Introduction to Time and Frequency: Using Digital Filters
25.108 Introduction to Engineering II
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Experiment Objectives:

• Introduce concepts of time and frequency
• Build complicated waveforms from harmonics
• Introduce concepts of digital filtering
• Reduce complicated waveform to simple sinusoid
Basic Theory

• In the early 1800’s Fourier showed that any periodic waveform could be expressed as an infinite sum of sine’s and cosines.
  – He developed a mathematical relationship between a waveform in the time domain and it’s component sines and cosines called the Fourier Series and The Fourier Transform
Time Frequency Plane
Wave Generation from Components

\[ v(t) = \sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \ldots \]
Example: Waveforms and Spectrum

A simple sinusoid in time and frequency
Example: Spectrum of a Triangle Wave
Basic Theory (You will learn this in 16.362 so do not worry now)

\[ x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos n\omega t + b_n \sin n\omega t \right] \]

where \( a_0 = \frac{2}{T} \int_a^T x(t) dt \)

\( a_n = \frac{2}{T} \int_a^T x(t) \cos n\omega t dt \)

\( b_n = \frac{2}{T} \int_a^T x(t) \sin n\omega t dt \)

3 Polar Fourier Series

- Another form of FS is obtained by combining the sine and cosine terms to give a single component with a phase angle

\[ x(t) = d_0 + \sum_{n=1}^{\infty} \left[ d_n \cos (n\omega t + \Theta_n) \right] \]

where \( d_0 = a_0 = C_0 \)

\( d_n = \sqrt{a_n^2 + b_n^2} = 2|C_n| \quad \Theta_n = -\tan^{-1}\left( \frac{b_n}{a_n} \right) \)
What is a Digital Spectrum Analyzer

**Fourier Analyzer**

- Parallel filters measured simultaneously
- CRT shows full spectral display

A

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An Example
Try it Yourself

```matlab
>> time=0:0.001:5;
>> sin5=cos(time.*2*pi*5);
>> plot(time,sin5)
>> plot(time(1:1000),sin5(1:1000))
>> SpectrumAnalyzer(sin5,1000)
>>
```

\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1 \]

\[-1 \quad -0.8 \quad -0.6 \quad -0.4 \quad -0.2 \quad 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \]

\[0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1\]

\[-2.5 \quad -2 \quad -1.5 \quad -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \]

\[-40 \quad -20 \quad 0 \quad 20 \quad 40 \quad 60 \]

\[-250 \quad -200 \quad -150 \quad -100 \quad -50 \quad 0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \]

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Square Wave has infinite bandwidth

>> sq5 = sign(sin5);
>> plot(time(1:1000), sq5(1:1000))
>> SpectrumAnalyzer(sq5, 1000)
What is a Filter?

- Passes a band of frequencies and rejects other frequencies
  - Three Bands of Interest
    - Pass band, The frequencies which get through
    - Stop band, the frequencies which don’t get through
    - Transition bands, the bands in which part of the frequencies get through, between stop and pass bands
Tuning a Radio, an example of Filtering

What does it do? Passes the frequencies you want and rejects those that you do not want.
Types of Digital Filters

• Low Pass: Passes low frequencies, rejects high frequencies
• High Pass: Passes high frequencies, rejects low frequencies
• Band Pass: Passes a band of frequencies
• Band Stop: Rejects a band of frequencies
Filter Implementations

• Infinite Impulse Response (IIR)
  – Feedback filter

• Finite Impulse response
  – Feed Forward

• Hybrid IIR/FIR
Key parameters in filter design

- Sampling rate
- Number of Taps
- Pass band
- Stop Band
Using FDA tool

- Type “FDATOOL” at command prompt
Step 2: Enter Parameters

• Enter Sampling Frequency
• Pass Band
• Stop Band
• Leave Everything else the same
Step 3: Design Filter

- Push "design Filter Button"

Filter Response Shown
Step 4: Export Coefficients

- On “File Menu” Type Export

Create “Num”, and “Den”. If you have matlab 6.5, type “Den=1”
Step 5: do the filtering

• Type

• “>> Output=filter(Num,Den,Input)” to apply the filter you have created. It is simple as that.
Quick Question?

- If you take a triangle wave with spectrum shown and low pass filter remove all frequencies except the fundamental, what will you see?

Answer: A sine wave at the fundamental frequency