Settlement Criteria and Concept of Analysis

(text Sections 5.1 through 5.20, pp. 283-285)

1. **Tolerance Criteria of Settlement and Differential Settlement**
   - Settlement most often governs the design as allowable settlement is exceeded before B.C. becomes critical.
   - Concerns of foundation settlement are subdivided into 3 levels of associated damage:
     - Architectural damage - cracks in walls, partitions, etc.
     - Structural damage - reduced strength in structural members
     - Functional damage - impairment of the structure functionality
   The last two refer to stress and serviceability limit states, respectively.
Tolerance Criteria of Settlement and Differential Settlement (cont’d.)

In principle, two approaches exist to determine the allowable displacements.

(a) **Rational Approach to Design**

Design Building ➔ Determine allowable deformation & displacements ➔ Design found. accordingly ➔ Check cost ➔ ok

Problems:  
- expensive analysis  
- limited accuracy in all predictions especially settlement & differential settlement
1. Tolerance Criteria of Settlement and Differential Settlement (cont’d.)

(b) Empirical Approach (see text section 5.20, “Tolerable Settlement of Buildings”, pp. 283-285)

- based on performance of many structures, provide a guideline for maximum settlement and maximum rotation

![Diagram of settlement criteria]

- $S_{\text{max}} = \text{maximum settlement}$
- $\delta = \Delta s = \text{differential settlement (between any two points)}$
- $\left(\frac{\delta}{\ell}\right)_{\text{max}} = \text{maximum rotation}$
1. **Tolerance Criteria of Settlement and Differential Settlement (cont’d.)**

(b) **Empirical Approach**

Angular Distortion = \( \tan \beta = \left( \frac{\Delta s}{\ell} \right)_{\text{max}} = \frac{\delta}{\ell} = \frac{s_A - s_B}{\ell} \)

\[
\left( \frac{\delta}{\ell} \right)_{\text{max}} \geq \frac{1}{300} & \quad \text{architectural damage}
\]

\[
\left( \frac{\delta}{\ell} \right)_{\text{max}} \geq \frac{1}{250} & \quad \text{tilting of high structures become visible}
\]

\[
\left( \frac{\delta}{\ell} \right)_{\text{max}} \geq \frac{1}{150} & \quad \text{structural damage likely}
\]
1. **Tolerance Criteria of Settlement and Differential Settlement (cont’d.)**

   **(b) Empirical Approach**

   maximum settlement ($S_{\text{max}}$) leading to differential settlement

   - Masonry wall structure 1 - 2”
   - Framed structures 2 - 4”
   - Silos, mats 3 - 12”
   - Lambe and Whitman “Soil Mechanics” provides in Table 14.1 and Figure 14.8 (see next page) the allowable maximum total settlement, tilting and differential movements as well as limiting angular distortions.
1. **Tolerance Criteria of Settlement and Differential Settlement (cont’d.)**

**Correlation Between Maximum Settlement to Angular Distortion**

Grant, Christian & Van marke (ASCE - 1974) correlation between angular settlement to maximum settlement, based on 95 buildings of which 56 were damaged.

<table>
<thead>
<tr>
<th>Type of Found</th>
<th>Type of Soil</th>
<th>$s_{\text{max}} (\text{in})$</th>
<th>$\rho_{\text{all}} (\text{in})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isol. Footings</td>
<td>Clay</td>
<td>1200</td>
<td>4&quot;</td>
</tr>
<tr>
<td></td>
<td>Sand</td>
<td>600</td>
<td>2&quot;</td>
</tr>
<tr>
<td>Mat</td>
<td>Clay</td>
<td>$\geq 138$ ft</td>
<td>$\geq 0.044$ B (ft)</td>
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<tr>
<td></td>
<td>Sand</td>
<td>no relationship</td>
<td></td>
</tr>
</tbody>
</table>

Limiting values of serviceability are typically $s_{\text{max}} = 1$ ” for isolated footing and $s_{\text{max}} = 2”$ for a raft which is more conservative than the above limit based on architectural damage. Practically serviceability needs to be connected to the functionality of the building and the tolerable limit.
Settlement Criteria and Concept of Analysis

(Lambe & Whitman, *Soil Mechanics*)

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### Table 14.1 Allowable Settlement

<table>
<thead>
<tr>
<th>Type of Movement</th>
<th>Limiting Factor</th>
<th>Maximum Settlement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total settlement</td>
<td>Drainage</td>
<td>6–12 in.</td>
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<tr>
<td></td>
<td>Accreting</td>
<td>12–24 in.</td>
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<tr>
<td></td>
<td>Probability of nonuniform settlement:</td>
<td></td>
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<tr>
<td></td>
<td>Masonry walled structure</td>
<td>1–2 in.</td>
</tr>
<tr>
<td></td>
<td>Framed structures</td>
<td>2–4 in.</td>
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<tr>
<td></td>
<td>Smokeystacks, silos, etc.</td>
<td>3–12 in.</td>
</tr>
<tr>
<td></td>
<td>Stability against overturning</td>
<td>Depends on height and width</td>
</tr>
<tr>
<td>Tilting</td>
<td>Tilting of smokeystacks, towers</td>
<td>0.004'</td>
</tr>
<tr>
<td></td>
<td>Rolling of trucks, etc.</td>
<td>0.01'</td>
</tr>
<tr>
<td></td>
<td>Stacking of goods</td>
<td>0.01'</td>
</tr>
<tr>
<td></td>
<td>Machine operation-cotton boom</td>
<td>0.003'</td>
</tr>
<tr>
<td></td>
<td>Machine operation-turbogenerator</td>
<td>0.0002'</td>
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<tr>
<td></td>
<td>Crane rails</td>
<td>0.003'</td>
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<tr>
<td></td>
<td>Drainage of floors</td>
<td>0.01–0.02'</td>
</tr>
<tr>
<td>Differential movement</td>
<td>High continuous brick walls</td>
<td>0.0605–0.001'</td>
</tr>
<tr>
<td></td>
<td>One-story brick mill building, wall</td>
<td>0.001–0.002'</td>
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<tr>
<td></td>
<td>Cracking</td>
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<tr>
<td></td>
<td>Plaster cracking (gyprock)</td>
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<tr>
<td></td>
<td>Reinforced-concrete building frame</td>
<td>0.0025–0.004'</td>
</tr>
<tr>
<td></td>
<td>Reinforced-concrete building curtain</td>
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<tr>
<td></td>
<td>walls</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Steel frame, continuous</td>
<td>0.002'</td>
</tr>
<tr>
<td></td>
<td>Simple steel frame</td>
<td>0.005'</td>
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</tbody>
</table>

From Sowers, 1962.

*Note.* 1 = distance between adjacent columns that settle different amounts, or between any two points that settle differently. Higher values are for regular settlements and more tolerant structures. Lower values are for irregular settlements and critical structures.

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### Angular distortion γ/L

- **Limit where difficulties with machinery operations in settlements are to be feared.**
- **Limit of danger for houses with chimneys.**
- **Safelimit for buildings where cracking is not permissible.**
- **Limit where first cracking in panel walls is to be expected.**
- **Limit where difficulties with plastered caves are to be expected.**
- **Limit where tilting of high, rigid buildings might become visible.**
- **Considerable cracking in panel walls and brick walls.**
- **Safe limit for flexible brick walls, b/L < 0.14.**
- **Limit where structural damage of general buildings is to be feared.**

*Fig. 14.3 Limiting angular distortions (From Bjerrum, 1963a).*
2. **Types of Settlement and Methods of Analysis**

- **Si** = Granular Soils
- **Sc**, **Sc(S)** - Cohesive Soils

<table>
<thead>
<tr>
<th>Settlement</th>
<th>Time</th>
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<tbody>
<tr>
<td><strong>Si</strong> (immediate)</td>
<td></td>
</tr>
<tr>
<td><strong>Sc</strong> (consolidation)</td>
<td></td>
</tr>
<tr>
<td><strong>Sc(S)</strong> (secondary compression = creep)</td>
<td></td>
</tr>
</tbody>
</table>

In principle, both types of settlement; the immediate and the long term, utilize the compressibility of the soil, one however, is time dependent (consolidation and secondary compression).
3. General Concept of Settlement Analysis

Two controlling factors influencing settlements:

- Net applied stress - $\Delta q$
- Compressibility of soil - $c = \frac{\text{settlement}}{\text{load}}$

when dealing with clay $c = f(t)$ as it changes with time

$$s = \Delta q \times c \times f(B)$$

where $s =$ settlement [L],
$\Delta q =$ net load [F/L$^2$],
$c =$ compressibility [L/(F/L$^2$)],
$f(B) =$ size effect [dimensionless]

obtain $c$ by $\rightarrow$ lab tests, plate L.T., SPT, CPT

c will be influenced by:
- width of footing = $B$
- depth of footing =
- location of G.W. Table =
- type of loading $\rightarrow$ static or repeated
- soil type & quality affecting the modulus

14.533 Advanced Foundation Engineering – Samuel Paikowsky
1. **Principle**

(a) **Required:** Vertical stress (pressure) increase under the footing in order to assess settlement.

(b) **Solution:** Theoretical solution based on theory of elasticity assuming load on $\infty$, homogeneous, isotropic, elastic half space.

- **Homogeneous** Uniform throughout at every point we have the same qualities.
- **Isotropic** Identical in all directions, invariant with respect to direction
- **Orthotropic** (tend to grow or form along a vertical axis) different qualities in two planes
- **Elastic** capable of recovering shape

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Das 7th ed., Sections 5.2 – 5.6 (pp. 224 - 239)
Bowles sections 5.2 – 5.5 (pp. 286-302)
1. **Principle (cont’d.)**

(c) Why can we use the elastic solutions for that problem?

- Is the soil elastic? no, but…

1. We are practically interested in the service loads which are approximately the dead load.
   - The ultimate load = design load $\times$ F.S.
   - Design load = $(DL \times F.S.) + (LL \times F.S.)$
   - Service load $\approx DL \rightarrow$ within the elastic zone

ii. The only simple straightforward method we know
2. Stress due to Concentrated Load

Boussinesq, 1885

\[ \Delta p = \Delta \sigma_v = \frac{3P}{2\pi z^2 \left[1 + (r/z)^2 \right]^{5/2}} \quad r = \sqrt{x^2 + y^2} \]  

(eq. 5.1)
3. **Stress due to a Circularly Loaded Area**

- referring to flexible areas as we assume uniform stress over the area. Uniform stress will develop only under a flexible footing.
- integration of the above load from a point to an area.
  - see equations 5.2, 5.3 (text 225)

\[
\Delta p = \Delta \sigma_v = q_0 \left( 1 - \frac{1}{\left[ 1 + \left( \frac{B}{2z} \right)^2 \right]^{3/2}} \right)
\]

vertical stress under the center

see Table 5.1 (p.226) for \( \frac{\Delta \sigma_v}{q_0} = f \left( \frac{r}{(B/2)} \& \frac{z}{(B/2)} \right) \)
4. **Stress Below a Rectangular Area**

\[ \Delta p = \Delta \sigma_v = q_0 \times I \]

below the corner of a flexible rectangular loaded area

\[ m = \frac{B}{z} \quad n = \frac{L}{z} \]

Table 5.2 (p.228-229) → \( I = f(m,n) \)
## Corner of a Foundation

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<th>$m$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
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</table>
4. **Stress Below a Rectangular Area (cont’d.)**

Stress at a point under different locations

\[
\Delta p = \Delta \sigma_v = q_o (l_1 + l_2 + l_3 + l_4)
\]

**Figure 5.4** Stress below any point of a loaded flexible rectangular area (text p.196)

- use \( B_1 \times L_1 \rightarrow m_1, n_1 \rightarrow l_2 \)
- \( B_1 \times L_2 \rightarrow m_1, n_2 \rightarrow l_1 \)
- \( B_2 \times L_1 \rightarrow m_2, n_1 \rightarrow l_3 \)
- \( B_2 \times L_2 \rightarrow m_2, n_2 \rightarrow l_4 \)

Stress at a point under **the center** of the foundation

\[
\Delta p = \Delta \sigma_v = q_c \times l_c
\]

\[ l_c = f(m_1, n_1) \quad m_1 = L/B \quad n_1 = z/(B/2) \]

- Table 5.3 (p.230) provides values of \( m_1 \) and \( n_1 \).
- See next page for a chart \( \Delta p/q_0 \) vs. \( z/B, f(L/B) \)
## Center of a Foundation

### Table 5.3 Variation of $I_c$ with $m_1$ and $n_1$

<table>
<thead>
<tr>
<th>$n_1$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<tbody>
<tr>
<td>0.20</td>
<td>0.994</td>
<td>0.997</td>
<td>0.997</td>
<td>0.997</td>
<td>0.997</td>
<td>0.997</td>
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<td>0.997</td>
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</tr>
<tr>
<td>0.40</td>
<td>0.960</td>
<td>0.976</td>
<td>0.977</td>
<td>0.977</td>
<td>0.977</td>
<td>0.977</td>
<td>0.977</td>
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</tr>
<tr>
<td>0.60</td>
<td>0.892</td>
<td>0.932</td>
<td>0.936</td>
<td>0.936</td>
<td>0.937</td>
<td>0.937</td>
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<tr>
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<td>0.878</td>
<td>0.880</td>
<td>0.881</td>
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<td>0.240</td>
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<td>0.089</td>
<td>0.097</td>
<td>0.103</td>
<td>0.108</td>
<td>0.112</td>
</tr>
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5. General Charts of Stress Distribution Beneath Rectangular and Strip Footings

(a) \( \frac{\Delta p}{q_0} \) vs. \( \frac{z}{B} \) under the center of a rectangular footing with \( L/B = 1 \) (square) to \( L/B = \infty \) (strip)

Stress Increase in a Soil Mass Caused by Foundation Load

Figure 3.41 Increase of stress under the center of a flexible loaded rectangular area
Das “Principle of Foundation Engineering”, 3rd Edition
Vertical Stress Increase in the Soil Due to a Foundation Load

5. General Charts of Stress Distribution Beneath Rectangular and Strip Footings (cont’d.)

(b) Stress Contours (laterally and vertically) of a strip and square footings. Soil Mechanics, DM 7.1 – p. 167

Navy Design Manual
**Example:** size 8 x 8m, depth \( z = 4m \)

Find the additional stress under the center of the footing loaded with \( q_0 \)

**Table 5.2, \( I = 0.17522 \)**

1. Generic relationship: \[ \Delta p = (4 \times 0.17522)q_0 = 0.7q_0 \]

2. Specific to center, \( m_1 = 1, n_1 = 1 \) → **Table 5.3, \( I_c = 0.701 \)**
3. Use Figure 3 of the Navy → Square Footing \( z = B/2 \), \( \sigma_z \approx 0.7p \)
4. Use figure 3.41 (class notes p.12) \( L/B = 1, Z/B = 0.5 \) → \( \Delta p / q_0 \approx 0.7 \)
6. **Stress Under Embankment**

