PROBLEM 7.1

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION

\[ F = 0: \sigma A - 15A \sin 30^\circ \cos 30^\circ - 15A \cos 30^\circ \sin 30^\circ + 10A \cos 30^\circ \cos 30^\circ = 0 \]

\[ \sigma = 30 \sin 30^\circ \cos 30^\circ - 10 \cos^2 30^\circ \]

\[ \sigma = 5.49 \text{ksi} \]

\[ \Sigma F = 0: \tau A + 15A \sin 30^\circ \sin 30^\circ - 15A \cos 30^\circ \cos 30^\circ - 10A \cos 30^\circ \sin 30^\circ = 0 \]

\[ \tau = 15(\cos^2 30^\circ - \sin^2 30^\circ) + 10 \cos 30^\circ \sin 30^\circ \]

\[ \tau = 11.83 \text{ksi} \]
**PROBLEM 7.4**

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

**SOLUTION**

\[ \sigma \Sigma F = 0: \sigma A + 18A \cos 15^\circ \sin 15^\circ = + 45A \cos 15^\circ \cos 15^\circ - 27A \sin 15^\circ \sin 15^\circ + 18A \sin 15^\circ \cos 15^\circ = 0 \]

\[ \sigma = -18 \cos 15^\circ \sin 15^\circ - 45 \cos^2 15^\circ + 27 \sin^2 15^\circ - 18 \sin 15^\circ \cos 15^\circ \]

\[ \sigma = -49.2 \text{ MPa} \]

\[ \tau \Sigma F = 0: \tau A + 18A \cos 15^\circ \cos 15^\circ = -45A \cos 15^\circ \sin 15^\circ - 27A \sin 15^\circ \cos 15^\circ - 18A \sin 15^\circ \sin 15^\circ = 0 \]

\[ \tau = -18(\cos^2 15^\circ - \sin^2 15^\circ) + (45 + 27) \cos 15^\circ \sin 15^\circ \]

\[ \tau = 2.41 \text{ MPa} \]
PROBLEM 7.5

For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

SOLUTION

\[ \sigma_x = -60 \text{ MPa}, \quad \sigma_y = -40 \text{ MPa}, \quad \tau_{xy} = 35 \text{ MPa} \]

(a) \[ \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35)}{-60 + 40} = -3.50 \]

\[ 2\theta_p = -74.05^\circ \quad \theta_p = -37.0^\circ, \quad 53.0^\circ \]

(b) \[ \sigma_{\text{max, min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

\[ = \frac{-60 - 40}{2} \pm \sqrt{\left(\frac{-60 + 40}{2}\right)^2 + (35)^2} \]

\[ = -50 \pm 36.4 \text{ MPa} \]

\[ \sigma_{\text{max}} = -13.60 \text{ MPa} \]

\[ \sigma_{\text{min}} = -86.4 \text{ MPa} \]
PROBLEM 7.8

For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

SOLUTION

\[ \sigma_x = -8 \text{ ksi} \quad \sigma_y = 12 \text{ ksi} \quad \tau_{xy} = 5 \text{ ksi} \]

(a) \[ \tan 2 \theta_p = \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(5)}{-8 - 12} = -0.5 \]

\[ 2 \theta_p = -26.565^\circ \]

\[ \theta_p = -13.3^\circ, \ 76.7^\circ \]

(b) \[ \sigma_{\text{max,min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

\[ = \frac{-8 + 12}{2} \pm \sqrt{\left(\frac{-8 - 12}{2}\right)^2 + (5)^2} \]

\[ = 2 \pm 11.1803 \]

\[ \sigma_{\text{max}} = 13.18 \text{ ksi} \]

\[ \sigma_{\text{min}} = -9.18 \text{ ksi} \]
PROBLEM 7.19

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of $22.5^\circ$ with a plane perpendicular to the axis of the pipe. Knowing that a 40-kip axial force $P$ and an 80-kip in. torque $T$, each directed as shown, are applied to the pipe, determine $\sigma$ and $\tau$ in directions, respectively, normal and tangential to the weld.

