Treasury Bill Prices and Returns

In this document we will explain how U. S. Treasury bills (T-bills) are quoted by dealers and reported in financial publications like the Wall Street Journal. T-bills are issued by the U.S. Treasury at regularly scheduled Treasury auctions with maturities of 4, 13, or 26 weeks, and entitles the holders of the T-bill to receive a single payment at the T-bill’s maturity called the face value or redemption value.

The face values can have various denominations, some as low as $1,000. T-bills, along with T-notes and T-bonds, which are securities with longer maturities, is the method through which the U.S. federal government borrows money. Since there has never been a default on these securities, the investment public usually considers them to be free of default risk. After they are issued by the government, they need not be held by the holder until their maturity. Instead, they can be sold in the secondary market for government securities through a dealer.

The table below would indicate the rates available for the trading day May 20, 2003.

<table>
<thead>
<tr>
<th>May 20, 2003</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Treasury Bills</th>
<th>DAYS TO MAT</th>
<th>BID</th>
<th>ASKED</th>
<th>CHG</th>
<th>ASK YLD</th>
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<tr>
<td>May 22 03</td>
<td>2</td>
<td>1.10</td>
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<td>0.04</td>
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<td>1.80</td>
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<td>0.98</td>
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<td>0.98</td>
<td>0.03</td>
<td>0.99</td>
</tr>
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<td>1.04</td>
<td>1.03</td>
<td>0.07</td>
<td>1.05</td>
</tr>
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<td>1.03</td>
<td>-0.01</td>
<td>1.05</td>
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<tr>
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<td>1.03</td>
<td>-0.01</td>
<td>1.05</td>
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<td>1.03</td>
<td>1.02</td>
<td>-0.01</td>
<td>1.04</td>
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<td>1.02</td>
<td>-0.01</td>
<td>1.04</td>
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<td>...</td>
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<td>1.03</td>
<td>...</td>
<td>1.05</td>
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<td>Nov 13 03</td>
<td>177</td>
<td>1.04</td>
<td>1.03</td>
<td>-0.01</td>
<td>1.05</td>
</tr>
</tbody>
</table>

The rates in the table all appear to be around 1%, which reflects the interest rate in 2003. You might wonder if all the detail in this document is worth the slight differences that are
obtained, but small differences in interest rates matter with large face values, and further, interest rates can be much higher, at which point these differences are magnified. Around 1980, most interest rates exceeded 10%.

A T-bill has only two cash flows. The price you pay today and the face value at maturity. Assume below that a T-bill will have a $10,000 face value. The price for that T-bill will be per $10,000 “face value.” Suppose you could buy that T-bill for \( P \) dollars. The cash flows of would be:

\[
\text{Cash Flows:} \quad P \quad \text{at time 0} \quad 10,000 \quad \text{at time } n \text{ days}
\]

If you were given the price of this T-bill, you could easily calculate the holding period return and the annualized rate of the return. At this point, you might want to review the web document “Return Calculations”. From that document, you can see that in general, the holding period of return for two cash flows is given as:

\[
r_h = \frac{C_1 - C_0}{C_0}
\]

which for the above cash flow would be:

\[
r_h = \frac{10,000 - P}{P}
\]

and annualized, the annual rate would be:

\[
r_a = (1 + r_h)^{365/n} - 1
\]

This calculation includes compounding, and is also the same as the effective annual rate or EAR.
But T-bills rates are quoted and reported using two slightly different rates, and one of the main objectives of this document is to explain these differences. The first way is called the **discount**. This is also an annual rate, but differs from our usual calculation as follows:

1) The holding period return uses the face value in the denominator of the return calculation instead of the price.

2) In annualizing this calculation, a 360 day year is used rather than 365.

3) Compounding or interest is ignored.

Example: Suppose that the T-bill has 180 days before maturity, and costs 9,900. The discount rate using the above rules would be obtained as follows: After calculating a holding period rate of \((10,000 - 9,900)\/10000 = 0.01\) or 1\%, the 180 day 1\% return is simply doubled to yield 2\%. This last step is the annualization based on following step 2 and 3 above. While the discount rate is slightly different from our more exact holding period calculation and annualization which takes into account compounding, it was a faster way to make these calculations before the existence of calculators. Showing all three steps in one, the discount rate for this T-bill would be calculated as:

\[
d = \left[ \frac{10,000 - 9,900}{10,000} \right] \frac{360}{180} = .02
\]

Notice that in calculating the holding period return, 10,000 is used in the denominator instead of the starting price 9,900, so it is not a true return calculation. Also, to annualize, you simply multiplied by how often the holding period divides into a 360 day year (360/180). In addition, compounding is ignored, because we simply multiply the holding period rate by the number of times it occurs in the 360 day year. In this case, it is exactly two, so we just double the holding period return.