Figure 5.10 Embankment loading (text p.236)

\[ \Delta p = \Delta \sigma = q_o l' \quad (\text{eq.5.23}) \]

\[ l' = f \left( \frac{B_1}{z}, \frac{B_2}{z} \right) \Rightarrow \text{Figure 5.11 (p.237)} \]

Example:

\[ \gamma = 20 \, \text{kN/m}^3 \]

\[ H = 3 \, \text{m} \quad \Rightarrow \quad q_o = \gamma H = 60 \, \text{kPa} \]

\[ B_1 = 4 \, \text{m} \quad \Rightarrow \quad \frac{B_1}{z} = \frac{4}{5} = 0.80 \]

\[ B_2 = 4 \, \text{m} \quad \Rightarrow \quad \frac{B_2}{z} = \frac{4}{5} = 0.80 \]

\[ z = 5 \, \text{m} \]

Fig. 5.11 (p.237) \Rightarrow \quad l' \approx 0.43 \quad \Rightarrow \quad \Delta p = 0.43 \times 60 = 25.8 \, \text{kPa} \]
7. **Average Vertical Stress Increase due to a Rectangular Loaded Area**

Average increase of stress over a depth $H$ under the corner of a rectangular foundation:

$$I_a = f(m, n)$$

$m = B/H$

$n = L/H$

use Figure 5.7, p. 234
7. **Average Vertical Stress Increase due to a Rectangularly Loaded Area (cont’d.)**

For the average depth between $H_1$ and $H_2$

Use the following:

$$\Delta p_{avg} = \Delta \sigma_{avg} = q_0 \left[ H_2 I_a(H_2) - H_1 I_a(H_1) \right] / (H_2 - H_1)$$

(eq. 5.19, p.233 in the text)
7. **Average Vertical Stress Increase due to a Rectangular Loaded Area (cont’d.)**

**Example:** 8x8m footing  
H = 4m (H₁=0, H₂=4m)

Use 4x4x4 squares  
m = 1, n = 1

Figure 5.7 (p.234)  
\[ I_a \approx 0.225 \]
\[ \Delta p_{avg} = 4 \times 0.225 \times q_o = 0.9 \times q_o \]

0.9 \( q_o \) is compared to 0.7\( q_o \) (see previous example) which is the stress at depth of 4m (0.5B). The 0.9 \( q_o \) reflects the average stress between the bottom of the footing (\( q_o \)) to the depth of 0.5B.
Figure 5.7 Griffiths’ influence factor $I_a$
Vertical Stress Increase in the Soil Due to a Foundation Load

Figure 5.7 Griffiths’ Influence factor $I_a$ (text p.234)
8. **Influence Chart – Newmark’s Solution**

Perform numerical integration of equation 5.1

Influence value = \( \frac{1}{200} \) (# of segments)

Each segment contributes the same amount:

1. Draw the footing shape to a scale where \( z = \text{length AB} \) (2 cm = 20 mm)
2. The point under which we look for \( \Delta \sigma_v' \), is placed at the center of the chart.
3. Count the units and partial units covered by the foundation
4. \( \Delta \sigma_v' = \Delta p = q_o \times m \times I \)

where
- \( m = \# \text{ of counted units} \)
- \( q_o = \text{contact stress} \)
- \( I = \text{influence factor} = \frac{1}{200} = 0.005 \)
Vertical Stress Increase in the Soil Due to a Foundation Load

Fig. 3.50 Influence chart for vertical stress $\sigma_z$ (Newmark, 1942) (All values of $\nu$) (Poulos and Davis, 1991)

$\sigma_z = 0.001N_p$ where $N = \text{no. of blocks}$
8. **Influence Chart – Newmark’s Solution**

**Example**

What is the additional vertical stress at a depth of 10 m under point A?

1. $z = 10$ m  
   scale $20 \text{ mm} = 10$ m
2. Draw building in scale with point A at the center
   No. of elements – is (say) 76
   $\Delta \sigma_v = \Delta p = 100 \times 76 \times \frac{1}{200} = 38 \text{kPa}$
9. **Using Charts Describing Increase in Pressure**

See figures from the Navy Design Manual and Das 3rd edition Fig 3.41 (notes pp. 12 & 13)
Many charts exist for different specific cases like Figure 5.11 (p.237) describing the load of an embankment (for extensive review see “Elastic Solutions for Soil and Rock Mechanics” by Poulus and Davis)

**Most important to note:**
1. What and where is the chart good for? e.g. under center or corner of footing?
2. When dealing with lateral stresses, what are the parameters used (mostly $\mu$) to find the lateral stress from the vertical stress
10. **Simplified Relationship**

Back of an envelope calculations

2 : 1 Method (text p.231)

\[
\Delta \sigma_v = \Delta P = \frac{Q}{(B + z)(L + z)}
\]

**Figure 5.5, (p.231)**
10. **Simplified Relationship (cont’d.)**

**Example:**

What is the existing, additional, and total stress at the center of the loose sand under the center of the foundation?

\[ \sigma_v = (2 \times 19) + (0.5 \times 17) = 46.5 \text{kPa} \]

Using 2:1 method:

\[ \Delta \sigma_v = \frac{1000kN}{(3+1.5)(4+1.5)} = 40kPa \quad q_{contact} = 83.3kPa \left( \frac{\Delta q}{q_0} \approx 0.50 \right) \]
10. **Simplified Relationship (cont’d.)**

**Example:**

Total average stress at the middle of the loose sand $\sigma_t = 86.5 \text{ kPa}$

Using Fig. 3.41 of these notes (p.12):

$$\frac{z}{B} = \frac{1.5}{3} = 0.5$$

$$\frac{L}{B} = \frac{4}{3} = 1.33 \quad \frac{\Delta p}{q_0} \approx 0.75$$

$\Delta p = 0.75 \times 83.3 = 62.5 \text{ kPa}$

The difference between the two values is due to the fact that the stress calculated by the 2:1 method is the average stress at the depth of 1.5m while the chart provides the stress at a point, under the center of the foundation.
10. **Simplified Relationship (cont’d.)**

**Example:**

This can be checked by examining the stresses under the corner of the foundation.

\[ m = \frac{3}{1.5} = 2 \quad \text{and} \quad n = \frac{4}{1.5} = 2.67 \]

**Table 5.2 (p.228-229)** \( \Delta p \approx 0.23671 \) interpolated between

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23614</td>
<td>0.23782</td>
</tr>
<tr>
<td>n = 2.5</td>
<td>n = 3</td>
</tr>
</tbody>
</table>

\( \Delta p = 0.23671 \times 83.3 = 19.71 \)

Checking the average stress between the center and the corner:

\[
\frac{\Delta p_{\text{corner}} + \Delta p_{\text{center}}}{2} = \frac{62.5 + 19.71}{2} = 41.1 \text{ kPa}
\]

the obtained value is very close to the stress calculated by the 2:1 method that provided the average stress at the depth of 1.5m. (40kPa).
1. **Immediate Settlement Analysis**

(text Sections 5.9-5.14, pp. 243-273)

1. **General Elastic Relations**

Different equations follow the principle of the analysis presented on class notes pg. 6. For a uniform load (flexible foundation) on a surface of a deep elastic layer, the text presents the following detailed analysis:

\[
S_e = q_0 \left( \alpha B' \right) \frac{1 - \frac{\mu_s^2}{E_s}}{I_s I_f} \quad \text{(eq. 5.33)}
\]

- \( q_0 \) = contact stress
- \( B' = B \) for settlement under the corner
  - \( B' = B/2 \) for settlement under the center
- \( E_s, \mu \) = soil’s modulus of elasticity and Poisson’s ratio within zone of influence
- \( \alpha \) = factor depending on the settlement location
  - for settlement under the center; \( \alpha = 4, m' = L/B, n' = H/(B/2) \)
  - for settlement under the corner; \( \alpha = 1, m' = L/B, n' = H/B \)
- \( I_s \) = shape factor, \( I_s = F_1 + \frac{1 - 2\mu}{1 - \mu} F_2 \)
  - \( F_1 \) & \( F_2 \) f(n’ & m’) use Tables 5.8 and 5.9, pp. 248-251
- \( I_f \) = depth factor, \( I_f = f \left( \frac{D_f}{B}, \mu_s, \frac{L}{B} \right) \), use Table 5.10 (pp.252), \( I_f = 1 \) for \( D_f = 0 \)

For a rigid footing, \( S_e \approx 0.93S_e \) (flexible footing)
2. Finding $E_s$, $\mu$: the Modulus of Elasticity and Poisson’s Ratio

For $E_s$: direct evaluation from laboratory tests (triaxial) or use general values and/or empirical correlation. For general values, use Table 5.8 from Das (6th ed., 2007).

**Table 5.8 Elastic Parameters of Various Soils**

<table>
<thead>
<tr>
<th>Type of Soil</th>
<th>Modulus of elasticity, $E_s$ (MN/m², lb/in²)</th>
<th>Poisson’s ratio, $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose sand</td>
<td>10.5 – 24.0, 1500 – 3500</td>
<td>0.20 – 0.40</td>
</tr>
<tr>
<td>Medium dense sand</td>
<td>17.25 – 27.60, 2500 – 4000</td>
<td>0.25 – 0.40</td>
</tr>
<tr>
<td>Dense sand</td>
<td>34.50 – 55.20, 5000 – 8000</td>
<td>0.30 – 0.45</td>
</tr>
<tr>
<td>Silty sand</td>
<td>10.35 – 17.25, 1500 – 2500</td>
<td>0.20 – 0.40</td>
</tr>
<tr>
<td>Sand and gravel</td>
<td>69.00 – 172.50, 10,000 – 25,000</td>
<td>0.15 – 0.35</td>
</tr>
<tr>
<td>Soft clay</td>
<td>4.1 – 20.7, 600 – 3000</td>
<td></td>
</tr>
<tr>
<td>Medium clay</td>
<td>20.7 – 41.4, 3000 – 6000</td>
<td>0.20 – 0.50</td>
</tr>
<tr>
<td>Stiff clay</td>
<td>41.4 – 96.6, 6000 – 14,000</td>
<td></td>
</tr>
</tbody>
</table>

For $\mu$ (Poisson’s Ratio):

- **Cohesive Soils**
  - Saturated Clays $\Delta V = 0$, $\mu = \nu = 0.5$
  - Other Soils, usually $\mu = \nu \approx 0.3$ to 0.4
2. Finding $E_s$, $\mu$: the Modulus of Elasticity and Poisson’s Ratio (cont’d.)

Empirical Relations of Modulus of Elasticity

$$\frac{E_s}{p_a} = \alpha N_{60} \quad \alpha = 5 \text{ to } 15 \quad (\text{eq. 2.29})$$

(5–sands with fine s, 10–Clean N.C. sand, 15–clean O.C. sand)

Navy Design Manual (Use field values): $E_s/N$

- Silts, sandy silts, slightly cohesive silt-sand mixtures 4
- Clean, fine to medium, sands & slightly silty sands 7
- Coarse sands & sands with little gravel 10
- Sandy gravels with gravel 12
Finding $E_s$, $\mu$: the Modulus of Elasticity and Poisson’s Ratio (cont’d.)

$E_s = 2$ to $3.5q_c$ (cone resistance) CPT General Value

(Some correlation suggest 2.5 for equidimensional foundations and 3.5 for a strip foundation.)

General range for clays:
- N.C. Clays $E_s = 250c_u$ to $500c_u$
- O.C. Clays $E_s = 750c_u$ to $1000c_u$

See Table 5.7 for $E_s = \beta \cdot C_u$ and $\beta = f(PI, OCR)$
Immediate Settlement Analysis

3. Improved Equation for Elastic Settlement (Mayne and Poulos, 1999)

Considering: foundation rigidity, embedment depth, increase of $E_s$ with depth, location of rigid layers within the zone of influence.

\[ E_s = E_o + kz \]
3. **Improved Equation for Elastic Settlement (Mayne and Poulos, 1999) (cont’d.)**

The settlement below the center of the foundation:

\[ S_e = \frac{q_0 B_e I_G F E}{E_0} (1 - \mu_s^2) \]  

(eq. 5.46)

- \( B_e = \sqrt{\frac{4BL}{\pi}} \) or for a circular foundation, \( B_e = B \)
- \( E_s = E_0 + k\zeta \) being considered through \( I_G \)
- \( I_G = f(B, H/B_e), \quad \beta = \frac{E_0}{kB_e} \)

**Figure 5.18 (p.255)**

Variation of \( I_G \) with \( \beta \)
3. Improved Equation for Elastic Settlement (Mayne and Poulos, 1999) (cont’d.)

- Effect of foundation rigidity is being considered through \( I_F \)

\[
I_F = f(k_f) \text{ flexibility factor } \quad k_F = \left( \frac{E_f}{E_0 + \frac{B_e^2 k}{2}} \right) \left( \frac{2t}{B_e} \right)^3
\]

- \( k \) needs to be estimated

- \( E_f \) = modulus of foundation material

- \( t \) = thickness of foundation

Figure 5.19 (p.256) Variation of rigidity correction factor \( I_F \) with flexibility factor \( k_F \) [Eq.(5.47)]
3. **Improved Equation for Elastic Settlement (Mayne and Poulos, 1999) (cont’d.)**

- Effect of embedment is being considered through $I_E$

$$I_E = f(\mu_s, D_f, B_e)$$

---

**Figure 5.20** (p.256) Variation of embedment correction factor $I_E$ with $D_f/B_e$ [Eq.(5.48)]

Note: Figure in the text shows $I_F$ instead of $I_E$. 
Immediate Settlement Analysis

4. Immediate (Elastic) Settlement of Sandy Soil – The Strain Influence Factor (Schmertmann and Hartman, 1978)

(See Section 5.12, pp. 258-263)

The surface settlement

\[ s_i = \int_{z=0}^{\infty} \varepsilon_z dz \]

From the theory of elasticity, the distribution of vertical strain \( \varepsilon_z \) under a linear elastic half space subjected to a uniform distributed load over an area:

\[ \varepsilon_z = \frac{\Delta q}{E} I_z \]

\( \Delta q = \) the contact load
\( E = \) modulus of elasticity - the elastic medium
\( I_z = \) strain influence factor = \( f(\mu, \text{point of interest}) \)
Immediate Settlement Analysis

4. Immediate (Elastic) Settlement of Sandy Soil – The Strain Influence Factor (Schmertmann and Hartman, 1978) (cont’d.)

- From stress distribution (see Figure 3.41, p.12 of notes):
  - Influence of a square footing ≈ 2B
  - Influence of a strip footing ≈ 4B
  (both for \( \frac{\Delta q}{q_{contact}} \approx 10\% \))

- From FEM and test results. The influence factor \( I_z \):
Immediate Settlement Analysis

4. Immediate (Elastic) Settlement of Sandy Soil – The Strain Influence Factor (Schmertmann and Hartman, 1978) (cont’d.)

- Immediate Settlement Analysis
- Equidimensional footing (square, circle)
- Strip footing (L/B ≥ 10)

Diagram: Graph showing settlement factors with depth (Z) and influence factor (Iz).