SOLUTION

$\begin{align*}
d_2 &= 12 \text{ in.}, \quad c_2 = \frac{1}{2}d_2 = 6 \text{ in.}, \quad t = 0.25 \text{ in.} \\
c_1 &= c_2 - t = 5.75 \text{ in.} \\
A &= \pi \left( c_2^2 - c_1^2 \right) = \pi (6^2 - 5.75^2) = 9.2284 \text{ in}^2 \\
J &= \frac{\pi}{2} \left( c_2^4 - c_1^4 \right) = \frac{\pi}{2} (6^4 - 5.75^4) = 318.67 \text{ in}^4
\end{align*}$

Stresses:

$\begin{align*}
\sigma &= \frac{-P}{A} \\
&= -\frac{40}{9.2284} = -4.3344 \text{ ksi} \\
\tau &= \frac{TC_2}{J} \\
&= \frac{(80)(6)}{318.67} = 1.5063 \text{ ksi}
\end{align*}$

$\begin{align*}
\sigma_x &= 0, \quad \sigma_y = -4.3344 \text{ ksi}, \quad \tau_{xy} = 1.5063 \text{ ksi}
\end{align*}$

Choose the $x'$ and $y'$ axes, respectively, tangential and normal to the weld.

Then

$\begin{align*}
\sigma_{x'} &= \sigma_y, \quad \text{and} \quad \tau_{x'y'} = \tau_{xy} \cos \theta - \tau_{xy} \sin \theta \\
\sigma_{y'} &= \frac{\sigma_x + \sigma_y - \sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \\
&= \left(\frac{-4.3344}{2}\right) \cos 45^\circ - \tau_{xy} \sin 45^\circ \\
&= -4.76 \text{ ksi} \\
\tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\
&= \left\{\begin{array}{l}
-\left[\frac{-4.3344}{2}\right] \sin 45^\circ + 1.5063 \cos 45^\circ \\
\tau_w = -0.467 \text{ ksi}
\end{array}\right.
\end{align*}$
**PROBLEM 7.22**

Two steel plates of uniform cross section \(10 \times 80 \text{ mm}\) are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (a) the angle \(\beta\), (b) the corresponding normal stress perpendicular to the weld.

**SOLUTION**

Area of weld:

\[
A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos \beta} = \frac{800 \times 10^{-6}}{\cos \beta} \text{ m}^2
\]

(a) \[\Sigma F_s = 0: \quad F_s - 100 \sin \beta = 0 \quad F_s = 100 \sin \beta \text{ kN} = 100 \times 10^3 \sin \beta \text{ N} \]

\[
\tau_w = \frac{F_s}{A_w} \quad 30 \times 10^6 = \frac{100 \times 10^3 \sin \beta}{800 \times 10^{-6} / \cos \beta} = 125 \times 10^6 \sin \beta \cos \beta
\]

\[
\sin \beta \cos \beta = \frac{1}{2} \sin 2\beta = \frac{30 \times 10^6}{125 \times 10^6} = 0.240
\]

\[\beta = 14.34^\circ \]

(b) \[\Sigma F_n = 0: \quad F_n - 100 \cos \beta = 0 \quad F_n = 100 \cos 14.34^\circ = 96.88 \text{ kN} \]

\[
A_w = \frac{800 \times 10^{-6}}{\cos 14.34^\circ} = 825.74 \times 10^{-6} \text{ m}^2
\]

\[
\sigma = \frac{F_n}{A_w} = \frac{96.88 \times 10^3}{825.74 \times 10^{-6}} = 117.3 \times 10^6 \text{ Pa}
\]

\[\sigma = 117.3 \text{ MPa} \]
PROBLEM 7.31

Solve Probs. 7.5 and 7.9, using Mohr’s circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

SOLUTION

\[ \sigma_x = -60 \text{ MPa}, \]
\[ \sigma_y = -40 \text{ MPa}, \]
\[ \tau_{xy} = 35 \text{ MPa} \]
\[ \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa} \]

