Necessary Skills

<table>
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<th>You should be able to</th>
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| 1.1     | • determine whether a given function is a solution of a given d.e.  
                    • use initial conditions to determine the values of arbitrary constants in the general solution of a d.e.  
                    • translate a verbal description of a physical system into a d.e. |
| 1.2     | • solve d.e.’s of the form $\frac{dy}{dx} = f(x)$  
                    • find the position of an object as a function of time given its acceleration, its initial position, and its initial velocity |
| 1.4     | • recognize and solve separable first-order d.e.’s/initial value problems  
                    • formulate and solve separable first-order d.e.’s to analyze such problems as radioactive decay, compound interest, simple population models, and cooling/heating. |
| 1.5     | • recognize and solve linear first-order d.e.’s/initial value problems  
                    • formulate and solve first-order linear d.e.’s to analyze mixture problems |

Answers to Practice Exam Questions

1. $y = e^{2x} + x$ is not a solution of the d.e. $y'' - 4y' + 4y = 4x$.

2. The car travels 50 meters before coming to a stop.

3. The population will reach 2500 in approximately 4.35 years.

4. $y = x^4 + 4x$.

5. $y = \sqrt{6 - 2x^{-1} - 4\ln(x)}$.

6. Let $t$ denote time, let $s$ denote the salt concentration in the cell, and let $a$ denote the salt concentration in the surrounding tissue. Then $\frac{ds}{dt} = k(a - s)$, where $k$ is a proportionality constant.
Problem #1 (10 points)
Is \( y = e^{2x} + x \) a solution of the d.e. \( y'' - 4y' + 4y = 4x \)? Why or why not?

Problem #2 (15 points)
A car’s brakes provide a constant deceleration of 4 m/s\(^2\). If the car is traveling at a speed of 20 m/s when its brakes are applied, how far will it travel before coming to a stop?

Problem #3 (15 points)
The population of a town is 2000. Last year the population was 1900. When will the population equal 2500? (Assume exponential growth.)

Problem #4 (25 points)
Find the solution of the following initial value problem:
\[
\frac{dy}{dx} = \frac{y}{x} + 3x^3 \quad \text{with} \quad y(1) = 5.
\]

Problem #5 (25 points)
Find the solution of the following initial value problem:
\[
x^2 \frac{dy}{dx} = 1 - \frac{2x}{y} \quad \text{with} \quad y(1) = 2.
\]

Problem #6 (10 points)
Write down a differential equation describing the following system. Be sure to define your variables. **DO NOT SOLVE THE DIFFERENTIAL EQUATION, JUST WRITE IT DOWN.**
The rate of change of the salt concentration in a cell is proportional to the difference between the salt concentration in the cell and the salt concentration in the surrounding tissue (assumed constant).