Equations and diagrams illustrate the relationship between settlement and footing dimensions.
4. Immediate (Elastic) Settlement of Sandy Soil – The Strain Influence Factor (Schmertmann and Hartman, 1978) (cont’d.)

substituting the above into Eq. (i).

For square

\[ s_i = \Delta q \int_0^{2B} \frac{I_z}{E} \, dz \]

Approximating the integral by summation and using the above simplified \( \varepsilon \) vs. D/B relations we get to equation 5.49 of the text.

\[ S_e = C_1 C_2 \Delta q \sum_{i=1}^{n} \left( \frac{I_z}{E_s} \right) \Delta z_i \]

\( \Delta q \) = contact stress (net stress = stress at found – \( q_0 \))

\[ c_1 = 1 - 0.5 \left[ \frac{\sigma'_v}{\Delta q} \right] \]

\( \sigma'_v \) is calculated at the foundation depth

\( I_z \) = strain influence factor from the distribution

\( E_s \) = modulus in the middle of the layer

\( C_2 \) - (use 1.0) or \( C_2 = 1 + 0.2 \log (10t) \)

Creep correction factor \( t = \) elapsed time in years, e.g. \( t = 5 \) years, \( C_2 = 1.34 \)
The Preferable Iz Distribution for the Strain Influence Factor

The distribution of Iz provided in p.28 of the notes is actually a simplified version proposed by Das (Figure 5.21, p.259 of the text). The more complete version of Iz distribution recommended by Schmertmann et al. (1978) is

\[ I_{zp} = 0.5 + 0.1 \frac{\Delta q}{\sigma'_{vp}} \]

Where \(\sigma'_{vp}\) is the effective vertical stress at the depth of Izp (i.e. 0.5B and 1B below the foundation for axisymmetric and strip footings, respectively).
Immediate Settlement Analysis

6. Immediate Settlement in Sandy Soils using Burland and Burbridge’s (1985) Method

(Section 5.13, pp. 265-267)

\[ \frac{S_e}{B_R} = \alpha_1 \alpha_2 \alpha_3 \left[ \frac{1.25(L/B)}{0.25+(L/B)} \right]^2 \left( \frac{B}{B_R} \right)^{0.7} \left( \frac{q'}{p_a} \right) \]  

(eq. 5.70)

1. Determine N SPT with depth (eq. 5.67, 5.68)
2. Determine the depth of stress influence - z’ (eq. 5.69)
3. Determine \( \alpha_1, \alpha_2, \alpha_3 \) for NC or OC sand (p.266)
7. Case History – Immediate Settlement in Sand

A rectangular foundation for a bridge pier is of the dimensions L=23m and B=2.6m, supported by a granular soil deposit. For simplicity it can be assumed that L/B ≈ 10 and, hence, it is a strip footing.

- Provided qc with depth (next page)
- Loading = 178.54kPa, q = 31.39kPa (at Df=2m)

Find the settlement of the foundation

(a-1) The Strain Influence Factor (as in the text)

\[ C_1 = 1 - 0.5 \frac{q}{\bar{q} - q} = 1 - 0.5 \frac{31.39}{178.54 - 31.39} = 0.893 \]

\[ C_2 0.2 \log \left( \frac{t}{0.1} \right) \rightarrow \begin{array}{c}
 t = 5 \text{ years } \\
 C_2 = 1.34 \\
 t = 10 \text{ years } \\
 C_2 = 1.40 
\end{array} \]
Immediate Settlement Analysis

7. Case History – Immediate Settlement in Sand (cont’d.)

Using the attached Table for the calculation of $\Delta z$ (see next page)

$$S_e = C_1 C_2 (\bar{q} - q) \sum \frac{I_z}{E_s} \Delta z = (0.893)(1.34)(178.54 - 31.39)(18.95 \times 10^{-5} m)$$

$$S_e = 0.03336 m \approx 33 mm$$

For $t = 10$ years $\rightarrow$ $S_e = 34.5 mm$

For the calculation of the strain in the individual layer and it’s integration over the entire zone of influence, follow the influence chart (notes p.28) and the figure and calculation table below.
Immediate Settlement Analysis

7. Case History – Immediate Settlement in Sand (cont’d.)

Example

$z = 0 \rightarrow I_z = 0.2$

$z = 1B = 2.6m \rightarrow I_z = 0.5$

$z_1 = 0.5m \rightarrow I_z = 0.2 + \frac{0.5 - 0.2}{2.6} \times 0.5 = 0.2577$

Note: sublayer 1 has a thickness of 1m and we calculate the influence factor at the center of the layer.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\Delta z$ (m)</th>
<th>$q_c$ (kN/m$^2$)</th>
<th>$E_z^*$ (kN/m$^2$)</th>
<th>$z$ to the center of the layer (m)</th>
<th>$I_z$ at the center of the layer</th>
<th>$(I_z/E_z) \Delta z$ (m$^3$/kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2,450</td>
<td>8,575</td>
<td>0.5</td>
<td>0.258</td>
<td>$3.00 \times 10^{-3}$</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>3,430</td>
<td>12,005</td>
<td>1.8</td>
<td>0.408</td>
<td>$5.43 \times 10^{-5}$</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>3,430</td>
<td>12,005</td>
<td>2.8</td>
<td>0.487</td>
<td>$1.62 \times 10^{-3}$</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>6,870</td>
<td>24,045</td>
<td>3.25</td>
<td>0.458</td>
<td>$0.95 \times 10^{-5}$</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>2,950</td>
<td>10,325</td>
<td>4.0</td>
<td>0.410</td>
<td>$3.97 \times 10^{-3}$</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>8,340</td>
<td>29,190</td>
<td>4.75</td>
<td>0.362</td>
<td>$0.62 \times 10^{-5}$</td>
</tr>
<tr>
<td>7</td>
<td>1.5</td>
<td>14,000</td>
<td>49,000</td>
<td>5.75</td>
<td>0.298</td>
<td>$0.91 \times 10^{-3}$</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>6,000</td>
<td>21,600</td>
<td>7.0</td>
<td>0.247</td>
<td>$1.17 \times 10^{-3}$</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>10,000</td>
<td>35,000</td>
<td>8.0</td>
<td>0.154</td>
<td>$0.44 \times 10^{-5}$</td>
</tr>
<tr>
<td>10</td>
<td>1.9</td>
<td>4,000</td>
<td>14,000</td>
<td>9.45</td>
<td>0.062</td>
<td>$0.84 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

$\Sigma 10.4 \text{ m} = 4B$

$E_z = 3.5q_c$
Immediate Settlement Analysis

7. Case History – Immediate Settlement in Sand (cont’d.)

Variation of $I_z$ and $q_c$ below the foundation
Immediate Settlement Analysis

7. Case History – Immediate Settlement in Sand (cont’d.)

Find the settlement of the foundation
(a-2) The Strain Influence Factor (Schmertmann et al., 1978))

\[ I_{zp} = 0.5 + 0.1 \sqrt{\frac{\Delta q}{\sigma'_{vp}}} \]

\[ q = 31.39 \text{kPa} \rightarrow \gamma_t = 15.70 \text{kN/m}^3 \]
\[ \Delta q = 178.54 - 31.39 = 147.15 \]
\[ \sigma'_{vp} \text{ @ 1B below the foundation} = 31.39 + 2.6 (15.70) = 72.20 \text{kPa} \]

\[ I_{zp} = 0.5 + 0.1 \sqrt{\frac{147.15}{72.2}} = 0.50 + 0.14 = 0.64 \]
7. **Case History – Immediate Settlement in Sand (cont’d.)**

This change will affect the table on p. 28 in the following way:

<table>
<thead>
<tr>
<th>Layer</th>
<th>( I_z )</th>
<th>((I_z/E_z)\Delta z) [((m^2/kN)\times10^{-5})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.285</td>
<td>3.31</td>
</tr>
<tr>
<td>2</td>
<td>0.505</td>
<td>6.72</td>
</tr>
<tr>
<td>3</td>
<td>0.624</td>
<td>2.08</td>
</tr>
<tr>
<td>4</td>
<td>0.587</td>
<td>1.22</td>
</tr>
<tr>
<td>5</td>
<td>0.525</td>
<td>5.08</td>
</tr>
<tr>
<td>6</td>
<td>0.464</td>
<td>0.79</td>
</tr>
<tr>
<td>7</td>
<td>0.382</td>
<td>1.17</td>
</tr>
<tr>
<td>8</td>
<td>0.279</td>
<td>1.32</td>
</tr>
<tr>
<td>9</td>
<td>0.197</td>
<td>0.56</td>
</tr>
<tr>
<td>10</td>
<td>0.078</td>
<td>1.06</td>
</tr>
</tbody>
</table>

\[ \sum (I_z/E_z)\Delta z = 23.31 \times 10^{-5} \]

Using the \( I_{zp} \) for \( t = 10 \) years,

- \( S_e = 40.6 \text{mm} \)
- \( S_e = 42.4 \text{mm} \)
Using the previously presented elastic solutions for comparison:

(b) The elastic settlement analysis presented in section 5.10

\[ S_e = q_0 (\alpha B') \frac{E_s}{E_s} I_s I_f \]  

(eq. 5.33)

\[ B' = \frac{2.6}{2} = 1.3 \text{m for center} \]
\[ B = 2.6 \text{m for corner} \]
\[ q_0 = 178.54 \text{kPa (stress applied to the foundation)} \]

Strip footing, zone of influence \( \approx 4B = 10.4 \text{m} \)
From the problem figure \( q_c \approx 4000 \text{kPa}. \) Note the upper area is most important and the high resistance zone between depths 5 to 6.3m is deeper than 2B, so choosing 4,000kPa is on the safe side. Can also use weighted average (equation 5.34)
Case History – Immediate Settlement in Sand (cont’d.)

(b) The elastic settlement analysis presented in section 5.10 (cont’d.)

\[ q_c \approx 4,000 \text{kPa}, \text{ general, use notes p.24-25:} \]
\[ E_s = 2.5q_c = 104,000 \text{kPa}, \text{ matching the recommendation for a square footing} \]
\[ \mu_s \approx 0.3 \text{ (the material dense)} \]

**For settlement under the center:**

\[ \alpha = 4, \ m' = \frac{L}{B} = \frac{23}{2.6} = 8.85, \ n' = \frac{H}{(B/2)} = \frac{>30\text{m}}{2.6/2} > 23 \]

Table 5.8

<table>
<thead>
<tr>
<th>( m' )</th>
<th>( n' )</th>
<th>( F_1 )</th>
<th>( F_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>12</td>
<td>0.828</td>
<td>0.095</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>1.182</td>
<td>0.014</td>
</tr>
</tbody>
</table>

the difference between the values of \( m' = 8 \) or \( m' = 9 \) is negligible so using \( m' = 9 \) is ok. For \( n' \) one can interpolate. For accurate values one can follow equations 5.34 to 5.39.
7. **Case History – Immediate Settlement in Sand (cont’d.)**

(b) The elastic settlement analysis presented in section 5.10 (cont’d.)

interpolated values for \(n' = 23\) \(\Rightarrow F_1 = 0.872, F_2 = 0.085\)

for exact calculations:

\[
I_s = F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2 = 0.872 + \frac{1 - 2(0.3)}{1 - 0.3} (0.085) \approx 0.921
\]

As the sand layer extends deep below the footing \(H/B \gg\) and \(F_2\) is quite negligible.
For settlement under corner:

\( \alpha = 1, \quad m' = \frac{L}{B} = 8.85, \quad n' = \frac{H}{B} = (\frac{30}{2.6}) > 11.5 \)

Tables 5.8 & 5.9

\[
m' = 9 \quad n' = 12 \quad F_1 = 0.828 \quad F_2 = 0.095
\]

\[
I_s = 0.828 + \frac{1 - 2(0.3)}{1 - 0.3} (0.095) \approx 0.882
\]

\[
D_f/B = 2/2.6 = 0.70, \quad L/B = 23/2.6 = 8.85
\]

Table 5.10 \( \Rightarrow \mu_s = 0.3, \quad B/L = 0.2, \quad D_f/B = 0.6 \Rightarrow I_f = 0.85, \)
Immediate Settlement Analysis

7. Case History – Immediate Settlement in Sand (cont’d.)

(b) The elastic settlement analysis presented in section 5.10 (cont’d.)

- Settlement under the center (B’ = B/2, \( \alpha = 4 \))

\[
S_e = 178.54(4)(1.15) \frac{1 - (0.3)^2}{10,000} (0.921)(0.85) = 0.0585m = 58\text{mm}
\]

- Settlement under the corner (B’ = B, \( \alpha = 1 \))

\[
S_e = 178.54(1)(2.3) \frac{1 - (0.3)^2}{10,000} (0.882)(0.85) = 0.0280m = 28\text{mm}
\]

Average Settlement = \( 43\text{mm} \)

Using eq. 5.41 settlement for flexible footing = (0.93)(43) = \( 40\text{mm} \)
7. **Case History – Immediate Settlement in Sand (cont’d.)**

(c) The elastic settlement analysis presented in section 5.11

\[ S_e = \frac{q_0 B_e I_G I_F I_E}{E_0} (1 - \mu_s^2) \]  

(eq. 5.46)

\[ B_e = \sqrt{\frac{4BL}{\pi}} = \sqrt{\frac{4(2.6)(23)}{\pi}} = 8.73m \]

\[ \beta = \frac{E_0}{kB_e} \]

Using the given figure of \( q_c \) with depth, an approximation of \( q_c \) with depth can be made such that \( q_c = q_0 + z(q/z) \) where \( q_0 \approx 2200\text{kPa}, \) \( q/z \approx 6000/8 = 750\text{kPa/m} \)
7. **Case History – Immediate Settlement in Sand (cont’d.)**

   (c) The elastic settlement analysis presented in section 5.11 (cont’d.)