Plotted points for Mohr’s circle:

\[ X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa}) \]
\[ Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa}) \]
\[ C: (\sigma_{ave}, 0) = (-50 \text{ MPa, 0}) \]

\( (a) \) \[ \tan \beta = \frac{GY}{CG} = \frac{35}{10} = 3.500 \]
\[ \beta = 74.05^\circ \]
\[ \theta_b = -\frac{1}{2} \beta = -37.03^\circ \]
\[ \alpha = 180^\circ - \beta = 105.95^\circ \]
\[ \theta_a = \frac{1}{2} \alpha = 52.97^\circ \]
\[ R = \sqrt{CG^2 + GX^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa} \]

\( (b) \)
\[ \sigma_{\min} = \sigma_{ave} - R = -50 - 36.4 \]
\[ \sigma_{\max} = \sigma_{ave} + R = -50 + 36.4 \]

\( (a') \)
\[ \theta_d = \theta_b + 45^\circ = 7.97^\circ \]
\[ \theta_e = \theta_a + 45^\circ = 97.97^\circ \]
\[ \tau_{\max} = R = 36.4 \text{ MPa} \]

\( (b') \)
\[ \sigma' = \sigma_{ave} = -50 \text{ MPa} \]
**PROBLEM 7.35**

Solve Prob. 7.13, using Mohr’s circle.

**PROBLEM 7.13 through 7.16** For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through *(a)* 25° clockwise, *(b)* 10° counterclockwise.

**SOLUTION**

\[
\begin{align*}
\sigma_x &= 0, \\
\sigma_y &= 8 \text{ ksi}, \\
\tau_{xy} &= 5 \text{ ksi} \\
\sigma_{ave} &= \frac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi}
\end{align*}
\]

Plotted points for Mohr’s circle:

\[
\begin{align*}
X &: (0, -5 \text{ ksi}) \\
Y &: (8 \text{ ksi}, 5 \text{ ksi}) \\
C &: (4 \text{ ksi}, 0)
\end{align*}
\]

\[
\tan 2\theta_p = \frac{FX}{FC} = \frac{5}{4} = 1.25
\]

\[
2\theta_p = 51.34^\circ
\]

\[
R = \sqrt{FC^2 + FX^2} = \sqrt{4^2 + 5^2} = 6.40 \text{ ksi}
\]

*(a)* \( \theta = 25^\circ \). \( \theta = 50^\circ \)

\[
\phi = 51.34^\circ - 50^\circ = 1.34^\circ
\]

\[
\begin{align*}
\sigma_x' &= \sigma_{ave} - R \cos \phi \\
&= -2.40 \text{ ksi} \\
\tau_{x'y'} &= R \sin \phi \\
&= 0.15 \text{ ksi}
\end{align*}
\]

\[
\begin{align*}
\sigma_y' &= \sigma_{ave} + R \cos \phi \\
&= 10.40 \text{ ksi}
\end{align*}
\]

*(b)* \( \theta = 10^\circ \). \( \theta = 20^\circ \)

\[
\phi = 51.34^\circ + 20^\circ = 71.34^\circ
\]

\[
\begin{align*}
\sigma_x' &= \sigma_{ave} - R \cos \phi \\
&= 1.95 \text{ ksi} \\
\tau_{x'y'} &= R \sin \phi \\
&= 6.07 \text{ ksi}
\end{align*}
\]

\[
\begin{align*}
\sigma_y' &= \sigma_{ave} + R \cos \phi \\
&= 6.05 \text{ ksi}
\end{align*}
\]
**PROBLEM 7.39**

Solve Prob. 7.17, using Mohr’s circle.

