Today, we learn some tools derived from the 3 basic laws:

- Equivalent resistance
- Voltage division
- Current division

They help you develop intuition about circuits:

- To see relationship between variables
- To see several steps ahead
- To plan a solution to a circuit

Two terminal circuit with all resistors

Ways of connection

**Series connection:** Two or more elements are in series if every connected pair exclusively share a single node, i.e., one node connects only two elements.

By KCL, same current flows through elements in series

R₁ and R₂ are not in series because the node connecting them also connects a third element.

**Parallel connection:** Two or more elements are in parallel if they are connected between the same two nodes.

By KVL, same voltage across parallel elements:

\[ v_1 = v_2 = v_3 \]
Determine series and parallel connection:

- \( R_2 \) and \( R_3 \) are in parallel.
- How about \( R_1 \) and \( R_2 \)?
- \( R_1 \) and \( R_2 \) are not in parallel.
- \( R_1 \) is between “a” and “c”, \( R_2 \) between “b” and “d”
- Any series connection?
  - \( R_1 \) and \( R_5 \) are not in series
  - \( R_1 \) and \( R_4 \) are not in series
- No series connection between any two elements.

These wires cannot be thrown away.
They must be connected to somewhere to draw power.

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§ 2.5 Series resistors and voltage division

Given \( R_1, R_2 \).
What is \( \frac{v}{i} \)?

By KCL, same current in \( R_1, R_2 \)
By Ohm’s law, \( v_1 = R_1 i, v_2 = R_2 i \)
By KVL, \( v = v_1 + v_2. \)
Putting together:
\[
\begin{align*}
  v &= v_1 + v_2 = R_1 i + R_2 i = (R_1 + R_2)i \\
  \Rightarrow v &= (R_1 + R_2)i \quad (1)
\end{align*}
\]

\[
  v = R_{eq} i \quad (2) \quad R_{eq} =?
\]

Compare (1) and (2):
\[
  R_{eq} = R_1 + R_2
\]
In general:

\[ i = \frac{v}{R_1 + R_2 + \ldots + R_N} \]

\[ v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + \ldots + R_N} v \]

\[ v_k = R_k i = \frac{R_k}{R_1 + R_2 + \ldots + R_N} v \]

Voltage division: Total voltage \( v \) is divided among the resistors in direct proportion to the resistances. Larger resistance takes more voltage.

Special case with two resistors:

\[ v_1 = \frac{R_1}{R_1 + R_2} v \]

\[ v_2 = \frac{R_2}{R_1 + R_2} v \]
§ 2.6 parallel resistors and current division

What is $R_{eq} = \frac{v}{i}$?

By KVL, same voltage across $R_1, R_2$
By ohm's law: $i_1 = \frac{v}{R_1}, i_2 = \frac{v}{R_2}$
By KCL: $i = i_1 + i_2$

Put together:

$$i = \frac{v}{R_1} + \frac{v}{R_2} = \frac{1}{R_1} + \frac{1}{R_2} v$$  \hspace{1cm} (1)

$$i = \frac{1}{R_{eq}} v$$  \hspace{1cm} (2)  $R_{eq} = ?$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

In general

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}}$$
Equivalent resistance for parallel resistors:

\[ R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}} \]

Notation: \( R_{eq} = R_1 // R_2 // \ldots // R_N \)

Special cases:

**N=2:** \( R_1 // R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \)

R_1=R_2=\ldots=R_N=R: \( R_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{R} + \ldots + \frac{1}{R}} = \frac{R}{N} \)

Simple combinations:

<table>
<thead>
<tr>
<th>Simple rule:</th>
<th>Example:</th>
<th>Result:</th>
</tr>
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</table>
| \( aR_1 // aR_2 = \frac{aR_1 \times aR_2}{aR_1 + aR_2} = \frac{aR_2 R_2}{R_1 + R_2} = \alpha (R_1 // R_2) \) | \( aR_3 // aR_4 = \frac{aR_3 \times aR_4}{aR_3 + aR_4} = \frac{aR_4 R_4}{R_3 + R_4} = \alpha (R_3 // R_4) \) | \( 27 // 54 = 9 // 6 = 9 \times 2 = 18 \)

Common sense:

Adding more resistor to existing parallel ones reduces \( R_{eq} \):

\[
\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N + 1}} < \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}}
\]

\[ R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}} \]

\[ R_{eq} < \frac{1}{\frac{1}{R_k}} = R_k, \text{ for any } k = 1, 2, \ldots, N \]

Equivalent resistance for parallel connection is less than any individual resistance

Equivalent conductance:

\[
G_{eq} = \frac{i}{V} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} = G_1 + G_2 + \ldots + G_N
\]
Current division: Given total $i$, what is $i_1, i_2$?

$$i_1 = \frac{v}{R_1} = \frac{R_1 R_2}{R_1 + R_2} i = R_2 \frac{i}{R_1 + R_2}$$

$$i_2 = \frac{v}{R_2} = \frac{R_2 R_1}{R_1 + R_2} i = R_1 \frac{i}{R_1 + R_2}$$

The other resistance on top

The current is shared by resistors in inverse proportion to resistance. Larger resistor takes less current.