   Using the ratio of $E_s/q_c = 2.5$ used before, this relationship translates
to $E_0 = 5500\text{kPa}$ and $k = E/z = 1875\text{kPa/m}$

   $$\beta = \frac{5500}{(1875)(8.73)} = 0.336$$

   $H/B_e > 10/8.73 > 1.15$ no indication for a rigid layer and actually a
less dense layer starts at $\approx 9\text{m}$ ($q_c \approx 4000\text{kPa}$)

   Figure 5.18, $\beta \approx 0.34 \rightarrow I_G \approx 0.35$ (note; $H/B_e$ has almost no effect in
that zone when greater than 1.0)
7. Case History – Immediate Settlement in Sand (cont’d.)

(c) The elastic settlement analysis presented in section 5.11 (cont’d.)

\[ k_F = \frac{E_f}{E_0 + \frac{B_e}{2}} \left( \frac{zt}{B_e} \right)^3 \]

Using \( E_f = 15 \times 10^6 \text{kPa}, t = 0.5 \text{m} \)

\[ k_F = \frac{15 \times 10^6}{5500 + \frac{8.73}{2} \times 1875} \left( \frac{2 \times 0.5}{8.73} \right)^3 = 1.65 \]

\[ I_F = \pi + \frac{1}{4 \times 10k_F} = \pi + \frac{1}{4 \times 4.6 \times 10 \times 1.65} = 0.80 \]

\[ I_E = 1 - \frac{1}{3.5e^{(1.22\mu_s-0.4)} \left( \frac{B_e}{D_f} + 1.6 \right)} = 1 - \frac{1}{3.5e^{(1.22\mu_s-0.4)} \left( \frac{8.73}{2} + 1.6 \right)} = 1 - \frac{1}{20.18} = 0.95 \]

\[ S_e = \frac{178.54 \times 8.73 \times 0.35 \times 0.80 \times 0.95}{5500} (1 - 0.3^2) = 0.0686 \text{m} = 69 \text{mm} \]
Immediate Settlement Analysis

7. **Case History – Immediate Settlement in Sand (cont’d.)**

(d) Burland and Burbridge’s Method presented in Section 5.13, p.265

1. Using \( q_c \approx 4,000 \text{kPa} = 41.8 \text{tsf} \) and as \( E_s \approx 7 \text{N} \) and \( E_s \approx 2q_c \), we can also say that: \( N \approx q_c(\text{tsf})/3.5 \)

\[ \therefore N \approx 12 \]

2. The variation of \( q_c \) with depth suggests increase of \( q_c \) to a depth of \( \sim 6.5 \text{m} \) (2.5B) and then decrease. We can assume that equation 5.69 is valid as the distance to the “soft” layer \( (z”) \) is beyond 2B.

\[ \frac{z_I}{B_R} = 1.4 \left( \frac{B}{B_R} \right)^{0.75} \]

\[ B_R = 0.3 \text{m} \]

\[ B = 2.6 \text{m} \]
7. **Case History – Immediate Settlement in Sand (cont’d.)**

(d) Burland and Burbridge’s Method presented in Section 5.13, p.265

3. Elastic Settlement (eq. 5.70)

\[ S_e = B_R \alpha_1 \alpha_2 \alpha_3 \left[ \frac{1.25L}{B R} \right]^2 \left( \frac{B}{B_R} \right)^{0.7} \left( \frac{q'}{p_a} \right) \]

Assuming N.C. Sand:

\[ \alpha_1 = 0.14, \quad \alpha_2 = \frac{1.71}{(12)^{1.4}} = 0.049, \quad \alpha_3 = 1 \]

\[ S_e = 0.3 \times 0.14 \times 0.049 \times 1 \left[ \frac{1.25 \times 23}{0.25 + \frac{23}{2.6}} \right]^2 \left( \frac{2.6}{0.3} \right)^{0.7} \left( \frac{178.54}{100} \right) \]

\[ S_e = 0.00206 \left[ \frac{11.06}{9.1} \right]^2 \times (8.67)^{0.7} \times (1.7854) = 0.025m = 25mm \]
(e) Summary and Conclusions

<table>
<thead>
<tr>
<th>Method</th>
<th>Case</th>
<th>Settlement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain Influence Section 5.12, 5 years</td>
<td>Iz (Das)</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>Izp (Schmertmann et al.)</td>
<td>41</td>
</tr>
<tr>
<td>Elastic Section 5.10</td>
<td>Center</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>Corner</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>40</td>
</tr>
<tr>
<td>Elastic Section 5.11</td>
<td></td>
<td>69</td>
</tr>
<tr>
<td>B &amp; B Section 5.13</td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

- The elastic solution (section 5.10) and the improved elastic equation (section 5.11) resulted with a similar settlement analysis under the center of the footing (58 and 69mm). This settlement is about twice that of the strain influence factor method as presented by Das (text) and B&B (section 5.13) (33 and 25mm, respectively).

- Averaging the elastic solution method result for the center and corner and evaluating “flexible” foundation resulted with a settlement similar to the strain influence factor as proposed by Schmertmann (40 vs. 41mm). The improved method considers the foundation stiffness.
Immediate Settlement Analysis

7. **Case History – Immediate Settlement in Sand (cont’d.)**

(e) **Summary and Conclusions (cont’d.)**

- The elastic solutions of sections 5.10 and 5.11 are quite complex and take into considerations many factors compared to common past elastic methods.

- The major shortcoming of all the settlement analyses is the accuracy of the soil’s parameters, in particular the soil’s modulus and its variation with depth. As such, many of the refined factors (e.g. for the elastic solutions of sections 5.10 and 5.11) are of limited contribution in light of the soil parameter’s accuracy.
7. **Case History – Immediate Settlement in Sand (cont’d.)**

**(e) Summary and Conclusions (cont’d.)**

- **What to use?**
  1. From a study conducted at UML Geotechnical Engineering Research Lab, the strain influence method using $I_{zp}$ recommended by Schmertmann provided the best results with the mean ratio of load measured to load calculated for a given settlement being about $1.28 \pm 0.77$ (1 S.D.) for 231 settlement measurements on 53 foundations.
  
  2. Check as many methods as possible, make sure to examine the simple elastic method.
  
  3. Check ranges of solutions based on the possible range of the parameters (e.g. $E_0$).
For example, in choosing $q_c$ we could examine the variation between 3,500 to 6,000 and then the variation in the relationship between $q_c$ and $E_s$ between 2 to 3.5. The results would be:

$$E_{s_{\min}} = 2 \times 3,500 = 7,000 \text{kPa}$$
$$E_{s_{\max}} = 3.5 \times 6,000 = 21,000 \text{kPa}$$

As $S_e$ of equation 5.33 is directly inverse to $E_s$, this range will result with:

$$S_{e_{\min}} = 27 \text{mm}, \quad S_{e_{\max}} = 81 \text{mm} \text{ (compared to 57mm)}$$
Immediate Settlement Analysis

8. **Immediate (Elastic) Settlement of Foundations on Saturated Clays:** (Junbu et al., 1956), section 5.9, p.243

\[ \mu = \nu_s = 0.5 \quad \text{Flexible Footings} \]

\[
S_e = A_1 A_2 \frac{q_0 B}{E_s} \quad (\text{eq. 5.30})
\]

- \( A_1 = \text{Shape factor and finite layer} - A_1 = f(H/B, L/B) \)
- \( A_2 = \text{Depth factor} - A_2 = f(D_f/B) \)

Note: \( H/B \gg \gg \text{deep layer the values become asymptotic} \)

\( e.g. \) for \( L = B \) (square) and \( H/B \geq 10 \ A_1 \approx 0.9 \)
Immediate Settlement Analysis

8. **Immediate (Elastic) Settlement of Foundations on Saturated Clays:** (Junbu et al., 1956), section 5.9, p.243 (cont’d.)

**Figure 5.14** Values of $A_1$ and $A_2$ for elastic settlement calculation – Eq. (5.30) (after Christian and Carrier, 1978)
Consolidation Settlement -
Long Term Settlement

Consolidation General, text Section 1.13 (pp. 32-37)
Consolidation Settlement for Foundations, text Sections 5.15 – 5.20 (pp. 273-285)

1. Principle and Analogy

**model**

- $t = 0^+$
  - $P_{spring} = 0$
  - $u = u_0 = 0$

- $t = t_1$
  - $P_{spring} = \Delta H \times K_{spring}$
  - $u = \frac{P - P_{spring}}{A}$

- $t = \infty$
  - $P_{spring} = P$
  - $u = u_0 = 0$

---

14.533 Advanced Foundation Engineering – Samuel Paikowsky
1. Principle and Analogy (cont’d.)

We relate only to changes, i.e. the initial condition of the stress in the soil (force in the spring) and the water are being considered as zero. The water pressure before the loading and at the final condition after the completion of the dissipation process is hydrostatic and is taken as zero, ($u_0 = u_{\text{hydrostatic}} = 0$). The force in the spring before the loading is equal to the weight of the piston (effective stresses in the soil) and is also considered as zero for the process, $P_{\text{spring}} = P_0 = \text{effective stress before loading} = P_{\text{at rest}}$. The initial condition of the process is full load in the water and zero load in the soil (spring), at the end of the process there is zero load in the water and full load in the soil.
1. **Principle and Analogy (cont’d.)**

**Analogy Summary**

- model: soil
- water → water
- spring → soil skeleton/effective stresses
- piston → foundation
- hole size → permeability
- force P → load on the foundation or at the relevant soil layer due to the foundation

![Diagram](image)
2. **Final Settlement Analysis**

(a) **Principle of Analysis**

\[ e = \frac{V_v}{V_s} \]

\[ \omega = \frac{W_w}{W_s} \]

Initial soil volume: \( V_o = 1 + e_o \)

Final soil volume: \( V_f = 1 + e_o - \Delta e \)
2. **Final Settlement Analysis (cont’d.)**

(a) **Principle of Analysis (cont’d.)**

\[ \Delta V = V_o - V_f = \Delta e \]

As area \( A \) = Constant: \( V_o = H_o \times A \) and \( V_f = H_f \times A \)

\[ \Delta V = V_o - V_f = A(H_o - H_f) = A \times \Delta H \]

\[ \Delta H = \frac{\Delta V}{A} \]

for 1-D  (note, we do not consider 3-D effects and assume pore pressure migration and volume change in one direction only).

\[ \varepsilon_v = \frac{\Delta H}{H_o} = \frac{\Delta V/A}{V_o/A} = \frac{\Delta V}{V_o}, \text{ substituting for } V, \ v \text{ relations} \]

\[ \varepsilon_v = \frac{\Delta V}{V_o} = \frac{\Delta e}{V_0} = \frac{\Delta e}{1 + e_0} \]

\[ \Delta H = \varepsilon_v \times H_o = \frac{\Delta e}{1 + e_0} \times H_0 \]
2. **Final Settlement Analysis (cont’d.)**

(a) **Principle of Analysis (cont’d.)**

Calculating $\Delta e$

We need to know:

i. Consolidation parameters $c_c$, $c_r$ at a representative point(s) of the layer, based on odometer tests on undisturbed samples.

ii. The additional stress at the same point(s) of the layer, based on elastic analysis.
2. Final Settlement Analysis (cont’d.)

(b) Consolidation Test (1-D Test)

1. Oedometer = Consolidometer

2. Test Results

![Figure 1.15a Schematic Diagram of consolidation test arrangement (p.33)](image-url)
Consolidation Settlement - Long Term Settlement

2. Final Settlement Analysis (cont’d.)

(b) Consolidation Test (1-D Test) (cont’d.)

a) final settlement with load after 24 hours

b) settlement with time under a certain load

\[ e = \frac{V_v}{V_s} \]

\[ e << \rightarrow V_v << \rightarrow \text{denser material} \]

\[ \gamma_d \gg \gamma_d = \frac{W_s}{V} \quad (V <<) \]

14.533 Advanced Foundation Engineering – Samuel Paikowsky
2. **Final Settlement Analysis (cont’d.)**

(c) **Obtaining Parameters from Test Results**

**Analysis of e-log p results.**

1. **Stage 1** - Casagrande’s procedure to find max. past pressure. (see Figures 1.15 to 1.17, text pp.33 to 37, respectively)

   1. find the max. curvature.
      - use a constant distance and look for the max. normal.
      - draw tangent to the curve at that point.
   2. draw horizontal line through that point and divide the angle.
   3. extend (if doesn’t exist) the e-log p line to e = 0.42e₀
   4. extend the tangent to the curve and find its point of intersection with the bisector of stage 2. → P’ = max. past pressure.

**Figure 1.15** (b) e-log σ’ curve for a soft clay from East St. Louis, Illinois (note: at the end of consolidation, σ = σ’
Consolidation Settlement - Long Term Settlement

2. **Final Settlement Analysis** (cont’d.)

(c) **Obtaining Parameters from Test Results** (cont’d.)

analysis of e-log p results.

2\textsuperscript{nd} Stage - Reconstructing the full e-log $p'$ (undisturbed) curve (Schmertmann’s Method, See Figures 1.16 and 1.17, pp.35,37)

1. find the point $e_o$, $p'$
   
   \[ e_o = \omega_n \times G_s \quad p'_o = \gamma'z. \]

2. find the avg. recompression curve and pass a parallel line through point 1.

3. find point $p_c' & e$

4. connect the above point to $e = 0.42e_o$

\[ OCR = \frac{p'_c}{p'_o} \]
Consolidation Settlement - Long Term Settlement

2. Final Settlement Analysis (cont’d.)

(c) Obtaining Parameters from Test Results (cont’d.)

Compression index (or ratio)

\[ C_c = \frac{e_1 - e_2}{\log\left(\frac{p_2}{p_1}\right)} = \frac{e_1 - e_2}{\log \left(\frac{p_2}{p_c}\right)} \]

Recompression index (or ratio)

\[ C_r = \frac{e_0 - e_1}{\log\left(\frac{p_c}{p_0}\right)} = \frac{e_0 - e_1}{\log \left(\frac{p_c}{p_0}\right)} \]

- See p.35-37 of the text for \( C_s \) & \( C_c \) values.
- natural clay \( C_c \approx 0.09(LL -10) \)
  where \( LL \) is in (%) (eq.1.50)
- B.B.C \( C_c = 0.35 \)  \( C_s = 0.07 \)
Consolidation Settlement - Long Term Settlement

2. Final Settlement Analysis (cont’d.)

(d) Final Settlement Analysis

\[ \Delta e = C_s \log \frac{\sigma'_0 + \Delta \sigma'}{\sigma'_0} \]

\[ \Delta e = C_c \log \frac{\sigma'_c}{\sigma'_0} + C_c \log \frac{\sigma'_0 + \Delta \sigma}{\sigma'_c} \]

(for \( \sigma'_0 + \Delta \sigma' > \sigma'_c \))
(d) Final Settlement Analysis (cont’d.)

