Mechanical Vibrations

Misc Topics

Base Excitation & Seismic

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Seismic events are often modeled with an input applied as a ‘base excitation’
Seismic Response - Support Motion

With the motion of the base denoted as 'y' and the motion of the mass relative to the inertial reference frame as 'x', the differential equation of motion becomes

\[ m\ddot{x} = -k(x - y) - c(\dot{x} - \dot{y}) \]  \hspace{1cm} (3.5.1)

Substitute \[ z = x - y \]  \hspace{1cm} (3.5.2)

into the equations to give

\[ m\ddot{z} + c\dot{z} + kz = -\ddot{y} \]  \hspace{1cm} (3.5.3)

The equation is assumed to be in standard form with \( F/m \) equal to the negative of the acceleration.
Seismic Response - Support Motion

Consider the SDOF system with base excitation

The excitation shown is that of the 1940 N-S El Centro earthquake (commonly used)
Seismic Response - Support Motion

The response of the SDOF system (or MDOF system if desired) can be computed for any given MCK system. Obviously the response will be dictated by the natural frequency of the structural system.

Since the majority of the energy of a seismic event is well below 20 Hz, often times a system can be analyzed statically if the natural frequency of the structure is above 33 Hz.

When this is the case then the static forces are approximated by $F=ma$ for ease of computation.
Seismic Response - Support Motion

The response of the SDOF system with 1 Hz
Seismic Response - Support Motion

The response of the SDOF system with >33 Hz

![Graph showing El Centro Earthquake - N-S Acceleration - 1940](image1)

![Graph showing Seismic Response - Freq > 33 Hz](image2)
Seismic Response - Pseudo Response Analysis

The response of the SDOF system is dependent upon its natural frequency and damping. Obviously there are an infinite number of combinations that exist and the response of each SDOF system in each environment must be determined in this case.

This is an extremely time consuming analysis that must be performed for all equipment used in buildings and structures that are prone to seismic environments.

An alternate approach is typically used as discussed next.
Consider a building that is subjected to a seismic disturbance.
Seismic Response - Pseudo Response Analysis

A coarse model of the building is generated to represent the gross overall weight and effective building stiffness characteristics. The base motion is used as input to determine the amplification and filtering that occurs to the ground motion at various levels in the building (ie, VTB1_4).

This modified input is then used for each level of the building to determine the pseudo-displacement, pseudo-velocity and pseudo-acceleration that various SDOF systems will be subjected to when the ground excitation is applied.
Seismic Response - Pseudo Response Analysis

Using this approach, the actual equipment at each level is not specifically modeled. The effective response of a variety of “assumed” SDOF systems with various frequencies and dampings are computed to determine the response in the building.

In this way the equipment manufacturer is provided “response spectrums” that are used to design their particular equipment depending on the location in the building.
Seismic Response - Support Motion

With no damping, the relative response is computed using the convolution (Duhamel) integral as

\[ z(t) = -\frac{1}{\omega_n} \int_0^t \ddot{y}(\tau) \sin \omega_n (t - \tau) d\tau \]  

(4.2.5)

and when considering damping as

\[ z(t) = -\frac{1}{\omega_d} \int_0^t \ddot{y}(\tau) e^{-\zeta \omega_n (t - \tau)} \sin \omega_d (t - \tau) d\tau \]
Seismic Response - Support Motion

This equation is typically used for shock loading considerations as well as for seismic applications.

Typically, for earthquake analysis, the velocity spectra is used extensively. Differentiating

$$\dot{z}(t) = \frac{1}{\omega_d} \int_0^t \ddot{y}(\tau) e^{-\xi \omega_n (t-\tau)} \left[-\xi \omega_n \sin \omega_d (t-\tau) + \omega_d \cos \omega_d (t-\tau)\right] d\tau$$
Seismic Response - Support Motion

This equation can be written as

\[ \dot{z}(t) = \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sqrt{P^2 + Q^2} \sin(\omega_d - \phi) \]

where

\[
P = \int_0^t \ddot{y}(\tau)e^{-\zeta \omega_n t} \cos \omega_d \tau d\tau
\]

\[
Q = \int_0^t \ddot{y}(\tau)e^{-\zeta \omega_n t} \sin \omega_d \tau d\tau
\]

\[
\phi = \tan^{-1} \left( -\frac{P\sqrt{1-\zeta^2} + Q\zeta}{P\zeta - Q\sqrt{1-\zeta^2}} \right)
\]
Seismic Response - Support Motion

It is important to realize that in shock spectrum analysis, only the maximum response is computed.

This implies that there could be many different excitation shock spectrums that cause the same response.

It is this feature that makes shock response spectrum analysis so attractive.
Seismic Response - Support Motion

The spectrums can be developed and then used for design even though the actual loading may be different, the same response is achieved.

Its limitation is in fatigue analysis where the energy associated with different frequencies may cause different failures to occur even though the max response is the same.
Seismic Response - Support Motion

The maximum velocity spectrum can be written as

\[ S_v = |\dot{z}(t)|_{\text{max}} = \left| \frac{e^{-\zeta \omega_n t}}{\sqrt{1 - \zeta^2}} \sqrt{P^2 + Q^2} \right|_{\text{max}} \]

and the corresponding displacement and acceleration values are

\[ S_d = |z(t)|_{\text{max}} = \frac{S_v}{\omega_n} \]

\[ S_a = |\ddot{z}(t)|_{\text{max}} = \omega_n S_v \]
Seismic Response - Pseudo Response Analysis

Conceptually the SDOF response is shown in the figure below.
Seismic Response - Pseudo Response Analysis

**Typical Response Spectrum**

![Typical Response Spectrum Graph]

**Typical Design Spectrum**

![Typical Design Spectrum Graph]

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