Math 475, Problem Set #6  
(due 3/2/06)

A. (a) For each point \((a, b)\) with \(a, b\) non-negative integers satisfying \(a+b \leq 8\), count the paths from \((0,0)\) to \((a,b)\) where the legal steps from \((i,j)\) are to \((i+2,j)\), \((i,j+2)\), and \((i+1,j+1)\).

(b) Compute the coefficients of \((x^2 + x + 1)^n\) for \(n = 0, 1, 2, 3, 4\).

(c) Based on parts (a) and (b), formulate a precise conjecture of the form “for all non-negative integers \(a\) and \(b\), the number of paths from \((0,0)\) to \((a,b)\) is equal to the coefficient of . . . in the polynomial . . .”.

B. Chapter 5, problem 12.

C. Solve Brualdi, Chapter 5, problem 18 in two different ways: once using problem 16 as a model, and once using problem 17 as a model.

D. What is the coefficient of \(x_1^3x_2^3x_3x_4^2\) in the expansion of \((x_1 - x_2 + 2x_3 - 2x_4)^9\)?

E. Brualdi, Chapter 5, problem 46. Retain all terms that are greater than \(10^{-3}\); discard the rest.

F. Fix positive integers \(n, k \geq 3\). Consider a convex \(n\)-gon with vertices labelled 1 through \(n\). Call a convex \(k\)-gon, whose vertices are a subset of the vertices of the \(n\)-gon, an internal \(k\)-gon if all of its sides are diagonals of the \(n\)-gon.

(a) How many internal \(k\)-gons are there containing the vertex labelled 1?

(b) How many internal \(k\)-gons are there all together? (Hint: What do you know ahead of time about the ratio between the answer to (a) and the answer to (b)?)