Two extreme cases:

Case 1: A resistor in parallel with a short circuit

$$i_1 = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = \frac{R_1}{R_1 + R_2} i$$

Case 2: A resistor in parallel with an open circuit

$$i_1 = i$$

$$i_2 = 0$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 \times 0}{R_1 + 0} = 0$$
Equivalent resistance Examples

\[ R_{eq} = 4\Omega + 2\Omega + 8\Omega = 14.4\Omega \]

\[ R_{eq} = (2\Omega + 2\Omega) / 6 = \frac{4\Omega \cdot 6}{4\Omega + 6} = \frac{24\Omega}{10\Omega} = 2.4\Omega \]

\[ R_{eq} = \frac{3\Omega \cdot 6}{3\Omega + 6} = \frac{18\Omega}{9} = 2\Omega \]

Equivalent resistance Examples

\[ R_{eq} = 10\Omega + \frac{2\Omega}{1+2} = \frac{10\Omega + 2\Omega}{3} = \frac{12\Omega}{3} = 4\Omega \]

\[ R_{eq} = 10\Omega + \frac{2\Omega}{3} = 10\Omega + 1.2\Omega = 11.2\Omega \]
Example: Compute $I_s$

\[ R_{eq1} = \frac{30}{(20+50)} = \frac{30}{70} = 21 \Omega \]

\[ R_{eq2} = \frac{14+9}{18} = 14 \div 6 = 20 \Omega \]

Need to find the equivalent resistance with respect to 40V

\[ R_{eq} = \frac{8+20}{(21+9)} = 8+12 = 20 \Omega \]

\[ I_s = \frac{40}{20} = 2A \]

Example: Compute $R_{eq}$

\[ R_{eq} = \frac{10}{(12+\frac{4}{12})} = \frac{10}{15} = 6 \Omega \]
Practice 6: Find $R_{eq}$

Practice 7: Find $I_s$

Practice 8: Find $v_s$

Practice 9: Find $v_1$, $v_2$

Practice 10: Find $i_1$, $i_2$
Three useful tools derived from basic laws:

• Equivalent resistance
• Voltage division
• Current division

Used together to solve circuit problems

Voltage division:
\[ v_1 = \frac{R_1}{R_1 + R_2} v \]
\[ v_2 = \frac{R_2}{R_1 + R_2} v \]

Current Division:
\[ i_1 = \frac{R_2}{R_1 + R_2} i \]
\[ i_2 = \frac{R_1}{R_1 + R_2} i \]

Example: Find \( i_0 \) and \( v_1 \).

Approach 1: Assign auxiliary variable \( I \).

Use equivalent resistance with respect to 12V to find \( I \), then \( v_1 = 4I \), and \( i_0 \) can be computed by current division.

\[ R_{eq} = \frac{4 + 6}{3} = \frac{4 + 2}{3} = 6 \Omega \]
By Ohm's Law, \( I = \frac{12}{R_{eq}} = \frac{12}{6} = 2 \text{A} \)
Thus \( v_1 = 4I = 8 \text{V} \).

By current division:
\[ i_0 = \frac{6}{3 + 6} I = \frac{6}{3 + 6} \times 2 = \frac{4}{3} A \]

Be Careful: In this equivalent circuit, only \( I \) is the same as in the original circuit. Neither \( v_1 \) nor \( i_0 \) can be found in it. You need to go back to the original circuit to find \( v_1 \) and \( i_0 \).
Approach 2: Use equivalent resistance of 6/3, denoted as $R'_{eq}$.

$$R'_{eq} = \frac{6}{3} = 2 \Omega$$  

By voltage division,

$$v_1 = \frac{4}{4+2} \times 12 = 8V, \quad v_2 = \frac{2}{4+2} \times 12 = 4V$$

Be careful, $i_0$ cannot be found in the equivalent circuit. $i_0 \neq I$. You have to use the original circuit to find $i_0$.

Where is $v_2$ in the original circuit?

Since the voltage across 3Ω is $v_2$, by Ohm’s Law, $i_0 = \frac{v_2}{3} = \frac{4}{3}A$

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Example: Find $v_1$, $v_2$, ...

Approach 1: Which is the voltage across 3A? $v_1$ or $v_2$?

