Show all work for each problem.

1. (20 pts) \( \frac{dy}{dx} = \frac{x + 3y}{y - 3x} \)

a) Show that the differential equation is exact.

b) Solve this differential equation using the fact that it is exact.
2. (20 pts) \( \frac{dy}{dx} = xy^3 - xy \)

Solve this differential equation as a Bernoulli equation.
3. (15 pts) Solve the initial value problem $y'' + y' - 6y = 0$, $y(0) = 7$, $y'(0) = -1$. 
4. (15 pts) Given that a breed of rabbits with birth rate $\beta$ and death rate $\delta$ is proportional to the rabbit population $P(t)$. Assume that $\beta < \delta$. This scenario generates the differential equation

$$\frac{dP}{dt} = -kP^2$$

with the constant $k > 0$, and time $t$ is in months.

a) Solve this differential equation for $P$.

b) Assume the initial population is 4 rabbits and there are 3 rabbits after 1 month. Find and simplify the resulting model for $P(t)$.

c) Will doomsday or extinction occur and when?
5. (15 pts) A person steps out of an airplane with zero initial velocity and opens a parachute (assume immediately) at an altitude of 2,000 ft. Assume linear air resistance is $2 \frac{ft}{sec^2}$ which in turn generates the differential equation for velocity as $v' = -32 - 2v$. Find the velocity $v$ and height $y$ of the person above the ground at any time $t$. 
Show all work for each problem.

1. (10 pts) Solve as a homogeneous first order differential equation

\[ y' = \frac{-1}{4} \left( \frac{3x}{y} + \frac{2y}{x} \right) \]
2. (10 pts) \( y' = e^{-y+1} \), \( y(0) = -1 \).

Solve this differential equation analytically. Then apply the Runge-Kutta Method to approximate the solution on the interval \([0, 0.5]\) with step size \( h = 0.25 \). Construct a table showing five-decimal place values of the approximate solution and actual solution at the points \( x = 0, 0.25 \) and \( 0.5 \).