Solution:
1. Subdivide layers according to stratification and stress variation
2. In the center of each layer calculate $\sigma'_v(s'_o)$ and $\Delta\sigma'$
3. Calculate for each layer $\Delta e_i$

$$H = \sum_{i=1}^{n} H_i \frac{\Delta e_i}{1 + e_0}$$

replace $p_c$ by $\sigma'_{v,\max}$ and $p_o$ by $\sigma'_{v,o}$

The average increase of the pressure on a layer ($\Delta\sigma' = \Delta s'_{av}$) can be approximated using the text; eq. 5.84 (p.274)

$$\Delta \sigma'_{av} = \frac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

↑ ↑ ↑

top middle bottom
2. **Final Settlement Analysis** (cont’d.)

(d) **Final Settlement Analysis** (cont’d.)

Skempton - Bjerrum Modification for Consolidation Settlement
Section 5.16 p. 275 - 279

The developed equations are based on 1-D consolidation in which the increase of pore pressure = increase of stresses due to the applied load. Practically we don’t have 1-D loading in most cases and hence different horizontal and vertical stresses.

\[ \Delta u = \sigma_c + A[\sigma_1 - \sigma_c] \]

A = Skempton’s pore pressure parameter
For example: Triaxial Test

N.C.  OCR = 1  0.5<A<1
OCR < 4  0.25<A<0.5
OCR ≈ 5  0
OCR > 6  -0.5<A<0

considering the partial pore pressure build up, we can modify our calculations:

1) calculate the consolidation settlement the same way as was shown earlier
2) determine pore water pressure parameter → lab test or see the table on p. 52 in the text
3) \( H_c/B = \) consolidation depth / foundation width
4) use Fig. 5.31, p.276, \((A & H_c/B)\) → settlement ratio (<1) (Note circular or continuous)
5) \( S_c = S_{c \text{ calc}} \times \text{Settlement Ratio} \)

Note: Table 5.14, p.277 provides the settlement ratio as a function of \( B/H_c \) and OCR based on Leonards (1976) field work. It is an alternative to Figure 5.31 as \( A = f(OCR) \), (see above) for which an equivalent circular foundation can be calculated (e.g.)
Final Settlement Analysis (cont’d.)

(d) Final Settlement Analysis (cont’d.)

From Das, Figure 5.31 and Table 5.14

Table 5.14 Variation of $K_{cr(OC)}$ with OCR and $B/H_c$

<table>
<thead>
<tr>
<th>OCR</th>
<th>$B/H_c = 4.0$</th>
<th>$B/H_c = 1.0$</th>
<th>$B/H_c = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.986</td>
<td>0.957</td>
<td>0.929</td>
</tr>
<tr>
<td>3</td>
<td>0.972</td>
<td>0.914</td>
<td>0.842</td>
</tr>
<tr>
<td>4</td>
<td>0.964</td>
<td>0.871</td>
<td>0.771</td>
</tr>
<tr>
<td>5</td>
<td>0.950</td>
<td>0.829</td>
<td>0.707</td>
</tr>
<tr>
<td>6</td>
<td>0.943</td>
<td>0.800</td>
<td>0.643</td>
</tr>
<tr>
<td>7</td>
<td>0.929</td>
<td>0.757</td>
<td>0.586</td>
</tr>
<tr>
<td>8</td>
<td>0.914</td>
<td>0.729</td>
<td>0.529</td>
</tr>
<tr>
<td>9</td>
<td>0.900</td>
<td>0.700</td>
<td>0.493</td>
</tr>
<tr>
<td>10</td>
<td>0.886</td>
<td>0.671</td>
<td>0.457</td>
</tr>
<tr>
<td>11</td>
<td>0.871</td>
<td>0.643</td>
<td>0.429</td>
</tr>
<tr>
<td>12</td>
<td>0.864</td>
<td>0.629</td>
<td>0.414</td>
</tr>
<tr>
<td>13</td>
<td>0.857</td>
<td>0.614</td>
<td>0.400</td>
</tr>
<tr>
<td>14</td>
<td>0.850</td>
<td>0.607</td>
<td>0.386</td>
</tr>
<tr>
<td>15</td>
<td>0.843</td>
<td>0.600</td>
<td>0.371</td>
</tr>
<tr>
<td>16</td>
<td>0.843</td>
<td>0.600</td>
<td>0.357</td>
</tr>
</tbody>
</table>

Figure 5.31 Settlement ratios for circular ($K_{cir}$) and continuous ($K_{str}$) foundations

14.533 Advanced Foundation Engineering – Samuel Paikowsky
2. Final Settlement Analysis (cont’d.)

(e) EXAMPLE – Final Consolidation Settlement

Calculate the final settlement of the footing shown in the figure below. Note, OCR = 2 for all depths. Give the final settlement with and without Skempton & Bjerrum Modification.

\[ P = 1 \text{MN} \]

\[ \gamma_{\text{sat}} = 20 \text{kN/m}^3 \quad C_c = 0.20 \]

\[ \gamma_{\text{w}} \approx 10 \text{kN/m}^3 \quad C_r = 0.05 \]

\[ 3B = 12 \text{m} \]

\[ \text{OCR} = 2 \]

\[ G_s = 2.65 \]

\[ \omega_n = 37.7\% \]

(note: assume 1-D consolidation)
Consolidation Settlement - Long Term Settlement

2. **Final Settlement Analysis (cont’d.)**

(e) **EXAMPLE – Final Consolidation Settlement (cont’d.)**

P=1MN, B=4mx4m, $q_0 = \frac{1000}{16} = 62.5\text{kPa}$

<table>
<thead>
<tr>
<th>Layer</th>
<th>$z$ (m)</th>
<th>$\frac{z}{B}$</th>
<th>$\Delta q/q_0$</th>
<th>$\Delta q$</th>
<th>$P_0'$ (kPa)</th>
<th>$P_c'$ (kPa)</th>
<th>$P_0' + \Delta q$</th>
<th>$\frac{\Delta e}{1 + e_0} \times \Delta H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>0.25</td>
<td>0.90</td>
<td>56.3</td>
<td>10</td>
<td>20</td>
<td>66.3</td>
<td>0.1188</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>0.1188</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>3</td>
<td>0.75</td>
<td>0.50</td>
<td>31.3</td>
<td>30</td>
<td>60</td>
<td>61.3</td>
<td>0.0165</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td>0.0165</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>6</td>
<td>1.50</td>
<td>0.16</td>
<td>10.0</td>
<td>60</td>
<td>120</td>
<td>70.0</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td></td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>10</td>
<td>2.5</td>
<td>0.07</td>
<td>4.4</td>
<td>100</td>
<td>200</td>
<td>104.4</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\sum = 0.1433\text{m}$
2. **Final Settlement Analysis (cont’d.)**

(e) **EXAMPLE – Final Consolidation Settlement (cont’d.)**

1) From Figure 3.41, Notes p. 12
   → influence depth \( \{10\% \rightarrow 2B, \approx 5\% \rightarrow 3B\} = 12 \text{ m.} \)
2) Subdivide the influence zone into 4 sublayers 2 of 2m in the upper zone (major stress concentration) and 2 of 4 m below.
3) Calculate for the center of each layer: \( \Delta q, P_o', P_c', P_f' \)
4) \( e_o = \omega_n G_s = 1.0 \)
5) Calculate \( \Delta e \) for each layer:

\[
\Delta e_1 = c_r \log \frac{20}{10} + C_c \log \frac{66}{20} = 0.1188 \\
\Delta e_2 = c_r \log \frac{60}{30} + C_c \log \frac{61}{60} = 0.0165 \\
\Delta e_3 = c_r \log \frac{70}{60} = 0.003 \\
\Delta e_4 = c_r \log \frac{104}{100} = 0.001
\]
2. **Final Settlement Analysis (cont’d.)**

(e) **EXAMPLE – Final Consolidation Settlement (cont’d.)**

For the evaluation of the increased stress, use general Charts of Stress distribution beneath a rectangular and strip footings

Use Figure 3.41 (p.12 of notes)

\[ \Delta p / q_0 \text{ vs. } z / B \text{ under the center of a rectangular footing} \]

(use \( L / B = 1 \))
Consolidation Settlement - Long Term Settlement

2. **Final Settlement Analysis** (cont’d.)

(e) **EXAMPLE – Final Consolidation Settlement** (cont’d.)

Das “Principle of Foundation Engineering”, 3rd Edition

Figure 3.41 Increase of stress under the center of a flexible loaded rectangular area
2. Final Settlement Analysis (cont’d.)

(e) EXAMPLE – Final Consolidation Settlement (cont’d.)

6) The final settlement, not using the table:

\[ \Delta H = \sum \Delta H_i \frac{\Delta e_i}{1 + e_0} = 2m \times \frac{0.1188}{1 + 1} + 2m \times \frac{0.0165}{1 + 1} + 4m \times \frac{0.003}{1 + 1} + 4m \times \frac{0.001}{1 + 1} = 0.14m \]

\[ = 14cm \]

note: upper 2m contributes \( \approx 85\% \) of the total settlement
2. **Final Settlement Analysis (cont’d.)**

(e) **EXAMPLE – Final Consolidation Settlement (cont’d.)**

6) The final settlement, not using the table: (cont’d.)

**Skempton - Bjerrum Modification**

Use Figure 5.31, p. 276

\[ A \simeq 0.4 \quad Hc/B \gg 2 \quad \text{Settlement ratio} < 0.57 \]

\[ Sc < 0.57 \times 14 = 8cm \quad \text{Sc} < 8cm \]

- **Check solution when using equation 5.84 and the average stress increase:**

\[ \Delta \sigma'_{av} = \frac{1}{6}(\Delta \sigma'_{t} + 4\Delta \sigma'_{m} + \Delta \sigma'_{b}) \]

Like before, assume a layer of 3B = 12m

\[ \Delta \sigma'_{t} = q_{o} = \frac{1000kN}{16} = 62.5 \text{ kPa} \]

\[ \Delta \sigma'_{m} (\text{ @6m = 1.5B}) \simeq 0.16q_{o} \]

\[ \Delta \sigma'_{b} (\text{ @12m = 3B}) \simeq 0.04q_{o} \]

\[ \Delta \sigma'_{av} = 1/6 (1 + 4 \times 0.16 + 0.04)q_{o} = 1/6 \times 1.68 \times 62.5 = 0.28 \times 62.5 = 17.5 \text{ kPa} \]

\[ \Delta \sigma'_{av} = 17.5 \text{ kPa} \]
2. Final Settlement Analysis (cont’d.)

(e) EXAMPLE – Final Consolidation Settlement (cont’d.)

6) The final settlement, not using the table: (cont’d.)

\[ Z = 6m, \frac{\Delta q}{q_0} = 0.28 \quad \Delta q = 17.5 \text{ kPa} \]

\[ P'_o = 60kPa, P'_c = 120kPa \quad P'_f = 77.5kPa \]

\[ \Delta e = C_r \log \frac{77.5}{60} = 0.05 \times 0.111 = 0.0056 \]

\[ \frac{\Delta e}{1+e_0} \times \Delta H = \frac{0.0056}{1+1} \times 12m = 0.033m = 3.33 \text{ cm} \]

Why is there so much difference?

As OCR does not change with depth, the influence of the additional stresses decrease very rapidly and hence the concept of the "average point" layer does not work well in this case. The additional stresses at the representative point remain below the maximum past pressure and hence large strains do not develop. The use of equation 5.84 is more effective with a layer of a final thickness.
2. **Final Settlement Analysis (cont’d.)**

(f) **Terzaghi’s 1-D Consolidation Equation**

Terzaghi used the known diffusion theory (e.g. heat flow) and applied it to consolidation.

1) The soil is homogenous and fully saturated
2) The solid and the water are incompressible
3) Darcy’s Law governs the flow of water out of the pores
4) Drainage and compression are one dimensional
5) The strains are calculated using the small strain theory, i.e. load increments produce small strains
Consolidation Settlement - Long Term Settlement

2. **Final Settlement Analysis (cont’d.)**

(f) **Terzachi’s 1-D Consolidation Equation (cont’d.)**

\[
\Delta V_z = \left[ V_z - \left( V_z + \frac{\partial V_z}{\partial z} \right) \right] dx dy
\]

\( V_z = k_z \cdot \frac{\partial h}{\partial z} \quad k = \text{coeff. of permeability} \)

\[
\Delta V_z = -\frac{\partial V_z}{\partial z} \quad dx dy dz
\]

Subst (i) into (iv)

\[
\Delta V_2 = -\frac{\partial V_z}{\partial z} \quad dx dy dz
\]

The volume of the water in the element:

\[
V_w = \frac{s \cdot e}{1 + e_0} \quad V_7 = \frac{s \cdot e}{1 + e_0} \quad dx dy dz
\]

\[
\Delta V_2 = \frac{\partial V_w}{\partial z} = \frac{\partial}{\partial t} \left( \frac{s \cdot e}{1 + e_0} \right) \quad (5) \text{ rate of change of water volume}
\]

\[
V_s = \frac{V_7}{1 + e_0} = \frac{dx dy dz}{1 + e_0} \quad \text{Volume of solid in the element = constant.}
2. Final Settlement Analysis (cont’d.)

(f) Terzachi’s 1-D Consolidation Equation (cont’d.)
2. **Final Settlement Analysis (cont’d.)**
   
   (f) **Terzachi’s 1-D Consolidation Equation (cont’d.)**

\[
\begin{align*}
\frac{\partial u}{\partial z} &= 0 \\
\text{USS varies linearly so } \frac{\partial u_{SS}}{\partial z} &= \text{const} \quad \text{and } \frac{\partial^2 u}{\partial t^2} = 0
\end{align*}
\]

\[
\text{hence:}
\]

\[
\frac{k(1+e)}{\lambda u} \frac{\partial^2 u}{\partial z^2} = -\frac{\partial u}{\partial t}
\]

\[
\frac{k(1+e)}{\lambda w} \frac{\partial^2 w}{\partial z^2} = -\frac{\partial w}{\partial t}
\]

\[
C_v \frac{\partial^2 u}{\partial t^2} = \frac{\partial u}{\partial t}
\]

\[
C_v \frac{\partial^2 w}{\partial t^2} = \frac{\partial w}{\partial t}
\]

\[
C_v = \frac{\partial^2 u}{\partial t^2} = \frac{\partial u}{\partial t}
\]

\[
C_v = \frac{\partial^2 w}{\partial t^2} = \frac{\partial w}{\partial t}
\]

\[
\frac{C_v}{\lambda w} \frac{\partial^2 w}{\partial z^2} = 1-D \text{ consolidation}
\]

\[
C_v \text{ is a diffusion constant usually obtained directly from the consolidation test. It is actually not a constant but only due to our simplifications seen as } \lambda, \text{ and } \frac{1}{\lambda w} \text{ constants, it becomes one.}
\]
Consolidation Settlement - Long Term Settlement

2. **Final Settlement Analysis (cont’d.)**

(f) **Terzachi’s 1-D Consolidation Equation (cont’d.)**
3. **Time Rate Consolidation** (sections 1.15 and 1.16 in the text, pp.38-47)

(a) **Outline of Analysis**

The consolidation equation is based on homogeneous completely saturated clay-water system where the compressibility of the water and soil grains is negligible and the flow is in one direction only, the direction of compression.