It is $v_2$. If you know the $R_{eq}$ w.r.t 3A, you can obtain $v_2$ by Ohm’s law.

$$R_{eq} = R_1//R_2$$

$$v_2 = R_{eq} \times 3$$

$$= 4 \times 3 = 12V$$

How to find $v_1$?

Need the original circuit.

By voltage division,

$$v_1 = \frac{4}{4+8} \times v_2 = \frac{4}{12} \times 12 = 4V$$
Approach 2: Use current division. Assign $I_1, I_2$.

\[
R_1 = 4 + 8 = 12 \Omega \\
R_2 = 15//10 = 6 \Omega
\]

By current division,

\[
I_1 = \frac{R_2}{R_1 + R_2} \times 3 = \frac{6}{18} \times 3 = 1A \\
I_2 = \frac{R_1}{R_1 + R_2} \times 3 = \frac{12}{18} \times 3 = 2A
\]

By ohm’s law, $v_1 = 4I_1 = 4V$; $v_2 = R_2I_2 = 6 \times 2 = 12V$.

Example: Find the currents $i_1, ..., i_5$

Approach 1: Use equivalence resistance and current division.

\[
R_{eq} = 4 + 3//9//18 = 6 \Omega \\
i_1 = \frac{18}{6} = 3A
\]

\[
i_2 = \frac{6}{3 + 6} \times i_1 = \frac{6}{9} \times 3 = 2A \\
i_3 = \frac{3}{3 + 6} \times i_1 = \frac{3}{9} \times 3 = 1A \\
i_4 = \frac{18}{9 + 18} \times i_3 = \frac{18}{27} \times 1 = \frac{2}{3}A \\
i_5 = \frac{9}{9 + 18} \times i_3 = \frac{9}{27} \times 1 = \frac{1}{3}A
\]
Example: Find the currents $i_1, \ldots, i_5$

Approach 2: Use voltage division and Ohm's law. Assign $v_2$.

By voltage division $v_2 = \frac{2}{4+2} \times 18 = 6V$

By Ohm's law, $i_1 = \frac{v_2}{2} = \frac{6}{2} = 3A$

$i_2 = \frac{v_2}{3} = \frac{6}{3} = 2A$

$i_4 = \frac{v_2}{9} = \frac{6}{9} = \frac{2}{3}A$

$i_5 = \frac{v_2}{18} = \frac{6}{18} = \frac{1}{3}A$

$i_3 = i_4 + i_5 = \frac{2}{3} + \frac{1}{3} = 1A$

Practice 11: Find $i_1, i_2, i_3, i_4, I_x$

In all three circuits, $i_1, i_2, i_3, i_4$ are the same

How about $I_x, I_0$? By KCL, $I_0 = i_1 + i_2 = ?$

$I_0 = i_1 + i_2 = i_3 + i_4 = 3A$

$I_x = i_1 - i_3 = i_4 - i_2$

By current division:

$i_1 = \frac{20}{30} \times 3 = 2A$; $i_2 = 1A$

$i_3 = 1.8A$; $i_4 = 1.2A$

$I_x = i_1 - i_3 = 0.2A$

$i_4 - i_2 = 0.2A$
• Test 1 will be given on Oct. 2 (Monday), 11-11:50am.
In Ball Hall 214. There will be two versions in different colors.

Please arrive 5-10 minutes earlier

• A practice exam will be given on 9/27/2017(Wednesday), 11-11:50am
in Ball Hall 214

Solution to practice exams will be posted at website
Solution to all practice problems in lecture note will be posted.

Practice 12: Find $I_1$, $I_2$, $v_4$

Practice 13: Find $I_0$, $v_4$
Practice 14: Find $I_0, v_2$

Practice 15: Find the voltage $v_x$

Practice 16: Find $v_x, I_x$
Practice 16a: Find $v_1$, $I_0$.

\[
\begin{align*}
5\text{A} & \quad 20\Omega \quad 5\Omega \quad + \quad v_1 \\
9\Omega \quad 15\Omega & \quad - \\
\end{align*}
\]

Practice 17: Find $v_x$, $I$.

\[
\begin{align*}
36\text{V} & \quad 6\Omega \quad 15\Omega \\
14\Omega \quad 24\Omega & \quad 16\Omega \\
\end{align*}
\]
Practice 18: Find $i_0$

Practice 19: Find $i_0$

Practice 20: Find $R$ for the circuit

Practice 21: Find $V_s$ for the circuit
Practice 22: Find $R$ so that $I_x$ is 10A

$$\begin{align*}
I_x &= 10A \\
4\Omega &\quad 10\Omega \\
64V &\quad 8/3\Omega \\
R &
\end{align*}$$