1.3 Problems

In Problems 1 through 10, we have provided the slope field of the indicated differential equation, together with one or more solution curves. Sketch likely solution curves through the additional points marked in each slope field.

1. \( \frac{dy}{dx} = -y - \sin x \)

FIGURE 1.3.12.

2. \( \frac{dy}{dx} = x + y \)

FIGURE 1.3.13.

3. \( \frac{dy}{dx} = y - \sin x \)

FIGURE 1.3.14.

4. \( \frac{dy}{dx} = x - y \)

FIGURE 1.3.15.

5. \( \frac{dy}{dx} = y - x + 1 \)

FIGURE 1.3.16.

6. \( \frac{dy}{dx} = x - y + 1 \)

FIGURE 1.3.17.
In Problems 11 through 20, determine whether Theorem 1 does or does not guarantee existence of a solution of the given initial value problem. If existence is guaranteed, determine whether Theorem 1 does or does not guarantee uniqueness of the solution.

11. \( \frac{dy}{dx} = 2x^2y; \quad y(1) = -1 \)

12. \( \frac{dy}{dx} = x \ln y; \quad y(1) = 1 \)

13. \( \frac{dy}{dx} = \sqrt{y}; \quad y(0) = 1 \)

14. \( \frac{dy}{dx} = \sqrt{y}; \quad y(0) = 0 \)

15. \( \frac{dy}{dx} = \sqrt{x-y}; \quad y(2) = 2 \)

16. \( \frac{dy}{dx} = \sqrt{x-y}; \quad y(2) = 1 \)

17. \( \frac{dy}{dx} = x-1; \quad y(0) = 1 \)

18. \( \frac{dy}{dx} = x-1; \quad y(1) = 0 \)

19. \( \frac{dy}{dx} = \ln(1+y^2); \quad y(0) = 0 \)

20. \( \frac{dy}{dx} = x^2 - y^2; \quad y(0) = 1 \)

In Problems 21 and 22, first use the method of Example 2 to construct a slope field for the given differential equation. Then sketch the solution curve corresponding to the given initial condition. Finally, use this solution curve to estimate the desired value of the solution \( y(x) \).

21. \( y' = x + y; \quad y(0) = 0; \quad y(-4) = ? \)

22. \( y' = y - x; \quad y(4) = 0; \quad y(-4) = ? \)