Circuit Theory I (16.201): Test 1

Total 20 points + 4 (bonus)

1. (2pts) (a) Determine the charge $q(t)$ if $q(0) = 0.5 \text{C}$ and the current is given by

$$i(t) = -6 \sin(3t) - 3e^{-2t} + 4t \ A$$

$$q(t) = \int_0^t i(t) \, dt + q(0) = \int_0^t (-6 \sin(3t) - 3e^{-2t} + 4t) \, dt + 0.5$$

$$= \left[ 2 \cos(3t) + \frac{3}{2} e^{-2t} + 2t^2 \right]_0^t + 0.5$$

$$= 2 \cos(3t) + \frac{3}{2} e^{-2t} + 2t^2 - 2 - \frac{3}{2} + 0.5$$

$$= 2 \cos(3t) + 1.5 e^{-2t} + 2t^2 - 3 \text{C}$$

(b) Determine the current $i(t)$ flowing through an element if the charge is given by

$$q(t) = (-e^{-3t} + 4 \cos(2t + \frac{\pi}{3})) t^2 \text{C}$$

$$i(t) = \frac{d}{dt} \left( \frac{q(t)}{t} \right) = \left( -3e^{-3t} + 4(-2) \sin(2t + \frac{\pi}{3}) \right) t^2 + \left( e^{-3t} + 4 \cos(2t + \frac{\pi}{3}) \right) 2t$$

$$= \left( 3e^{-3t} - 8 \sin(2t + \frac{\pi}{3}) \right) t^2 - e^{-3t} (2t) + 4 \cos(2t + \frac{\pi}{3}) 2t$$

$$= (3t^2 - 2t)e^{-3t} - 8t^2 \sin(2t + \frac{\pi}{3}) + 8t \cos(2t + \frac{\pi}{3}) \text{A}$$

2. (3pts) Determine the total power supplied by the voltage source and the power consumed by the 15\(\Omega\) resistor

![Circuit Diagram](image)

**Reg** = 10 \(\Omega\) (9 + 10/115) = 10 \(\Omega\) (9 + 6) = 10 \(\Omega\) 15 = 65 \(\Omega\)

$$P_{total} = \frac{V_s^2}{R} = \frac{30 \times 30}{6} = 150 \text{W}$$

**Reg'** = 10 \(\Omega\) 15 = 6 \(\Omega\)

By voltage division,

$$V_1 = \frac{R_{reg}}{R_{reg} + 9} \times 30 \text{V}$$

$$= \frac{6}{6 + 9} \times 30 = 12 \text{V}$$

$$P_{15\Omega} = \frac{V_1^2}{15} = \frac{12 \times 12}{15} = 9.6 \text{W}$$
4. (4 pts) Find the current \( i_o \) and the voltage \( V_{ab} \) between the two nodes “a” and “b”.

\[ V_0 = -2i_0, \quad V_1 = 3i_0 \]

By KVL along big loop

\[ -V_0 - 36 + V_1 - 2V_0 + 3i_0 = 0 \]

\[ -(2i_0) - 36 + 3i_0 - 2(-2i_0) + 3i_0 = 0 \]

\[ 2i_0 + 3i_0 + 4i_0 + 3i_0 = 36 \]

\[ 12i_0 = 36, \quad i_0 = 3A \]

KVL along small loop \( L_1 \)

\[ -36 + V_1 - 2V_0 - V_{ab} = 0 \]

\[ V_{ab} = -36 + 3i_0 - 2(-2i_0) = -36 + 3*3 + 4*3 \]

\[ = -15V \]

5. (4 pts) Compute \( I_0, V_0 \).

By current division,

\[ I_1 = \frac{15}{15+4+6} \times 1.5 = \frac{15}{25.5} \times 1.5 = 0.9A \]

\[ I_2 = \frac{4+6}{15+4+6} \times 1.5 = \frac{10}{25} \times 1.5 = 0.6A \]

\[ I_3 = \frac{12}{12+6} \times 1.5 = \frac{12}{18} \times 1.5 = 1A \]

By KCL

\[ I_0 = I_1 - I_3 = 0.9 - 1 = -0.1A \]

By ohm's law,

\[ V_0 = 6I_1 = 6 \times 0.9 = 5.4V \]
5. (4pts) Compute $V_0$ and $I_s$.

By voltage division

$$V_0 = \frac{4}{4+6} \times 10 = 4V$$

$$I_s = \frac{10}{6+4} = 1A$$

6. (4pts) Determine $v_0$ and $v_x$ (across 6A current source).

By KCL at node 1,

$$I_0 + 0.5v_0 = 6$$

$$\frac{v_0}{4} + 0.5v_0 = 6$$

$$\frac{3}{4}v_0 = 6$$

$$v_0 = 8V$$

By KVL along left loop,

$$I_0 = \frac{v_0}{4} = 2A$$

$$v_x = v_1 + v_0 + v_2 = 2I_0 + 4I_0 + 6I_0 = 24V$$

Note: Applying KVL along right loop will give no answer since the voltage of 0.5$v_0$ is not known, even though it turned out to be 0. It is incorrect to ignore the voltage before computation, or take the voltage as 0.5$v_0$.
**Bonus Problem**: (4pts) The current $I_0$ flowing through the 6Ω resistor is 2A. Find $V_s$ for the voltage source and the total power supplied by the source.

Need to find the equivalent resistance w. r. t \( @ b \)

Alternatively,

\[
V_s = V_1 + 4I_0
\]

\[
= 24 + 4 \times 3 = 36 \text{V}
\]

\[
P_{V_3} = V_s I_0 = 36 \times 3 = 108 \text{W}
\]

\[
V_1 = (6 + 6) \times 2 = 24 \text{V}
\]

\[
I_1 = \frac{V_1}{24} = 1 \text{A}
\]

\[
I_0 = 1 + 2 = 3 \text{A}
\]

\[
V_s = (4 + 24/12) \times 3 = (4 + 8) \times 3 = 36 \text{V}
\]