Problem #1 (20 points)
Let \(a = (2, 1, -2)\) and let \(b = (3, 0, 4)\).

a. Compute \(a - b\). \(a - b = (2 - 3, 1 - 0, -2 - 4) = (-1, 1, -6)\).

b. Compute \(a \cdot b\). \(a \cdot b = 2 \cdot 3 + 1 \cdot 0 + (-2) \cdot 4 = -2\)

c. Compute \(a \times b\). \(a \times b = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 3 & 0 & 4 \end{vmatrix} = i[(1)(4) - (0)(-2)] + j[(-2)(3) - (2)(4)] + k[(2)(0) - (1)(3)] = \langle 4, -14, -3 \rangle\).

d. Compute \(\text{proj}_a b\) (the vector projection of \(b\) onto \(a\)). \(\text{proj}_a b = \frac{a \cdot b}{|a|^2} a = \frac{-2}{2^2 + 1^2 + (-2)^2} a = \frac{-2}{9} a = \langle -\frac{4}{9}, -\frac{2}{9}, \frac{4}{9} \rangle\).

Problem #2 (10 points)
Find the equation of the plane containing the point \((0, 3, 2)\) and perpendicular to the vector \(\langle 2, 1, -2 \rangle\).

The equation of the plane containing the point \((x_0, y_0, z_0)\) and perpendicular to the vector \(\langle a, b, c \rangle\) is \(a(x - x_0) + b(y - y_0) + c(z - z_0) = 0\). Therefore, the equation of the plane described in this problem is \(2(x - 0) + 1(y - 3) - 2(z - 2) = 0\), or \(2x + y - 2z = -1\).

Problem #3 (15 points)
Let \(P\) denote the point \((1, -1, 2)\) and let \(S\) denote the sphere with center at \(P\) and radius 3.

a. Let \(Q\) denote the point \((3, 0, 0)\). Compute the distance from \(P\) to \(Q\). Using the distance formula, we find that \(|PQ| = \sqrt{(3 - 1)^2 + (0 - (-1))^2 + (0 - 2)^2} = 3\).

b. Does \(Q\) lie on the sphere \(S\)? Why or why not? The sphere \(S\) consists of all points whose distance from \(P\) is 3. Since \(|PQ| = 3\), \(Q\) does lie on \(S\).

c. Find the equation of the sphere \(S\). The equation of a sphere with radius \(r\) and center \((x_0, y_0, z_0)\) is \((x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2\), so the equation of \(S\) is \((x - 1)^2 + (y - (-1))^2 + (z - 2)^2 = 3^2\), or \((x - 1)^2 + (y + 1)^2 + (z - 2)^2 = 9\).

Problem #4 (15 points)
Let \(C\) denote the curve described by the vector function \(r(t) = \langle e^{2t}, 2e^{2t}, 2e^{2t} \rangle\), \(0 \leq t \leq 1\).

a. Show that \(|r'(t)| = 6e^{2t}\). \(r(t) = \langle e^{2t}, 2e^{2t}, 2e^{2t} \rangle \Rightarrow r'(t) = \langle 2e^{2t}, 4e^{2t}, 4e^{2t} \rangle\)

(because \(\frac{d}{dt}[e^u] = e^u \frac{du}{dt}\). Therefore, \(|r'(t)| = \sqrt{(2e^{2t})^2 + (4e^{2t})^2 + (4e^{2t})^2} = \sqrt{4e^{4t} + 16e^{4t} + 16e^{4t}} = \sqrt{36e^{4t}} = 6e^{2t}\).

b. Find the length of \(C\). The length of \(C\) is given by \(L = \int_{a}^{b} |r'(t)| \ dt = \int_{0}^{1} 6e^{2t} \ dt = 3e^{2t} \bigg|_{0}^{1} = 3e^2 - 3 \approx 19.167\).
Problem #5 (20 points)
Let \( C \) denote the curve described by the vector function \( r(t) = \langle t^2, 2 - t \rangle \)

a. Find \( r'(t) \). \( r(t) = \langle t^2, 2 - t \rangle \Rightarrow r'(t) = \langle 2t, 1, -1 \rangle \). (Take the derivative of each component.)

b. Show that the point \((4, 2, 0)\) lies on \( C \). We must find a value of \( t \) for which \( t^2 = 4 \) and \( t = 2 \). \( t = 2 \) clearly satisfies all three conditions, so the point \((4, 2, 0)\) lies on \( C \).

c. Find parametric equations for the line tangent to \( C \) at the point \((4, 2, 0)\). The parametric equations of the line containing the point \((x_0, y_0, z_0)\) and parallel to the vector \( \langle a, b, c \rangle \) are \( x = x_0 + at, y = y_0 + bt, z = z_0 + ct \). The vector \( r'(t) \) is tangent to \( C \), so \( r'(2) = \langle 4, 1, -1 \rangle \) is tangent to \( C \) at \((4, 2, 0)\). (From part b we know that the point \((4, 2, 0)\) corresponds to \( t = 2 \).) The tangent line therefore contains point \((4, 2, 0)\) and is parallel to vector \( \langle 4, 1, -1 \rangle \), so its parametric equations are \( x = 4 + 4t, y = 2 + 1t, z = 0 + (-1)t \), or \( x = 4 + 4t, y = 2 + t, z = -t \).

Problem #6 (20 points)
Let \( C \) denote the curve described by the vector function \( r(t) = \langle 1 + \sin(4t), 3t, 2 - \cos(4t) \rangle \).

a. Find the unit tangent vector \( T(t) \). \( r(t) = \langle 1 + \sin(4t), 3t, 2 - \cos(4t) \rangle \Rightarrow r'(t) = \langle 4 \cos(4t), 3, 4 \sin(4t) \rangle \Rightarrow |r'(t)| = \sqrt{4 \cos^2(4t) + 3^2 + 4 \sin^2(4t)} = \sqrt{16 \cos^2(4t) + 9 + 16 \sin^2(4t) = \sqrt{\frac{16 \cos^2(4t) + \sin^2(4t)}{1}} = 9 = \sqrt{16 + 9} \). Therefore, \( T(t) = \frac{1}{|r'(t)|} r'(t) = \frac{1}{5} \langle 4 \cos(4t), 3, 4 \sin(4t) \rangle = \frac{2}{5} \langle \cos(4t), 4, 5 \sin(4t) \rangle \).

b. Find the unit normal vector \( N(t) \). \( T(t) = \langle \frac{4}{5} \cos(4t), \frac{3}{5}, \frac{4}{5} \sin(4t) \rangle \Rightarrow T'(t) = \langle -\frac{16}{5} \sin(4t), 0, \frac{16}{5} \cos(4t) \rangle \Rightarrow |T'(t)| = \sqrt{\left(\frac{16}{5} \sin(4t) \right)^2 + 0^2 + \left(\frac{16}{5} \cos(4t) \right)^2} = \sqrt{\left(\frac{16}{5} \right)^2 \left(\cos^2(4t) + \sin^2(4t) \right)} = \frac{16}{5}, \)
\[ \frac{1}{|T'(t)|} T'(t) = \frac{1}{16/5} \langle -\frac{16}{5} \sin(4t), 0, \frac{16}{5} \cos(4t) \rangle = \langle -\sin(4t), 0, \cos(4t) \rangle. \]

c. Find the curvature \( \kappa \). \( \kappa = \frac{|T'(t)|}{|r'(t)|} = \frac{16/5}{5} = \frac{16}{25}. \)