A. Chapter 3, problem 28. Do part (a) in two different ways: once by brute force (i.e., dynamic programming), and once by interpreting the counting of routes in terms of multiset permutations. Likewise, do part (b) in two different ways: once by dynamic programming, and once by multiset permutations (making use of Brualdi’s hint as well). You may use a calculator or computer to facilitate the dynamic programming computation.

B. Chapter 3, problem 40.

C. (a) Chapter 3, problem 48. Do this problem directly in terms of multiset permutations. (Hint: Look at the special case $m = n = 2$. What reversible operation might you perform on a string of 3 $A$’s and 2 $B$’s that would turn it into a string of 2 $A$’s and at most 2 $B$’s?)

(b) Use the addition principle (just once) to show that

$$p(m, m) + p(m + 1, m) + p(m + 2, m) + \ldots + p(m + n, m) = p(m + n + 1, m + 1),$$

where $p(\cdot, \cdot)$ is as section 5.1.

(c) Explain the relationship between parts (a) and (b) of this problem.

D. Chapter 3, problem 49. Find and fix Brualdi’s mistake. (Hint: Look at the special case $m = n = 1$. What reversible operation might you perform on a string of 2 $A$’s and 2 $B$’s that would turn it into a string of at most 1 $A$ and at most 1 $B$? If you’re stuck for ideas, take another look at part (a) of the preceding problem!)

E. Let $f(n)$ be the $n$th Fibonacci number, so that $f(1) = 1$, $f(2) = 2$, and $f(n) = f(n - 1) + f(n - 2)$ for all $n \geq 3$. Prove by induction that the sum $f(1) + f(2) + \ldots + f(n)$ is equal to $f(n + 2) - 2$, for all $n \geq 1$. 