Using the same cash flows but calculating the EAR rate which uses a proper basis for the divisor for the holding period return, and uses 365 days per year, and accounts for interest on interest (compounding) we would get:

\[
r_h = \frac{10,000 - P}{P} = \frac{10,000 - 9,900}{9900} = 0.010101
\]

\[
r_u = (1 + r_h)^{365/\text{n}} - 1 = (1 + 0.010101)^{365/180} - 1 = 1.010101^{2.027777} - 1 = 0.0206
\]

As you can see, the two methods produce similar but not identical results.
Let us develop the general formula for the discount rate. Consider the simple rate of return over \( n \) days that a T-bill would provide if it would pay a face value of 10,000 at maturity and if its price is \( P \). This rate would be:

\[
\frac{10,000 - P}{P}
\]

Since we want it to be a discount, we would modify it:

\[
\frac{10,000 - P}{10,000}
\]

To annualize we want to ignore interest on interest and use a 360 day year.

\[
d = \left[ \frac{10,000 - P}{10,000} \right] \left[ \frac{360}{n} \right]
\]

This is how the discount is calculated. Suppose that this discount is given to us (as in a table in the Wall Street Journal, as it was on page one of this document. If we need to obtain the price of the T-bill:

\[
\left[ \frac{10,000 - P}{10,000} \right] = \left[ \frac{nd}{360} \right]
\]

\[
10,000 - P = 10,000 \left[ \frac{nd}{360} \right]
\]

\[
P = 10,000 - 10,000 \left[ \frac{nd}{360} \right]
\]

\[
P = 10,000 \left[ 1 - \frac{nd}{360} \right]
\]

In the WSJ, both the bid and the ask discount is provided as a percentage. Consider the T-bill in the table on the first page of this document that has the **October 9 2003** maturity. (This is the sixth from the bottom). **DAYS TO MAT** is given as 142 days, and it is the number of days between May 20, 2003 and October 9, 2003. The column labeled **BID** is the bid discount, which means it is the rate that corresponds to the price you would obtain if you were selling the T-bill, while the **ASKED** column indicates the ask discount, which corresponds to the price you would pay if you were buying the T-bill. The difference between the asked and the bid price is a profit for the dealer. **CHG** is the change in the asked discount from the previous day. The **ASK YLD** will be explained later. The asked discount for this bill is 1.03 as a percent or 0.0103 as a rate. The table has everything in percentage terms, but the formulas we derived require the discount as a rate.

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\[1\] This is the same formula as 9.1 in Corrado and Jordan and Slide 19, Lecture 14, (61.300).
You would calculate the price of the T-bill as:

\[ P = 10,000 \left[ 1 - \frac{nd}{360} \right] = 10,000 \left[ 1 - \frac{(142)(0.0103)}{360} \right] = $9,959.37 \]

It is now easy to take this price and find the annualized rate of return using our standard method of calculation:

\[ r_h = \frac{10,000 - P}{P} = \frac{10,000 - 9,959.37}{9,959.37} = 0.004079575 \]

\[ r_{EAR} = (1 + r_h)^{\frac{365}{n}} - 1 = (1 + 0.004079575)^{\frac{365}{142}} - 1 = 1.004079575^{2.570422535} - 1 = 0.010519848 \]

This way of calculating is also called the equivalent annual return or EAR.

The final rate on the table (ASK YLD) is the bond equivalent yield. Only the ask version of the bond equivalent yield is given in the table. This calculation does assume a proper return calculation, and 365 days per year, but still ignores the interest on interest. It would be obtained as follows:

\[ \text{bond equivalent yield} = \left\lfloor \frac{10,000 - P}{P} \right\rfloor \times \frac{365}{n} \]

For our ask price $9,959.37 and 142 days to maturity:

\[ \text{bond equivalent yield} = \left\lfloor \frac{10,000 - 9,959.37}{9,959.37} \right\rfloor \times \frac{365}{142} = 0.010486 \]

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2 The answer you get is always a price for the given face value. If your face value is ten times greater, so is your price. You could repeat the calculation using the bid discount (1.04% or 0.0104) and obtain the price of $9,958.97, which is a lower price. This is the price you would get on that day, if you sold the T-bill instead of purchased it.