Calculus II-Practice Exam #3
Spring 2004 Section 201-K. Levasseur

Note: This is somewhat longer than the exam on April 23 will be. Any types of the problems that were assigned for practice or were discussed in class could be on the exam; so the questions here are not all "templates" for the exam questions.
1. Determine whether the following integrals converge and for those that converge, find their values, if possible.

(a) \( \int_{0}^{4} \frac{1}{\sqrt{x}} \, dx \)

(b) \( \int_{0}^{\infty} \frac{1}{x+1} \, dx \)

(c) \( \int_{0}^{\infty} \frac{1}{x^2+1} \, dx \)

(d) \( \int_{0}^{\infty} \frac{1}{e^x+x} \, dx \)

2. Consider the region bounded by \( y = \sin x \) and the \( x \)-axis, between \( x = 0 \) and \( x = \pi \). Find the volume of the solid obtained by rotating this region

(a) around the \( x \)-axis.  
(b) around the \( y \)-axis.

3. Suppose that you pulled a 20 kg bucket of sand attached with chain to the top of a 20 meter building. If the mass of the chain is 0.3 kg/m, how much work did you do?

(b) How much work does it take to pump the water out of a full hemispherical tank of radius 10 feet?

4. Suppose that the density function for the number of years needed to graduate from a university is

\[ p(t) = \begin{cases} 
0 & \text{if } t < 3 \\
k(t-3) & \text{if } 3 \leq t < 4 \\
-k(t-8)/4 & \text{if } 4 \leq t < 8 \\
0 & \text{if } t > 8 
\end{cases} \]

(a) What value must \( k \) have?

(b) Draw the cumulative distribution function of the time it takes to graduate.

(c) Estimate the percentage of students who graduate in less than 6 years.

5. Which of the following sequences converge? What are the limits of those that converge?

(a) \( \sum_{n=0}^{\infty} 0.99^n \)

(b) \( \sum_{k=1}^{\infty} (-1)^{k+1} \)

(c) \( \sum_{n=0}^{\infty} e^{-3n} \)

6. Suppose that the sequence of partial sums of an infinite series \( \sum_{k=1}^{\infty} a_k \) is \( s_n = 1 - \frac{1}{n} \).

(a) Does \( \sum_{k=1}^{\infty} a_k \) converge? Explain.

(b) What is \( a_5 \)? What is \( a_n \)?

7. (a) Use integral test to show that \( s = \sum_{n=1}^{\infty} \frac{1}{n^3} \) converges.

(b) How large must \( m \) be in order to be sure that \( s_m = \sum_{n=1}^{m} \frac{1}{n^3} \) is within \( \frac{1}{1000} \) of \( s \)?