Utilizing Darci’s Law and a mass conservation equation \( \text{rate of outflow} - \text{rate of inflow} = \text{rate of volume change} \); leads to a second order differential equation

\[
C_v \frac{\partial^2 u_e}{\partial z^2} = \frac{\partial u_e}{\partial t} - \frac{\partial \sigma_v}{\partial t}
\]

\( u_e = \) excess pore pressure
\( \sigma_v = \) vertical effective stress

Practically, we use either numerical solution or the following two relationships related to two types of problems:
Consolidation Settlement - Long Term Settlement

3. **Time Rate Consolidation** (cont’d.)

(a) Outline of Analysis (cont’d.)

**Problem 1**: Time and Average Consolidation

**Equation 1)**

\[ t_i = \frac{T_v H_{dr}^2}{C_v} \]

\( t_i \) - The time for which we want to find the average consolidation settlement.

See Fig. 1.21 (p.42) in the text, and the tables on p.56-58 in the notes.

\( T_v = \) time factor \( \rightarrow T = f (U_{avg}) \)

\( H_{dr} = \) the layer thickness of drainage path.

\( C_v = \) coeff. of consolidation = \( \frac{k}{\gamma_w m_v} \)

\( m_v = \) coeff. Of volume comp. = \( \frac{a_v}{1+e_0} \)

\( a_v = \) coeff. Of compression = \( \frac{\Delta e}{\Delta p} \)
3. **Time Rate Consolidation** (cont’d.)

(a) **Outline of Analysis** (cont’d.)

**Problem 1**: Time and Average Consolidation

Equation 2) \[ U_{avg} = \frac{S_t}{S_\infty} = \frac{\text{Settlement of the layer at time } t}{\text{Final settlement due to primary consolidation}} \]

---

For initial constant pore pressure with depth

---

![Graph](Figure 7.25 Variation of average degree of consolidation with time factor, $T_r$ (u0 constant with depth))
3. **Time Rate Consolidation** (cont’d.)

(a) **Outline of Analysis** (cont’d.)

**Problem 2:** Time related to a consolidation at a specific point

Equation 3) Degree of consolidation at a point

\[ U_{zt} = 1 - \frac{u_{z,t}}{u_{z,0}} \]

Pore pressure at a point (distance \( z \), time \( t \))

\[ U_{z,t} = \gamma_w \times h_w \]

For initial linear distribution of \( \Delta u_i \) the following distribution of pore pressures with depths and time is provided.

**Fig. 1.20 (c)**
Plot of \( \Delta u/\Delta u_0 \) with \( T_v \) and \( H/H_c \) (p.39)
Consolidation Settlement - Long Term Settlement
Table 1

One-Dimensional Consolidation Theory: Solutions for Four Cases of Initial Excess Pore Water Pressure Distribution in Double-Drained Stratum.

<table>
<thead>
<tr>
<th>Average Degree of Consolidation for Various Values of T</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (%)</td>
<td>0.004</td>
<td>0.008</td>
<td>0.012</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>7.14</td>
<td>6.49</td>
<td>6.98</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Fig. 7.8 - Initial excess pore water pressure distribution for double-drained and single drained strata for which Table 1 is applicable.

see Table 1 for initial excess pore pressure distribution
3. **Time Rate Consolidation (cont’d.)**

(b) **Obtaining Parameters from the Analysis of e-log t Consolidation Test Results**

1. **find** $d_0$ - 0 consolidation time $t = 0$
   set time $t_1, t_2 = 4t_1, t_3 = 4t_2$
   find corresponding $d_1, d_2, d_3$
   offset $d_1 - d_2$ above $d_1$ and $d_2 - d_3$ above $d_2$

2. **find** $d_{100}$ - 100% consolidation
   referring to primary consolidation (not secondary).

3. **find** $d_{50}$ and the associated $t_{50}$
3. **Time Rate Consolidation** (cont’d.)

(b) **Obtaining Parameters from the Analysis of e-log t Consolidation Test Results** (cont’d.)

**Coefficient of consolidation**

\[ C_v = \frac{T_i H_{dr}^2}{t_i} \]

- \( T_i \) = time factor (equation 1.75, p.41 of text)
- \( H_{dr} \) = drainage path = \( \frac{1}{2} \) sample
- \( t_i \) = time for \( i\% \) consolidation

Using 50% consolidation and case I

\[ C_v = \frac{0.197 H_{dr}^2}{t_{50}} \]

\( T \) for \( U_{avg} \) = 50%

and linear initial distribution
Consolidation Settlement - Long Term Settlement

3. **Time Rate Consolidation** (cont’d.)

(b) **Obtaining Parameters from the Analysis of e-log t Consolidation Test Results** (cont’d.)
3. **Time Rate Consolidation** (cont’d.)

(b) **Obtaining Parameters from the Analysis of e-log t Consolidation Test Results** (cont’d.)

**Coefficient of consolidation**

For simplicity we can write \( u(z_{iH}, t_j) = u_{i+1,j} \)

\[
C_v = \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}
\]

Substitute

\[
C_v \left( u_{i+1,j} - 2u_{i,j} + u_{i-1,j} \right) = \frac{u_{i,j+1} - u_{i,j}}{\Delta t}
\]

Which can easily be solved by a computer. For simplicity we can rewrite the above equation as:

\[
u_{i+1,j} = \alpha u_{i+1,j} + (1 - 2\alpha)u_{i,j} + \alpha u_{i-1,j}
\]

For which:

\[
\alpha = \frac{C_v \cdot \Delta t}{(\Delta z)^2} \leq 0.5
\]
3. **Time Rate Consolidation** (cont’d.)

(b) **Obtaining Parameters from the Analysis of e-log t Consolidation Test Results** (cont’d.)

**Coefficient of consolidation**

For $\alpha = 0.5$ we get:

$$u_{i,j+1} = \frac{1}{2}(u_{i-1,j} + u_{i+1,j})$$

This form allows for hand calculations

e.g. For $i=2, j=3$

$$u_{2,4} = \frac{1}{2} (u_{1,3} + u_{3,3})$$
3. **Time Rate Consolidation** (cont’d.)

(b) **Obtaining Parameters from the Analysis of e-log t Consolidation Test Results** (cont’d.)

---

**Example**

Find \( u(x,t) \) using the simplified finite differences solution for double drainage and rectangular initial pore pressure distribution.

\[
H = 10 \\
\text{no. of sublayers} \\
C_v = 10^{-5} \text{ m}^2/\text{min} \\
\Delta t = 50 \text{ m} \\
H = 25 \text{ m} \\
\Delta t = \frac{Q (C_v)^2}{0.5 \times 25} = \frac{0.5 \times 25^2}{10^{-5}} = 3.125 \times 10^5 \text{ min} = 217 \text{ days}
\]

---

<table>
<thead>
<tr>
<th>Layer</th>
<th>( u(x,t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
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<tr>
<td>3</td>
<td>7.5</td>
</tr>
<tr>
<td>4</td>
<td>10.0</td>
</tr>
<tr>
<td>5</td>
<td>12.5</td>
</tr>
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<td>6</td>
<td>15.0</td>
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<tr>
<td>7</td>
<td>17.5</td>
</tr>
<tr>
<td>8</td>
<td>20.0</td>
</tr>
<tr>
<td>9</td>
<td>22.5</td>
</tr>
<tr>
<td>10</td>
<td>25.0</td>
</tr>
</tbody>
</table>

---

14.533 Advanced Foun
4. Consolidation Example

The construction of a new runway in Logan Airport requires the pre-loading of the runway with approximately 0.3 tsf. The simplified geometry of the problem is as outlined below, with the runway length being 1 mile.
4. Consolidation Example (cont’d.)

1) Calculate the final settlement.

Assuming a strip footing and checking the stress distribution under the center of the footing using Fig. 3.41 (p. 12 of the notes)

<table>
<thead>
<tr>
<th>Location</th>
<th>z (ft)</th>
<th>z/B</th>
<th>( \Delta q /q_o )</th>
<th>( \Delta q ) (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of Clay</td>
<td>10</td>
<td>0.2</td>
<td>~0.98</td>
<td>588</td>
</tr>
<tr>
<td>Middle of Clay</td>
<td>25</td>
<td>0.5</td>
<td>~0.82</td>
<td>492</td>
</tr>
<tr>
<td>Bottom of Clay</td>
<td>40</td>
<td>0.8</td>
<td>~0.60</td>
<td>360</td>
</tr>
</tbody>
</table>

Using the average method

\[
\Delta \sigma'_{av} = \frac{1}{6} (\sigma'_t + 4\Delta \sigma'_m + \Delta \sigma'_b) = \frac{1}{6} (588 + 4 \times 492 + 360) = 486 \text{ psf}
\]
4. **Consolidation Example (cont’d.)**

1) Calculate the final settlement (cont’d.)

The average number agrees well with the additional stress found for the center of the layer, (492psf).

Assuming that the center of the layer represents the entire layer for a uniform stress distribution. At 25 ft:

\[ p'_o = \sigma'_v = 115 \times 5 + (115 - 62.4) \times 5 + (110 - 62.4) \times 15 \]
\[ = 575 + 263 + 714 = 1552 \text{psf} \]

\[ p'_t = p'_o + \Delta q = 1552 + 486 = 2038 \text{psf} \]

\[ \Delta e = C_c \log \left( \frac{p'_t}{p'_o} \right) = 0.35 \log \left( \frac{2038}{1552} \right) = 0.0414 \]

\[ s = \Delta H = H \left( \frac{\Delta e}{1 + e_0} \right) = 30 \text{ft} \times 12 \text{inch} \times \left( \frac{0.0414}{1 + 1.1} \right) = 7.1 \text{inch} \]
4. **Consolidation Example (cont’d.)**

2) Assuming that the excess pore water pressure is uniform with depth and equal to the pressure at the representative point, find:

(a) The consolidation settlement after 1 year

- Find the time factor:
  
  \[ t_i = \frac{T_v H_{dr}^2}{C_v} \quad \quad \quad T_v = \frac{t_i C_v}{H_{dr}^2} \]

  \( C_v = 0.05 \text{ cm}^2/\text{min} = 0.00775 \text{ in}^2/\text{min} \)
  
  \( H_{dr} = H/2 = 30 \text{ ft} / 2 = 15 \text{ ft} \)
  
  \( T_v = 12 \times 30 \times 24 \times 60 \times 0.00775 / (15 \times 12)^2 = 0.124 \)

- Find the average consolidation for the time factor.

For a uniform distribution you can use equation 1.74 (p.41) of the text or the chart or tables provided in the notes.
Consolidation Settlement - Long Term Settlement

4. **Consolidation Example (cont’d.)**

2) Assuming that the excess pore water pressure is uniform with depth and equal to the pressure at the representative point, find:

(a) The consolidation settlement after 1 year

- Find the average consolidation for the time factor.

Using the table in the class notes (p.56 & p.58)

\[ T = 0.125 \rightarrow \text{Case I - uniform or linear initial excess pore pressure distribution.} \rightarrow U = 39.89\% = 40\% \]

\[ U_{avg} = \frac{S_t}{S_\infty} \]

\[ S_t = U_{avg} \times S_\infty \]

\[ S_t = 0.40 \times 7.1 = 2.84 \text{ inch} \]
4. **Consolidation Example (cont’d.)**

2) Assuming that the excess pore water pressure is uniform with depth and equal to the pressure at the representative point, find: (cont’d.)

(b) What is the pore pressure 10 ft. above the till 1 year after the loading?

From above; \( t = 12 \) months, \( T = 0.124 \)
\( 2 \ H_d = 30 \) ft
\( z / H_{dr} = 20/15 = 1.33 \) (\( z \) is measured from the top of the clay layer)

Using the isochrones with \( T = 0.124 \) and \( z/H = 1.33 \)
We get \( u_e / u_i \approx 0.8 \)
\( u_e = 0.8 \times 486 = 389 \) psf
4. **Consolidation Example (cont’d.)**

2) Assuming that the excess pore water pressure is uniform with depth and equal to the pressure at the representative point, find: (cont’d.)

(c) What will be the height of a water column in a piezometer located 10 ft above the till: (i) immediately after loading and (ii) one year after the loading?

(i) \( u_i = 486 \text{ psf} \quad h_i = \frac{u}{\gamma_w} = \frac{486}{62.4} = 7.79 \text{ ft} \).

(ii) \( u_e = 389 \text{ psf} \quad h = \frac{u}{\gamma_w} = \frac{389}{62.4} = 6.20 \text{ ft} \).

The water level will be 2.79 ft. above ground and 1.2 ft above the ground level immediately after loading and one year after the loading, respectively.
Consolidation Settlement - Long Term Settlement

The 27th Terzaghi Lecture, 1991
Annual Convention
JGE, ASCE Vol. 119, No. 9, Sept. 1993

THE TWENTY-SEVENTH TERZAGHI LECTURE

Presented at the American Society of Civil Engineers
1991 Annual Convention
October 22, 1991

J. MICHAEL DUNCAN
5. Secondary Consolidation (Compression) Settlement

Figure 5.33 (p.279)
(a) Variation of e with log t under a given load increment, and definition of secondary compression index.
5. **Secondary Consolidation (Compression) Settlement (cont’d.)**

Following the full dissipation of the excess pore pressure, (primary consolidation) more settlement takes place, termed secondary compression or secondary consolidation. This settlement under constant effective stresses is analogous to creep in other materials. The secondary consolidation is relatively small in regular clays but can be dominant in organic soils, in particular peat.

\[ C_\alpha = \frac{\Delta e}{\log\left(\frac{t_2}{t_1}\right)} \]

Magnitude of secondary consolidation:

\[ S_{c(s)} = \frac{\Delta e}{1 + e_0} H_c \]

where: \( \Delta e = C_\alpha \log\left(\frac{t_2}{t_1}\right) \)

Clays \( C_\alpha /cc \approx 0.045 \pm 0.01 \)

Peats \( C_\alpha /cc \approx 0.075 \pm 0.01 \)
ENGINEERING PROPERTIES OF CRANBERRY BOG PEAT

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ABSTRACT

Peat is an organic complex soil, well known for its high compressibility and low stability. Peat forms naturally by the incomplete decomposition of plant and animal constituents under anaerobic conditions at low temperatures. A relocation of state highway No. 44 in Carver, Massachusetts requires the construction of sheet pile walls, fills and embankments through cranberry bog and ponds containing deep peat deposits. The engineering properties of Carver peat in southern Massachusetts (south of Boston) were investigated via laboratory testing including standard index tests, fiber content, engineering classification, consolidated undrained triaxial tests, and oedometer tests. The tests were carried out on vertically and horizontally oriented undisturbed samples. Unlike inorganic clays, the secondary compression of peat is of great significance as it dominates its deformation and takes place over a long period of time. The present test program examines the deformation properties of the peat and the ratio of the coefficient of secondary compression (C_{s}) to compression index (C_{c}). The data are compared to those reported in the literature. The obtained engineering properties were found to be overall within the range reported for other peat types. The peat structure and fiber orientation seem to affect the properties. The time for primary consolidation for horizontally oriented samples decreases due to an increase in the horizontal permeability and the time of secondary compression increases due to compression mostly normal to the fibers' orientation.

