Problem 1. (10 pts.)

Solve the following initial value problem: \( \frac{dy}{dx} = \frac{3x^2}{3y^2 + 1} \) with \( y(1) = 2 \).

Problem 2. (10 pts.)

Solve the following initial value problem: \( x^2 \frac{dy}{dx} = xy + 2 \), \( y(1) = 2 \).

Express your solution \( y \) explicitly in terms of \( x \). In other words, write your answer in the form \( y = \) something.

Problem 3. (15 points)

A cup of coffee at temperature 120° F is brought into a room where the temperature is 70° F. After 10 minutes the coffee temperature is 110° F. When will the coffee temperature reach 100° F?

Recall that the de modeling heating/cooling problems is \( \frac{dT}{dt} = -k (T - A) \).

Problem 4. (10 points)

Find the general solution to each of the following differential equations.

a. (4 points) \( y'' + 2y' + y = 0 \)

b. (6 points) \( y^{(4)} + y'' = 0 \)

OVER
Problem 5. (15 points)
Solve the following initial value problem:
\[ y'' - y' - 2y = 4x + 12e^{3x} \quad \text{with} \quad y(0) = 8 \quad \text{and} \quad y'(0) = 11. \]
Note: \( y' = dy/dx \) and \( y'' = d^2y/dx^2 \)

Problem 6. (15 points)
Find the position function \( x(t) \) for an unforced, damped mass-spring system with mass \( m = 1 \) kg, damping coefficient \( c = 4 \) Ns/m, and spring constant \( k = 5 \) N/m. Take \( x(0) = 2 \) m and \( x'(0) = 0 \) m/s.

Problem 7. (10 points)
\( a. \) (2 pts.) Find \( \mathcal{L} \{7 + \cos(2t)\} \)
\( b. \) (8 pts.) Find \( \mathcal{L}^{-1}\left\{\frac{4s + 8}{(s - 2)(s^2 + 4)}\right\} \).

Problem 8. (15 points)
Use the Laplace Transform to solve the following initial value problem:
\[ x'' - 4x' = 6e^t \quad \text{with} \quad x(0) = 0 \quad \text{and} \quad x'(0) = -2. \]
Solutions to this IVP not using the Laplace transform method will not receive any credit. Primes denote derivatives with respect to \( t \): \( x' = dx/dt \) and \( x'' = d^2x/dt^2 \).