E} (-\nu \sigma_x - \sigma_y - \nu \sigma_z)$$

$$= \frac{1}{105 \times 10^9} \left[ -(0.33)(-48 \times 10^6) + 0 - (0.33)(-48 \times 10^6) \right]$$

$$= 301.71 \times 10^{-6}$$

$(a)$ Change in length: only portion $BC$ is strained. $L = 240$ mm

$$\delta_y = L \varepsilon_y = (240)(-301.71 \times 10^{-6}) = -0.0724 \text{ mm}$$

$(b)$ Change in diameter: $d = 50$ mm

$$\delta_d = \delta_x = d \varepsilon_x = (50)(-306.29 \times 10^{-6}) = -0.01531 \text{ mm}$$
PROBLEM 2.68

A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 18$ ksi and $\sigma_z = 24$ ksi. Knowing that the properties of the fabric can be approximated as $E = 12.6 \times 10^6$ psi and $v = 0.34$, determine the change in length of (a) side $AB$, (b) side $BC$, (c) diagonal $AC$.

SOLUTION

$\sigma_x = 18$ ksi  $\sigma_y = 0$  $\sigma_z = 24$ ksi

$\varepsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{1}{12.6 \times 10^6} \left[ 18,000 - (0.34)(24,000) \right] = 780.95 \times 10^{-6}$

$\varepsilon_z = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_z) = \frac{1}{12.6 \times 10^6} \left[ -(0.34)(18,000) + 24,000 \right] = 1.41905 \times 10^{-3}$

(a) $\delta_{AB} = (AB)\varepsilon_x = (4 \text{ in.})(780.95 \times 10^{-6}) = 0.0031238 \text{ in.}$

(b) $\delta_{BC} = (BC)\varepsilon_z = (3 \text{ in.})(1.41905 \times 10^{-3}) = 0.0042572 \text{ in.}$

Label sides of right triangle $ABC$ as $a$, $b$, $c$.

Then $\ c^2 = a^2 + b^2$

Obtain differentials by calculus.

$2c dc = 2a da + 2b db$

$dc = \frac{a}{c} da + \frac{b}{c} db$

But $a = 4 \text{ in.}$, $b = 3 \text{ in.}$, $c = \sqrt{4^2 + 3^2} = 5 \text{ in.}$

$da = \delta_{AB} = 0.0031238 \text{ in.}$ $db = \delta_{BC} = 0.0042572 \text{ in.}$

(c) $\delta_{AC} = dc = \frac{4}{5}(0.0031238) + \frac{3}{5}(0.0042572)$

$0.00505 \text{ in.}$
PROBLEM 2.77

Two blocks of rubber with a modulus of rigidity $G = 12 \text{ MPa}$ are bonded to rigid supports and to a plate $AB$. Knowing that $c = 100 \text{ mm}$ and $P = 45 \text{ kN}$, determine the smallest allowable dimensions $a$ and $b$ of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.

SOLUTION

Shearing strain: $\gamma = \frac{\delta}{a} = \frac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(12 \times 10^6 \text{ Pa})(0.005 \text{ m})}{1.4 \times 10^6 \text{ Pa}} = 0.0429 \text{ m} \quad a = 42.9 \text{ mm}$$

Shearing stress: $\tau = \frac{1}{2} \frac{P}{A} = \frac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{45 \times 10^3 \text{ N}}{2(0.1 \text{ m})(1.4 \times 10^6 \text{ Pa})} = 0.1607 \text{ m} \quad b = 160.7 \text{ mm}$$
PROBLEM 2.79

An elastomeric bearing \((G = 130 \text{ psi})\) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than \(\frac{3}{8} \text{ in.}\) when a 5-kip lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 60 psi, determine (a) the smallest allowable dimension \(b\), (b) the smallest required thickness \(a\).

SOLUTION

Shearing force:
\[ P = 5 \text{ kips} = 5000 \text{ lb} \]

Shearing stress:
\[ \tau = 60 \text{ psi} \]

\[ \tau = \frac{P}{A}, \text{ or } A = \frac{P}{\tau} = \frac{5000}{60} = 83.333 \text{ in}^2 \]

and \(A = (8 \text{ in.})(b)\)

\(a\) \(b = \frac{A}{8} = \frac{83.333}{8} = 10.4166 \text{ in.} \quad b = 10.42 \text{ in.} \)

\[ \gamma = \frac{\tau}{G} = \frac{60}{130} = 461.54 \times 10^{-3} \text{ rad} \]

\(b\) \(\text{But } \gamma = \frac{\delta}{a}, \text{ or } a = \frac{\delta}{\gamma} = \frac{0.375 \text{ in.}}{461.54 \times 10^{-3}} \quad a = 0.813 \text{ in.} \)