1. INTRODUCTION

1.1 Background

US Route 44 spans east west across southeastern Massachusetts into Rhode Island. The Massachusetts Highway Department (MHD) is relocating Route 44 under project no. 113300. Parts of the new highway alignment span across ponds and cranberry bogs in the town of Carver, located about 40 miles southeast of Boston. The proposed roadway is a four lane divided highway with a typical median width of 60 feet. Environmental concerns dictated that sheet piles need to be placed at the ponds and bogs roadway sections, in order to excavate and replace the underlying organic soils and construct the embankments and roadway. The design of sheet piles supported by organic soils raises the difficulties of assigning engineering parameters to peat. These difficulties prevail whenever other engineering alternatives are considered. The objective of the presented work is to assess the engineering properties of the peat found along the proposed highway, and which is currently supporting the sheet piling. The investigated properties are to be utilized in the analysis of the supporting sheet piles and compared with the wall performance during construction as monitored by instrumentation.

1.2 Subsurface Conditions

Extensive subsurface investigation shows that the soil type and density is relatively consistent throughout the project and the wetland areas. The soil profile consists primarily of fibrous peat within a fine to coarse sand layer. The thickness of the Peat deposits range from 0 to 10.7 m (35 feet) and the ground
“Engineering Properties of Cranberry Bog Peat”

by

2nd International Conference on Advances in Soft Soil Engineering and Technology
2-4 July 2003, Patongjaya, MALAYSIA.

Fig. 1 Typical subsurface conditions in the wetland area along a section of Route 44 between stations 138 and 152.
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2.2 Carver Peat Characteristics

The obtained peat, termed Carver peat is classified by the different peat classifications in the following way: Fibrous according to the plasticity chart for peat suggested by Cauzminde (1966), Fibric according to Lynn et al. (1974) and Hemic, high sulf, moderately acidic, and highly absorbent according to ASTM D-4447. The color of carver peat is dark brown to brownish-orange, it has strong odor and contains small woody elements. The humification degree of Carver peat is H4 to H5 using Von Post’s Humification Scale, (ASTM D5715). The principal characteristics of the Carver peat are summarized in Table 2.

3. CONSOLIDATION TESTS

3.1 General Details

Three vertically oriented samples and four horizontally oriented samples were tested in odometer cells with the details outlined in Table 3. The effective overburden pressure for the sampled peat (mid point) was approximately 1.2 kPa with effective preconsolidation pressure of approximately 9 kPa, and a resulting over consolidation ratio of about 7.5. Sample preparation of peat is more difficult than that of the typical inorganic soils due to the presence of fibers, the high initial water content and voids ratio. To minimize sample disturbance the samples were trimmed using a very sharp razor knife, and special care was taken in its placement. The porous stones were fully saturated before the test and filter papers were used to margin the biodegradation and decomposition of the samples. This is necessary considering the long period of time required for the consolidation tests in which each applied increment was sustained for about 10,000 minutes (1 week). A thin film of Silicon grease was applied to the cell wall in order to minimize the side friction. The consolidation tests were carried out at approximately constant temperature of 22 ± 4 °C.

<table>
<thead>
<tr>
<th>Test Type</th>
<th>No. Of Planned Tests</th>
<th>No. Of Performed Tests</th>
<th>Comments</th>
</tr>
</thead>
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<tr>
<td>(Bulk Unit Weight)</td>
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<td>2</td>
<td>One test for each block</td>
</tr>
<tr>
<td>Specific Gravity</td>
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<td>2</td>
<td></td>
</tr>
<tr>
<td>Organic Content</td>
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<td>2</td>
<td></td>
</tr>
<tr>
<td>pH</td>
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<td>1</td>
<td></td>
</tr>
<tr>
<td>(Liquid Limit), (Plastic Limit)</td>
<td>2</td>
<td>2</td>
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<th>Horizontal Samples</th>
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<th>Performed</th>
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<td>Direct Shear Test</td>
<td>4</td>
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<td>4</td>
<td>4</td>
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</tr>
</tbody>
</table>

Table 1: Testing program of Carver peat
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3.3 Time – Settlement Relationship

The formulation of the consolidation process for fully saturated soils, (Terzaghi, 1923) assumes that the soil particles and water are incompressible and deformation takes place due to expulsion of water from the pores under the influence of hydrodynamic effects upon loading. This compression process, termed primary consolidation, assumes relationship between effective stresses to void ratio and should cease therefore when the dissipation of the excess pore-water pressure is completed. In fine-grained soils the compression continues after the dissipation of pore water pressure is completed and takes place under a constant effective stress in what is termed secondary compression or creep. Due to the high permeability of peat, the primary consolidation is relatively short but the secondary compression takes place over a lengthy period of time and hence is of great significance. The secondary compression has been attributed to the plastic deformation of the highly viscous adsorbed double layer and continuous adjustment and arrangement of soil constituents after they have been distributed during the primary consolidation, (Dhowian, 1978). Accordingly, the primary consolidation method of settlement analysis developed by Terzaghi seems to be inappropriate to address the secondary compression. Many investigators have assumed and used different relationships and models to describe the secondary compression. Gibson and Lo (1961) identified three types of secondary compression curves relating to the relationship between settlement and time on a logarithm scale; type 1 shows a gradual decrease in the rate of secondary compression until ultimate settlement is finally reached; type 2 exhibits a proportional relationship between secondary compression and logarithm of time for a significantly long period of time before reaching the final settlement, and type 3 shows a proportional relationship to a certain point at which an acceleration of the rate of secondary compression takes place, believed to be the result of bond breakage of between particles. The compression-log time curve of type 3 materials consists of four components of strain, instantaneous strain which takes place immediately after load application, primary strain which lasts in most cases for several minutes, secondary strain which has a constant rate with log time and lasts for a considerable period of time and tertiary strain which is a higher rate secondary strain. This phenomenon is believed to be due to the breakage of bonds between particles and a curved transition zone usually exists from the secondary to the tertiary zones.

Fig. 2. Void ratio versus consolidation pressure for samples oriented vertically
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3.4 Secondary and Tertiary Compression

Obtained relations and the related parameters

Figures 5 and 6 describe some of the relationships between the void ratio and time for the vertically oriented samples under different loads. The obtained relations show that Currier peat behavior is in agreement with the aforementioned type 3 curves, exhibiting an accelerated rate of secondary compression. Dhovam and Shih (1980) and Meier and Choi (1985) suggested that secondary compression begins after the primary compression ends, this hypothesis was adopted in this research study finding the related parameters in the following way:

(i) \( t_P \) - the time at the end of primary consolidation, (EDP employing Taylor’s square root method (Taylor, 1942).)

(ii) \( C_s \) - the coefficient of secondary compression, defining the tangential slope (\( k_o \) \( b_o \)).

(iii) \( t_D \) - the designated time for the end of secondary compression and the beginning of the tertiary compression, defined by the interception of the tangents to the curves in the secondary and tertiary zones.
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The time for secondary and tertiary compression

Figures 7a,b present the time in which the primary compression is completed and the secondary compression starts, $t_1$, versus the consolidation pressure for vertically and horizontally oriented samples, respectively. In all cases, the primary consolidation takes place within 3 minutes, and for the horizontally oriented samples, the time is about one half of the time required to complete the primary consolidation in the vertically oriented samples. Figures 8a,b present the time in which the secondary compression is completed and the tertiary compression starts, $t_2$, versus the consolidation pressure for vertically and horizontally oriented samples, respectively. The time of the secondary compression is measured in hundreds to thousands minutes with distinctive peak(s) at particular stress levels. Overall, the time required for secondary compression is longer in the horizontally oriented samples compared with the time required for the vertically oriented samples under the same consolidation pressure.

It seems that the behavior observed in figures 7 and 8 is associated with the structure of the peat and its deposition process, having the majority of the fibers oriented horizontally. Such structure results with permeability in the horizontal direction being higher than in the vertical direction, and hence the time for the primary consolidation being shorter. In contrast, the structure in the vertical direction is more easily
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Fig. 7. The time to the end of primary consolidation and beginning of the secondary compression versus consolidation pressure for vertically (a) and horizontally (b) oriented samples.
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Fig. 8. The time to the end of secondary consolidation and beginning of the tertiary compression (tE) versus consolidation pressure for vertically (a) and horizontally (b) oriented samples.

Coefficients of secondary and tertiary compression of vertically oriented samples

Figures 9 and 10 present the values of the coefficient of the secondary compression (C_q) and tertiary compression (C_s) versus the consolidation pressure for test no. 4, respectively. Beyond a pressure of about 1 kPa, approximately a linear increase exists between the stress and the value of the coefficient of secondary compression, on a log stress axis. Variations of the values of the coefficient of tertiary compression exist with the increase of the consolidation stresses. Figures 11 and 12 present the values of the coefficient of secondary compression (C_q) and tertiary compression (C_s) versus the consolidation pressure in the range of 10 to 100 kPa, respectively. The data in Figure 11 suggests that the values of C_s
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Figures 13 and 14 present the values of the coefficient of secondary compression ($C_{s}$) and tertiary compression ($C_{t}$) versus the consolidation pressure for the consolidation tests of the horizontally oriented samples, respectively. The coefficient of secondary compression show an approximate constant value of 0.02 to the pressure of 5.0 kN/m$^2$ from which a linear increase is observed up to a pressure of about 100 kPa beyond which the data is scattered. The coefficient of secondary compression $C_{s}$ has the average value of (0.03) within the consolidation pressure range of 0.10 to 10.0 kPa and a range of values between 0.05 to 0.15 for the stress levels of 10 to 100 kPa. These values are about two third of the values observed for the vertically loaded samples under the same pressure range (Fig. 11). Mesri (1973) reported on conflicting relationships that have been proposed regarding the coefficient of secondary compression.
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Newland and Allely (1960) indicate $C_a$ independent of consolidation pressure. Wahls, (1962) indicates $C_a$ decreases with pressure. Ladd and Preston, (1965) indicate $C_a$ increases slightly with consolidation pressure. In this paper, $C_a$ may be assumed to be constant within some levels of stresses, but generally $C_a$ increases with consolidation pressure for both horizontally and vertically oriented samples. The data in figure 14 suggests a gradual consistent increase in the coefficient of tertiary compression with the increase of the consolidation pressure. This trend is opposite to that observed for the vertically loaded samples (Fig. 12).

As tertiary compression is an accelerated rate of the secondary compression, a ratio between the two may be both feasible and practical. Figure 15 presents this ratio ($C_a/C_n$) for all the tests. While the horizontally oriented samples show a larger scatter (open symbols) the ratio remains limited in magnitude for most consolidation pressures, resulting in $C_a/C_n = 3.4 \pm 1.8$ (± 1SD, 22 points) for the consolidation pressures between 10 to 100 kPa.
3.5 The Relationship between The Primary And The Secondary Compression Indices ($C_d/C_s$)

Mesri and Godlewski (1977) suggested that for natural soils, there seems to be a unique relationship between $C_d$ and $C_s$ that holds good at any effective pressure, void ratio, and time during secondary compression. Fox et al. (1992), however, reported that the ratio $C_d/C_s$ is not constant because $C_d$ increases with time under constant effective stress. Very often tertiary compression is also seen following secondary compression. Figure 16 shows the variation of the ratio $C_d/C_s$ with the consolidation pressure for test 4. It can be seen that the ratio $C_d/C_s$ ranges from 0.0026 to 0.058 and is not constant. Table 4 summarizes the range of values for the ratio $C_d/C_s$ found in the different tests, referring to all stresses tested and to a range between 10 to 100 kPa. When referring to a limited range of stresses (mostly beyond 10 kPa) the ratio of $C_d/C_s$ seem to remain in a relatively small range for all practical proposes. This range did not differ much between the vertically and the horizontally oriented samples.
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4. CONSOLIDATED UNDRAINED TRIAXIAL TESTS

Isotropically consolidated undrained triaxial compression tests were performed on samples obtained from a depth of 1.80 m. The undisturbed triaxial specimens were approximately 7.0 cm in diameter and 15.25 cm in height with an aspect ratio of 2.17. The specimens were taken from larger block samples and carefully trimmed to size using a razor knife. The porous stones were fully desired and saturated with water. The drainage lines were flushed with water to eliminate air bubbles. Full saturation of the samples is essential in order obtain reliable pore pressure readings. The soft peat samples obtained from the field were essentially saturated. However, the triaxial specimens enclosed in the membrane were flushed with desired water under a low hydraulic gradient to remove any trapped air bubbles. The saturated samples yielded B values higher than 0.598.

Deviator stress versus axial strain for triaxial tests performed on vertically oriented peat samples are shown in Figure 17a. Results of excess pore-water pressure versus axial strain are shown in Figure 17b. The tests were performed at four different confining pressures (0.1 psi, 5 psi, 10 psi, and 20 psi). Apparently, the higher the consolidation stress the higher the strength. The following effective stress shear strength parameters were obtained from the Mohr-Coulomb failure envelopes presented in Figure 18: \( \phi = 12^\circ, c = 12 \text{ kN/m}^2 \). These preliminary triaxial test results differ from those reported by Edil and Wang (2000), that suggest higher friction angles and negligible cohesion for normally consolidated peats. Future testing of Curver peat will further address this issue.