**PROBLEM 7.17** The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

**SOLUTION**

\[ \sigma_x = -4 \text{ MPa} \quad \sigma_y = -1.6 \text{ MPa} \quad \tau_{xy} = 0 \]

\[ \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -2.8 \text{ MPa} \]

Plotted points for Mohr’s circle:

- X: \((\sigma_x, -\tau_{xy}) = (-4 \text{ MPa}, 0)\)
- Y: \((\sigma_y, \tau_{xy}) = (-1.6 \text{ MPa}, 0)\)
- C: \((\sigma_{\text{ave}}, 0) = (-2.8 \text{ MPa}, 0)\)

\[ \theta = -15^\circ \quad 2\theta = -30^\circ \]

\[ CX = 1.2 \text{ MPa} \quad R = 1.2 \text{ MPa} \]

(a) \[ \tau_{xy}' = -CX \sin 30^\circ = -R \sin 30^\circ = -1.2 \sin 30^\circ \]

\[ \tau_{xy}' = -0.600 \text{ MPa} \]

(b) \[ \sigma_x' = \sigma_{\text{ave}} - CX \cos 30^\circ = -2.8 - 1.2 \cos 30^\circ \]

\[ \sigma_x' = -3.84 \text{ MPa} \]
**PROBLEM 7.44**

Solve Prob. 7.22, using Mohr’s circle.

**PROBLEM 7.22** Two steel plates of uniform cross section $10 \times 80$ mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (a) the angle $\beta$, (b) the corresponding normal stress perpendicular to the weld.

**SOLUTION**

\[
\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10 \times 10^{-3})(80 \times 10^{-3})} = 125 \times 10^6 \text{ Pa} = 125 \text{ MPa}
\]

\[
\sigma_y = 0 \quad \tau_{xy} = 0
\]

From Mohr’s circle:

(a) \[\sin 2\beta = \frac{30}{62.5} = 0.48 \quad \beta = 14.3^\circ \]

(b) \[\sigma = 62.5 + 62.5 \cos 2\beta \]

\[\sigma = 117.3 \text{ MPa} \]

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PROBLEM 7.98

A spherical gas container made of steel has a 5-m outer diameter and a wall thickness of 6 mm. Knowing that the internal pressure is 350 kPa, determine the maximum normal stress and the maximum shearing stress in the container.

SOLUTION

\[
d = 5 \text{ m} \quad t = 6 \text{ mm} = 0.006 \text{ m}, \quad r = \frac{d}{2} - t = 2.494 \text{ m}
\]

\[
\sigma = \frac{pr}{2t} = \frac{(350 \times 10^3 \text{ Pa})(2.494 \text{ m})}{2(0.006 \text{ m})} = 72.742 \times 10^6 \text{ Pa}
\]

\[
\sigma_{\text{max}} = 72.742 \text{ MPa}
\]

\[
\sigma_{\text{min}} = 0 \quad \text{(Neglecting small radial stress)}
\]

\[
\tau_{\text{max}} = \frac{1}{2}(\sigma_{\text{max}} - \sigma_{\text{min}}) = 36.4 \text{ MPa}
\]

\[
\sigma = 72.7 \text{ MPa} \quad \tau_{\text{max}} = 36.4 \text{ MPa}
\]
PROBLEM 7.109

The unpressurized cylindrical storage tank shown has a \( \frac{3}{16} \)-in. wall thickness and is made of steel having a 60-ksi ultimate strength in tension. Determine the maximum height \( h \) to which it can be filled with water if a factor of safety of 4.0 is desired. (Specific weight of water = 62.4 lb/ft\(^3\).)

SOLUTION

\[ d_0 = (25)(12) = 300 \text{ in.} \]
\[ r = \frac{1}{2}d - t = 150 - \frac{3}{16} = 149.81 \text{ in.} \]
\[ \sigma_{\text{all}} = \frac{\sigma_{\text{u}}}{F.S.} = \frac{60 \text{ ksi}}{4.0} = 15 \text{ ksi} = 15 \times 10^3 \text{ psi} \]
\[ \sigma_{\text{all}} = \frac{pr}{t} \]
\[ p = \frac{t\sigma_{\text{all}}}{r} = \frac{(1/16)(15 \times 10^3)}{149.81} = 18.77 \text{ psi} = 2703 \text{ lb/ft}^2 \]

But \( p = \gamma h \),
\[ h = \frac{p}{\gamma} = \frac{2703 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} \]
\[ h = 43.3 \text{ ft} \]