Chapter 6, Problem 2.

A 20-μF capacitor has energy \( w(t) = 10 \cos^2 377t \) J. Determine the current through the capacitor.

Chapter 6, Solution 2.

\[
w = \frac{1}{2} CV^2 \quad \rightarrow \quad v^2 = \frac{2W}{C} = \frac{20 \cos^2 377t}{20 \times 10^{-6}} = 10^6 \cos^2 377t
\]

\( v = \pm 10^3 \cos(377t) \) V, let us assume the \( v = +\cos(377t) \) mV, this then leads to,

\[
i = C\frac{dv}{dt} = 20 \times 10^{-6}(-377\sin(377t) 10^{-3}) = -7.54\sin(377t) \text{ A}
\]

*Please note that if we had chosen the negative value for \( v \), then \( i \) would have been positive.*

Chapter 6, Problem 8.

A 4-mF capacitor has the terminal voltage

\[
v = \begin{cases} 
50 \text{ V}, & t \leq 0 \\
Ae^{-100t} + Be^{-600t} \text{ V}, & t \geq 0
\end{cases}
\]

If the capacitor has initial current of 2A, find:
(a) the constants \( A \) and \( B \),
(b) the energy stored in the capacitor at \( t = 0 \),
(c) the capacitor current for \( t > 0 \).

Chapter 6, Solution 8.

\[
(a) \quad i = C \frac{dv}{dt} = -100ACe^{-100t} - 600BCe^{-600t} 
\]

\[
i(0) = 2 = -100AC - 600BC 
\rightarrow \quad 5 = -A - 6B
\]

\[
v(0^+) = v(0^-) \quad \rightarrow \quad 50 = A + B
\]

Solving (2) and (3) leads to

\( A = 61, \quad B = -11 \)

\[
(b) \quad \text{Energy} = \frac{1}{2} Cv^2(0) = \frac{1}{2} x4\times10^{-3} x2500 = 5 \text{ J}
\]

\[
(c) \quad \text{From (1),}
\]

\[
i = -100x61x4x10^{-3} e^{-100t} - 600x(-11)x4x10^{-3} e^{-600t} = -24.4e^{-100t} + 26.4e^{-600t} \text{ A}
\]

Chapter 6, Problem 9.
The current through a 0.5-F capacitor is $6(1-e^{-t})$A. Determine the voltage and power at $t=2$ s. Assume $v(0) = 0$.

**Chapter 6, Solution 9.**

$$v(t) = \frac{1}{1/2} \int_0^t 6(1-e^{-t}) \, dt + 0 = 12\left(t + e^{-t}\right) V = 12(t + e^t) - 12$$

$$v(2) = 12(2 + e^{-2}) - 12 = 13.624 \text{ V}$$

$$p = iv = [12 \left( t + e^{-t}\right) - 12]6(1-e^{-t})$$

$$p(2) = [12 \left( 2 + e^{-2}\right) - 12]6(1-e^{-2}) = 70.66 \text{ W}$$

**Chapter 6, Problem 13.**

Find the voltage across the capacitors in the circuit of Fig. 6.49 under dc conditions.

**Figure 6.49**

**Chapter 6, Solution 13.**

Under dc conditions, the circuit becomes that shown below:

$$i_2 = 0, \quad i_1 = \frac{60}{30+10+20} = 1\text{ A}$$
\[ v_1 = 30i_1 = 30V, v_2 = 60 - 20i_1 = 40V \]

Thus, \[ v_1 = 30V, v_2 = 40V \]

Chapter 6, Problem 37.

The current through a 12-mH inductor is \(4\sin 100t\) A. Find the voltage, and also the energy stored in the inductor for \(0 < t < \pi/200\) s.

Chapter 6, Solution 37.

\[
\begin{align*}
v &= L \frac{di}{dt} = 12 \times 10^{-3} \times 4(100) \cos 100t \\
&= 4.8 \cos 100t \text{ V}
\end{align*}
\]

\[
p = vi = 4.8 \times 4 \sin 100t \cos 100t = 9.6 \sin 200t
\]

\[
w = \int_0^t pdt = \int_0^{11/200} 9.6 \sin 200t
\]

\[
= -\frac{9.6}{200} \cos 200t \bigg|_0^{11/200} \text{ J}
\]

\[
= -48(\cos \pi - 1) \text{mJ} = 96 \text{mJ}
\]

Please note that this problem could have also been done by using \((\frac{1}{2})L i^2\).

Chapter 6, Problem 39.

The voltage across a 200-mH inductor is given by

\[ v(t) = 3t^2 + 2t + 4 \text{ V for } t > 0. \]

Determine the current \(i(t)\) through the inductor. Assume that \(i(0) = 1\) A.

Chapter 6, Solution 39

\[
\begin{align*}
v &= L \frac{di}{dt} \Rightarrow i = \frac{1}{L} \int_0^t i dt + i(0)
\end{align*}
\]

\[
i = \frac{1}{200 \times 10^{-3}} \int_0^t (3t^2 + 2t + 4) dt + 1
\]

\[
= 5(t^3 + t^2 + 4t) \bigg|_0^t + 1
\]

\[i(t) = 5t^3 + 5t^2 + 20t + 1 \text{ A}\]
Chapter 6, Problem 47.

For the circuit in Fig. 6.70, calculate the value of $R$ that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions.

Figure 6.70

Chapter 6, Solution 47.

Under dc conditions, the circuit is equivalent to that shown below:

\[ i_L = \frac{2}{R+2} (5) = \frac{10}{R+2}, \quad v_C = R_i = \frac{10R}{R+2} \]

\[ w_C = \frac{1}{2} C v_C^2 = 80 \times 10^{-6} \times \frac{100R^2}{(R+2)^2} \]

\[ w_L = \frac{1}{2} L i_L^2 = 2 \times 10^{-3} \times \frac{100}{(R+2)^2} \]

If $w_C = w_L$, 

\[
80 \times 10^{-6} \times \frac{100R^2}{(R + 2)^2} = \frac{2 \times 10^{-3} \times 100}{(R + 2)^2} \quad \Rightarrow \quad 80 \times 10^{-3}R^2 = 2
\]

\[R = 5 \Omega\]

**Chapter 6, Problem 48.**

Under steady-state dc conditions, find i and v in the circuit in Fig. 6.71.

**Figure 6.71** For Prob. 6.48.

**Chapter 6, Solution 48.**

Under steady-state, the inductor acts like a short-circuit, while the capacitor acts like an open circuit as shown below.

Using current division,

\[i = \frac{30k}{(30k+20k)} \times 10 \text{ mA} = 6 \text{ mA}\]

\[v = 20ki = 120 \text{ V}\]