Table 4. Values of \( C_a/C_e \) for the various tests

<table>
<thead>
<tr>
<th>Test No.</th>
<th>( C_a/C_e )</th>
<th>( 10 &lt; \sigma &lt; 100 \text{ kPa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.026-0.038</td>
<td>0.026-0.038</td>
</tr>
<tr>
<td>2</td>
<td>0.028-0.047</td>
<td>0.028-0.047</td>
</tr>
<tr>
<td>3</td>
<td>0.0056-0.058</td>
<td>0.009-0.059</td>
</tr>
<tr>
<td>4</td>
<td>0.0075-0.085</td>
<td>0.019-0.041</td>
</tr>
<tr>
<td>5</td>
<td>0.0020-0.035</td>
<td>0.020-0.035</td>
</tr>
<tr>
<td>6</td>
<td>0.0074-0.055</td>
<td>0.029-0.047</td>
</tr>
<tr>
<td>7</td>
<td>0.0034-0.037</td>
<td>0.012-0.032</td>
</tr>
</tbody>
</table>

Fig. 17. (a) Axial strain versus deviator stress for the peat samples
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5. SUMMARY AND CONCLUSIONS

Peat samples from Carver, Massachusetts, were tested to characterize their engineering properties. Carver peat is fibrous, over consolidated and was tested on vertically and horizontally oriented samples. Conventional long duration oedometer tests showed that primary consolidation was very rapid, (especially for the horizontally oriented samples) and creep effect counted for the majority of the compression. The different long-term behavior curve related to the heterogeneity of the peat, the different fibrous content and the orientation of the loading relative to the orientation of the peat deposition.

The compression index for the vertically oriented samples was between 3.4 to 5.2. These values are within the range observed for other vertically loaded peat type at similar natural water contents. The compression index for the horizontally oriented samples was lower, at the approximate ratio of \( C_{v} / C_{h} \approx 0.75 \).

The time for primary consolidation for horizontally oriented samples is shorter compared to that in the vertically oriented samples. The time of secondary compression is longer in the horizontally oriented samples (while scattered was overall significantly longer) than that in the vertically oriented samples.
“Engineering Properties of Cranberry Bog Peat”

by


under the same consolidation stresses. These observations seem to be explained through the peat structure and fiber orientation such that the permeability increases along the fibers and the compressibility increases normal to the fibers’ orientation. Further research is required and will be carried out to examine these observations.

The coefficient of secondary compression ($C_s$) increases with the consolidation pressure once exceeding a threshold stress level between the overburden pressure and the preconsolidation pressure. A coefficient of secondary compression of $C_s = 0.15$ was found for the consolidation pressure in the range of 10 to 100 kPa. The coefficient of tertiary compression ($C_t$) decreased with the increase of the consolidation pressure for the vertically oriented samples and increased for the horizontally oriented samples. The trends and absolute values of the vertically oriented samples matched those reported in the literature for other peat types.

The ratio between the primary and secondary compression indices $C_s/C_h$ is not constant as $C_s$ varies with the consolidation pressure. This ratio seems to remain, however, within a relatively limited range of 0.03 ± 0.01 for stresses between 10 to 100 kPa, regardless of the orientation of the sample. The ratio between the tertiary to secondary compression indices ($C_t/C_s$) was found to be within the range of $3.4 \pm 1.8$ for both, vertically and horizontally oriented samples within a limited zone of consolidation pressure between 10 to 100 kPa.

Isotropically consolidated undrained triaxial compression tests were performed on Curver peat, showing that the peat has apparent cohesion of 12.0 kN/m² at 45 % fibers content, undrained angle of friction of 8°, and a drained angle of friction of 12°. These initial tests were performed without backpressure. Future planned tests will be performed using backpressure, and the results will be closely compared to those available for other peat types.

ACKNOWLEDGMENT

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BIographies

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Dr. Praveep U. Kurup is an Associate Professor in the Department of Civil and Environmental Engineering at the University of Massachusetts Lowell. He has vast expertise in advanced experimental techniques and in analytical modeling. He has done extensive research in the areas of site characterization and monitoring, application of novel sensing technologies to geotechnical and geo-environmental engineering, calibration chamber testing, soil-structure interaction, "smart-ahead techniques" for trenchless technologies, and artificial neural network modeling. He has published his research contributions in several journals and conferences proceedings. Dr. Kurup is an active member in several professional societies, and is also a registered Professional Engineer.

Assam Elsayed holds a BSc. in Civil Engineering from Alexandria University, Egypt in 1992. His engineering experience started in Alexandria and Cairo, Egypt, when he worked for several consulting offices in the field of structural and geotechnical engineering. After 4 years of working in Egypt, he joined Aramco Construction Inc. in Saudi Arabia, where he supervised the foundation and structural work of power plants in Dihlahan. In fall 2001 he has become a graduate research assistant at the University of Massachusetts Lowell during pursuing his master degree. Assam is the 1992 recipient of excellence award by Alexandria University for the best senior project in Highways and Airports design. He presented "The Engineering Properties of Peat" at the Northeast Geo-technical Graduate Research Symposium, Amherst Massachusetts 2002.
5. **Secondary Consolidation (Compression) Settlement (cont’d.)**

**Example**

Excavation and replacement of the organic soils was carried out between the sheet piles in Rt. 44 relocation project. Due to various reasons, a monitoring program has detected a remnant peat layer, 4ft thick as shown in the figure. Using the expected loads due to the fill and the MSE (Mechanically Stabilized Earth) Walls, estimate the settlement of the peat:

(a) During primary consolidation, and
(b) During secondary consolidation over a 30 year period.
Consolidation Settlement - Long Term Settlement

5. **Secondary Consolidation (Compression) Settlement (cont’d.)**

**Example (cont’d.)**

Peat Parameters:
Based on Table 2 of the paper, $\gamma_{sat} = 10.2\text{kN/m}^3 = 65\text{pcf}$
Based on Tables 3 and 4 for vertically loaded samples, $e_0 \approx 13$, $C_c \approx 4.3$, $C_s \approx 0.68$, $C_\alpha/C_c \approx 0.036 \rightarrow C_\alpha \approx 0.15$

(see Figure 11)
5. **Secondary Consolidation (Compression) Settlement (cont’d.)**

**Example (cont’d.)**

Assuming a 2-D problem and a peat cross-section before the excavation,

\[ \sigma'_{vo} = (110-107) \times 65 + (107-98)(65-62.4) = 218.4 \text{psf} \]

\[ \Delta \sigma'_v = (123-114) \times 120 + (114-107) \times 120 + (107-100)(122-62.4) + (100-98) \times (65-62.4) = 2342.4 \text{psf} \]

\[
S_c = \frac{C_c}{1 + e_0} \log \left( \frac{\sigma'_f}{\sigma'_0} \right) H_0 = \frac{4.3}{1 + 13} \log \left( \frac{2342.4 + 218.4}{218.4} \right)^4 = (0.307)(1.07)^4
\]

\[ = 1.31 \text{ft} = 15.75 \text{inch} \]

\[
S_{c(s)} = \frac{C_\alpha}{1 + e_0} \log \left( \frac{t}{t_p} \right) H_0
\]
5. **Secondary Consolidation (Compression) Settlement (cont’d.)**

**Example (cont’d.)**

Evaluation of $t_p$ – end of primary consolidation

From the consolidation test result,

$t_p\approx 2\text{min}$ (Figure 7a, and section 3.4.2 of the paper)

$$t = \frac{T_vH_{dr}^2}{C_v}$$

As $C_v$ and $T_v$ are the same for the sample and the field material:

$$\frac{t_{p\text{ field}}}{t_{p\text{ lab}} = \frac{H_{dr\text{ field}}^2}{H_{dr\text{ lab}}^2 = \left(\frac{H_{dr\text{ field}}}{H_{dr\text{ lab}}\right)^2}}

H_{dr\text{ lab}} = 2.89/2 = 1.45\text{inch} \quad H_{dr\text{ field}} = 2\text{ft} = 24\text{ inch} \quad \text{(see table 3)}

$t_{p\text{ field}}\approx 2\text{min} \times (24/1.45)^2 = 548\text{min} \approx 9.1\text{hours}$

$$S_c = \frac{0.15}{1 + 13} \log \left(\frac{(30)(365)(24)}{9.1}\right)4 = (0.011)(4.46)4 = 0.20\text{t} = 2.3\text{inch}$$

14.533 Advanced Foundation Engineering – Samuel Paikowsky
5. **Secondary Consolidation (Compression) Settlement (cont’d.)**

**Conclusions:**

1. A relatively thin layer of peat, 4ft thick, will undergo a settlement of 18 inches, 38%, of its thickness.

2. Most of the settlement will occur within a very short period of time, theoretically within 9 hours, practically within a few weeks.

3. The secondary settlement, which is significant, will continue over a 30-year period and may become a continuous source of problem for the road maintenance.
Consolidation Settlement - Long Term Settlement

5. **Secondary Consolidation (Compression) Settlement (cont’d.)**
5. Secondary Consolidation (Compression) Settlement (cont’d.)
Consolidation Settlement - Long Term Settlement

5. Secondary Consolidation (Compression) Settlement (cont’d.)
1. **Allowable Bearing Pressure in Sand Based on Settlement Consideration** (Section 5.13, pp. 263-267)

- Using an empirical correlation between N SPT and allowable bearing pressure which is associated with a standard maximum settlement of 1 inch and a maximum differential settlement of \( \frac{3}{4} \) inch.
- Relevant Equations (modified based on the above)

**SI Units**

\[
q_{net} = 19.16 \times N \times F_d \times \left( \frac{S_e}{25.4} \right) \quad B \leq 1.22m \quad \text{(eq. 5.63)}
\]

\[
q_{net} = 11.98 \times N \times F_d \times \left( \frac{S_e}{25.4} \right) \times \left( \frac{3.28B+1}{3.28B} \right)^2 \quad B > 1.22m \quad \text{(eq. 5.63)}
\]

- \( q_{net} \) (\( q_{all} - \gamma D_f \)) is the allowable stress, \( N = N \) corrected depth factor
- \( F_d = 1 + \frac{1}{3} \left( \frac{D_f}{B} \right) \leq 1.33 \)
- \( S_e = \) tolerable settlement in mm
1. **Allowable Bearing Pressure in Sand Based on Settlement Consideration** (cont’d.)

**English Units**

- The same equations in English units:

\[
q_{net} = \left( \frac{N}{2.5} \right) \times F_d \times S_e \quad \text{B} \leq 4\text{ft} \quad \text{(eq. 5.59)}
\]

\[
q_{net} \text{ [kips/ft}^2\text{]} \quad S_e \text{ [inches]}
\]

\[
q_{net} = \frac{N}{4} \left( \frac{B+1}{B} \right)^2 \times F_d \times S_e \quad \text{B} > 4\text{ft} \quad \text{(eq. 5.60)}
\]
1. **Allowable Bearing Pressure in Sand Based on Settlement Consideration (cont’d.)**

The following figure is based on equations 5.59 and 5.60: $q_{\text{net}}$ over the depth factor vs. foundation width for different $N_{\text{corrected}}$ SPT.

Find B to satisfy a given $Q_{\text{load}}$ following the procedure below:

- Correct NSPT with depth for approximately 2-3B below the base of the foundation (use approximated B).
- Choose a representative $N_{\text{corrected}}$ value
- Assume B → Calculate $F_d$ → Calculate $q_{\text{net}}$ using B&N or find from the above figure $q_{\text{net}}/(F_d \times S_e)$
- Use iterations:
1. **Allowable Bearing Pressure in Sand Based on Settlement Consideration** (cont’d.)

Find B to satisfy a given $Q_{\text{load}}$ following the procedure below:

1. Correct NSPT with depth for approximately 2-3B below the base of the foundation (use approximated B).
2. Choose a representative $N_{\text{corrected}}$ value.
3. Assume B → Calculate $F_d$ → Calculate $q_{\text{net}}$ using B&N or find from the above figure $q_{\text{net}}/(F_d \times S_e)$.
4. Use iterations:

   - Calculate $Q_{\text{load}} = q_{\text{net}} \times B^2$.
   - If Calculated $\approx$ Required, Use B, otherwise continue iterations.

\[\text{Diagram:}\]

- Calculate $Q_{\text{load}} = q_{\text{net}} \times B^2$
- If Calculated $\approx$ Required, Use B
- Otherwise, continue iterations.
EXAMPLE 4.9

A shallow square foundation for a column is to be constructed. It must carry a net vertical load of 1000 kN. The foundation soil is sand. The standard penetration numbers obtained from field exploration are given in Figure 4.34. Assume that the depth of the foundation will be 1.5 m and the tolerable settlement is 25.4 mm. Determine the size of the foundation.

Solution  The field standard penetration numbers need to be corrected by using the Liao and Whitman relationship (Table 2.4). This is done in the following table:

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Field value of $N_F$</th>
<th>$c'$ (kN/m$^2$)</th>
<th>Corrected $N'_{cor}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>31.4</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>62.8</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>94.2</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>125.8</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>157.0</td>
<td>13</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>188.4</td>
<td>9</td>
</tr>
<tr>
<td>14</td>
<td>12</td>
<td>206.4</td>
<td>8</td>
</tr>
<tr>
<td>16</td>
<td>14</td>
<td>224.36</td>
<td>9</td>
</tr>
<tr>
<td>18</td>
<td>18</td>
<td>242.34</td>
<td>11</td>
</tr>
</tbody>
</table>

* Rounded off

From the table, it appears that a corrected average $N'_{cor}$ value of about 10 would be appropriate. Using Eq. (4.53)

$$q'_{s\text{allow}} = 11.98 N'_{cor} \left( \frac{3.28 B + 1}{3.28 B} \right) F_d \left( \frac{S_t}{25.4} \right)$$

Allowable $S_t = 25.4$ mm and $N'_{cor} = 10$, so

$$q'_{s\text{allow}} = 119.8 \left( \frac{3.28 B + 1}{3.28 B} \right) F_d$$

![Field standard penetration number, $N_F$](image)
Additional Topics

The following table can now be prepared for trial calculations:

<table>
<thead>
<tr>
<th>$F_B$ (m)</th>
<th>$F_P$</th>
<th>$q_B$ (kN/m²)</th>
<th>$q_{col}$ (kN/m²)</th>
<th>$Q$ (kN)</th>
<th>$Q_{min}$ = $R_{min}$ × $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.25</td>
<td>1.245</td>
<td>197.34</td>
<td>187.19</td>
<td>788.56</td>
<td></td>
</tr>
<tr>
<td>2.3</td>
<td>1.215</td>
<td>185.46</td>
<td>947.65</td>
<td>981.1</td>
<td></td>
</tr>
<tr>
<td>2.4</td>
<td>1.206</td>
<td>182.29</td>
<td>981.1</td>
<td>981.1</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>1.195</td>
<td>181.15</td>
<td>1212.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Because $Q$, required is 1000 kN, $B$ will be approximately equal to 2.4 m.

The column load = net load + $\bar{g}$ × $A$

Column load = 1050 kN + 15.7 × 1.5 × 2.4 = 1050 + 72.6 = 1122.6 kN

Field corrected

Field standard penetration number, $N_F$

Ground water table

Depth (m)

$\gamma_{sat}$ = 18.8 kN/m³

$\gamma = 15.7$ kN/m³

FIGURE 4.34